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Theoretical Aspects of Automated Assembly of Cylindrical and Threaded Joints Using the Pneumowhirl Method

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Part II. Dynamical Behavior of the Assembly Part after Its Separation from the Intermediate Sleeve in the Mounting Head

1. Introduction

In this paperwork the researches are continued from [1]. *The main task here is to examine the motion of the assembly part (screw, trunk) since its separation from the edge point A of the intermediate sleeve* and the opportunity of self-centering and screw on in the hole of the assembly part to be shown.

2. Precession motion of the part after its separation from the intermediate sleeve

After separating the part from the intermediate sleeve, *the contact in the edge point disappear* and the reaction N_2 (i.e. $N_2 = 0$) stops acting on it. **The relation (3.11) from [1]** between the precession and angular velocity of the part **also disappear.** Then, if the fulcrum *O* remains steady and no extra constrains appear, *one of the opportunities of further motion is* **regular precession** of the part, such as the one of a heavy gyroscope. In this case **the task is:** for a given values of its own angular velocity ω_3 and angle of nutation θ, the precession angular velocity ω_1 *arround the vertical axis Oz, is to be found,* according to which this precession motion appears.

We use the formula for the gyroscopic moment according to the fixed fulcrum *O* [2]

$$\vec{L} = J_{z'}\vec{\omega}_3 \times \vec{\omega}_1 \left(1 + \frac{J_{z'} - J_{z'}}{J_z} \frac{\omega_1}{\omega_3} \cos\theta \right) \tag{2.1}$$

and kinetic-static equation

(2.2) $$\vec{M}_0^{(e)} + \vec{L} = 0,$$

describing the motion of the part. Here $J_{z'}$ is the inertia moment of the part with respect to its symmetry axis *Oz* and $J_{x'}$ is the inertia moment according to transverse axis *Ox'*. Taking into account that in the given case the main moment $\vec{M}_0^{(e)}$ is considered only the moment of the weight force $\vec{G}$, i.e. $\vec{M}_0^{(e)} = \overrightarrow{OC} \times \vec{G}$ or $M_0^{(e)} = Gl\sin\theta$, where $|OC| = l$, after the projection of (2.2) on the line of nodes we obtain

$$-Gl\sin\theta + J_{z'}\omega_3\omega_1\sin\theta(1 + \frac{J_{z'} - J_{x'}}{J_{z'}}\frac{\omega_1}{\omega_3}\cos\theta) = 0.$$

After reduction of the common multiplier $\sin\theta$, this equation transforms into

(2.3) $$(J_{z'} - J_{x'})\omega_1^2\cos\theta + J_{z'}\omega_3\omega_1 - Gl = 0,$$

from here for a given ω_3 and θ we find two values of the precession angular velocity ω_1, i.e.

(2.4) $$\omega_1^{(1,2)} = \frac{-J_{z'}\,\omega_3 \pm \sqrt{J_{z'}^2\omega_3^2 + 4Gl(J_{z'} - J_{x'})\cos\theta}}{2(J_{z'} - J_{x'})\cos\theta}.$$

Regular precession is possible if

(2.5) $$J_{z'}^2\omega_3^2 + 4Gl(J_{z'} - J_{x'})\cos\theta > 0.$$

If its own angular velocity ω_3 is significant, we expand the square root into series

$$\sqrt{J_{z'}^2\omega_3^2 + 4Gl(J_{z'} - J_{x'})\cos\theta} = J_{z'}\,\omega_3(1 + \frac{2Gl(J_{z'} - J_{x'})\cos\theta}{J_{z'}^2.\omega_3^2} + \ldots).$$

We can stop after the second addend. Then, taking the upper sign in front of the radical in (2.4), we get

(2.6) $$\omega_1^{(1)} = \frac{Gl}{J_{z'}\,\omega_3}.$$

This angular velocity corresponds to the so called *slow precession of the part.*

Taking the lower sing in front of the radical into (2.4.), we obtain

(2.7) $$\omega_1^{(2)} = \frac{J_{z'}\omega_3}{(J_{z'} - J_{x'})\cos\theta}.$$

This angular velocity corresponds to *fast precession of the part.*

3. Influence of the friction on the motion of the part

In fact the steady plane, which the part touches, is not absolutely smooth, therefore a friction force $\vec{T}$ occurs. In this case, on the part not only the weight $\vec{G}$ will act, but also the normal reaction $\vec{N}$ and friction force $\vec{T}$, applied in the fulcrum *O*.

We apply the mass center motion theorem

(3.1) $$M\vec{a}_{C*} = \vec{G} + \vec{N} + \vec{T},$$
where M is the mass of the assembly part (screw) and

(3.2) $$\vec{a}_{C*} = \vec{\varepsilon} \times \overrightarrow{OC}* + \omega_1^2 \overrightarrow{C^* C_1^*}$$

is the acceleration of its weight C. Here $\vec{\omega}_1$ is the precession angular velocity , directed along the vertical axis Oz; C_1^* is the orthogonal projection of the weight center C on Oz axis and $\vec{\varepsilon} = \vec{\omega}_1 \times \vec{\omega}_3$ with magnitude $\varepsilon = \omega_1 \omega_3 \sin\theta$ is the angular acceleration of the part, where $\vec{\omega}_3$ is its own angular velocity directed along Oz' axis. It is obvious that the angular acceleration $\vec{\varepsilon}$ is directed along the line of the nodes ON (Fig.1) so the centripetal acceleration $\vec{a}_{C*}^{(c)} = \omega_1^2 \overrightarrow{CC_1}$ lays in the plane Ozz' and its normally directed to the axis Oz'.

Considering the equality (3.2), we project equation (3.1) over the vertical axis Oz and the horizontal axis ON', perpendicular to the line of the nodes ON placed in the plane Ozz'. It is obvious that all vector addend in (3.1), including the friction force $\vec{T}$, lay in this plane. We will differ two boundary states of a boundary kinetic-static equilibrium:

a) when the angular velocities ω_1 and ω_3 are such as the fulcrum O of the part will slip into the hole of the assembly part (nut);

b) when the angular velocities ω_1 and ω_3 are such as the fulcrum O of the part will slip out of the hole of the assembly part.

In the first case the friction force T is pointed out and in the second is pointed in to the hole.

Case a. As we project the equation (3.1) on the axes Oz and ON', we obtain the scalar equations:

(3.3) $$\begin{aligned} -Ml\omega_1\omega_3 \sin^2\theta &= N - G, \\ Ml\omega_1\omega_3 \sin\theta\cos\theta - Ml\omega_1^2 \sin\theta &= -T. \end{aligned}$$

We find

(3.4) $$\begin{aligned} N &= G - Ml\omega_1\omega_3 \sin^2\theta, \\ T &= Ml\omega_1 \sin\theta(\omega_1 - \omega_3\cos\theta). \end{aligned}$$

To prevent slip of the part in the fulcrum O along the axis Oz, the friction force T should satisfy the Colon’s law, i.e.

$$T \le \mu N \text{ or } Ml\omega_1 \sin\theta(\omega_1 - \omega_3\cos\theta) \le \mu(G - Ml\omega_1\omega_3 \sin^2\theta),$$

where μ is the coefficient of friction. After a transformation we obtain the following equation:

(3.5) $$\omega_1^2 \sin\theta - \omega_1\omega_3 \sin\theta(\cos\theta - \mu\sin\theta) - \mu\frac{g}{l} \le 0.$$

If its own angular velocity ω_3 is given we can determine ω_1. We find the solutions of the following quadratic equation:

(3.6) $$\omega_1^{(1,2)} = \frac{\omega_3}{2}(\cos\theta - \mu\sin\theta) \pm \sqrt{\frac{\omega_1^2}{4}(\cos\theta - \mu\sin\theta)^2 + \mu\frac{g}{l\sin\theta}},$$

where the smaller solution $\omega_1^{(1)}$ is obtained when the sign in front of the radical I minus and the received value is negative and the bigger solution $\omega_1^{(2)}$ is obtained when the sign in front of the radical is plus and the result is a positive value.

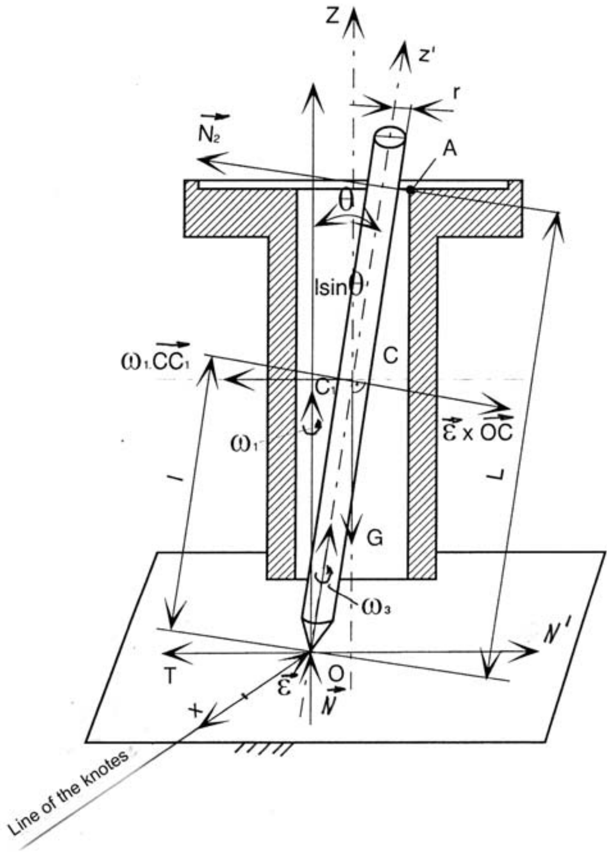

Fig. 1. Study of the piece motion on the fulcrum (supporting) plane (surface)

Factorizing the quadratic trinomial into a simple multipliers, the inequality (3.5) becomes

(3.7) $$\sin\theta(\omega_1 - \omega_1^{(1)})(\omega_1 - \omega_1^{(2)}) \le 0.$$

Here $\sin\theta > 0$ and $\omega_1 - \omega_1^{(1)} > 0$, because in a physical way $\omega_1 > 0$. Thus, for the inequality (3.7) to be correct, respectively the inequality (3.5), it is necessary

(3.8) $$\omega_1 \le \omega_1^{(2)},$$

where $\omega_1^{(2)}$ is the positive solution of (3.6).

Vice verse, if the precession angular velocity ω_1 is given, we find that that the own angular velocity ω_3 is

(3.9) $$\omega_3 \geq \frac{\omega_1^2 \sin\theta - \mu \frac{g}{l}}{\omega_1 \sin\theta(\cos\theta - \mu \sin\theta)}.$$

Case b. In this case the friction force T is directed inwards and after projection of (3.1) on *Oz* and *ON'*, we obtain:

(3.10) $$\begin{gathered} -Ml\omega_1\omega_3 \sin^2\theta = N - G, \\ Ml\omega_1\omega_3 \sin\theta\cos\theta - Ml\omega_1^2 \sin\theta = T. \end{gathered}$$

Therefore we determine *T* and *N*. We replaced them in the Colon's law $T \leq \mu N$, ensuring to prevent a slip in the fulcrum *O* inside out. We have the inequality

(3.11) $$\omega_1^2 \sin\theta - \omega_1\omega_3 \sin\theta(\cos\theta - \mu\sin\theta) + \mu\frac{g}{l} \geq 0.$$

Here the solutions of the quadratic equation are

(3.12) $$\omega_1^{(1,2)*} = \frac{\omega_3}{2}(\cos\theta + \mu\sin\theta) \pm \sqrt{\frac{\omega_3^2}{4}(\cos\theta + \mu\sin\theta)^2 - \mu\frac{g}{l\sin\theta}}.$$

They are positive as the smaller one is denoted with $\omega_1^{(1)*}$, and the bigger one with $\omega_1^{(2)*}$.

Through factorization of the quadratic trinomial we find that to satisfy the inequality (3.11) we need $\omega_1 \leq \omega_1^{(1)*}$ or $\omega_1 \geq \omega_1^{(2)*}$. Under technological circumstances we can realize the second case, i.e.

(3.13) $$\omega_1 \geq \omega_1^{(2)*}.$$

On the other hand, if the translational angular velocity ω_1 is given, for ω_3 we find

(3.14) $$\omega_3 \geq \frac{\omega_1^2 \sin\theta + \mu \frac{g}{l}}{\omega_1 \sin\theta(\cos\theta + \mu \sin\theta)}.$$

So based on (3.8) and (3.13) we conclude: *the fulcrum O of the part to stay fixed, it is necessary the values of the precession angular velocity* ω_1 *to be in the interval*

(3.15) $$\omega_1^{(2)*} \leq \omega_1 \leq \omega_1^{(2)},$$

or based on (3.9) and (3.14) the own angular velocity ω_3 to be within the interval

(3.16) $$\frac{\omega_1^2 \sin\theta - \mu \frac{g}{l}}{\omega_1 \sin\theta(\cos\theta - \mu \sin\theta)} \leq \omega_3 \leq \frac{\omega_1^2 \sin\theta + \mu \frac{g}{l}}{\omega_1 \sin\theta(\cos\theta + \mu \sin\theta)}.$$

Of great importance is the case when $\omega_1 \geq \omega_1^{(2)}$ or ω_3 is less than the left part in (3.16). Then *the fulcrum O of the part will slide on the horizontal plane towards the axis Oz and due to the existence of chamfers a self center will occur, while the axis of the part will try to reach a vertical position. Due to the gained angular velocity of rotation the assembly process will begin.* When we have high angular velocities of the pneumowhirl stream these conditions can be reached very quickly with a great hustle.

4. Movement of the particle along the support plane like a whipping-top

After separation of the particle from the edged point *A* such a condition can be made so that it will make a movement alike a heavy whipping-top. The osculating point is moving along the horizontal plane. Because the assembly part is symmetrical and its inertia ellipsoid according to its mass center is rotational, *the task is brought to the examination of motion of the particle in the field of the gravity(weight) force according to the assumption that one of its points, lying on the axis of dynamic symmetry is moving on the horizontal plane.*

We assume, that the part is with a very sharp end in point *D* from the axis of symmetry, which stays in static horizontal plane all the time. We also will consider this plane to be *absolutely smooth.*

Than the interaction with the part is brought to a vertical reaction $\vec{N}$. Because the active force is the force of weight therefore it also is vertical. Based on the theorem for motion of the mass center we conclude that the rotation of the mass center *C* on the horizontal plane is moving uniformly and rectilinearly or it is steady. Without limitation of the whole we will consider it to be steady. Then the mass center will move only on the given vertical axis.

Let us choose a steady coordinate system *Oxyz*. The axis *Oz* is vertical and passes through the mass center *C* of the part. The plane *Oxy* coincides with the horizontal plane on which the part osculate with its edge *D* (Fig. 2). The orientation of the part according to the fixed coordinate system will be determined by the three Euler's angles ψ, θ and φ.

Let *m* be the mass of the part, *l* –the distance between the mass center *C* and the contact point *D*, $J_{z'}$ – the inertia moment according to the axis of dynamic symmetry $C_{z'}$, $J_{x'}^{(C)}$ and $J_{y'}^{(C)}$ ($J_{x'}^{(C)} = J_{y'}^{(C)}$) are the inertia moments of the part according to any two connected to it perpendicular axes $C_{x'}$ and $C_{y'}$, also perpendicular to the axis $C_{z'}$. For the distance *h*, from the mass center to the support plane, the valid equation is $h=l\cos\theta$.

Because $J_{x'}^{(C)} = J_{y'}^{(C)}$ and the moment of the external forces (the reaction of the support plane $\vec{N}$ and the force of weight $\vec{G}$) according to the axis $C_{z'}$ is equal to zero, from the third equation of the Euler's dynamic equation system

(4.1) $$J_{z'}^{(C)}\dot{\omega}_{z'} + (J_{y'}^{(C)} - J_{x'}^{(C)})\omega_{y'}\omega_{z'} = M_z^{(E)}$$

it follows that $\dot{\omega}_{z'} = 0$, i.e. we have the integral

(4.2) $$\omega_{z'} = r_0 = \text{const.}$$

Thus the projection of the angular velocity $\vec{\omega}$ of the part according to its axis of dynamic symmetry remains constant.

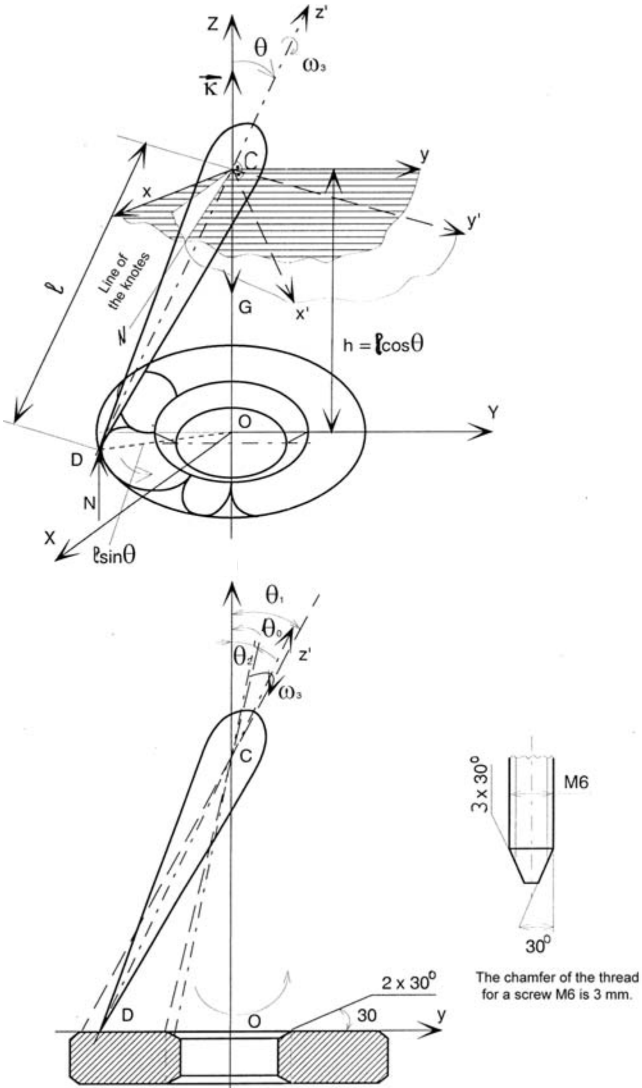

Fig. 2. Motion of the piece as spinning tap (sleeping tap)

As the external forces are pointed vertically, they don’t create a moment toward the vertical axis *Cz*. Therefore from the theorem for the change of kinematics moment follows that the projection of the kinetic moment of the part toward the vertical axis *Cz* remains constant, i.e.

(4.3) $$K_{C_z} = J_{x'}\omega_{x'}a_{31} + J_{y'}\omega_{y'}a_{32} + J_{z'}\omega_{z'}a_{33} = C\ ,$$

where $J_{x'} = J_{y'}$, and C is integrating constant.

Here, for the shown cosines a_{31}, a_{32} and a_{33} of the single vector $\vec{k}$ on the vertical axis *Cz*, we have

(4.4) $$a_{31} = \sin\theta.\sin\varphi\ ,\ a_{32} = \sin\theta.\cos\varphi\ ,\ a_{33} = \cos\theta\ ,$$

and for the projections of the angular velocity $\vec{\omega}$ of the part according to the axis *Cx'y'z'* the Euler's kinematics equations take place

(4.5) $$\begin{aligned} \omega_{x'} &= \dot{\psi}\sin\theta\sin\phi + \dot{\theta}\cos\phi, \\ \omega_{y'} &= \dot{\psi}\sin\theta\cos\phi - \dot{\theta}\sin\phi, \\ \omega_{z'} &= \dot{\psi}\cos\theta + \dot{\phi} = r_0\ . \end{aligned}$$

Here ψ, θ and φ are Euler's angles and the integral (4.2) is considered. Using (4.4) and (4.5), the equality (4.3) is written as

(4.6) $$J_{x'}\dot{\phi}\sin^2\theta + J_{z'}r_0\cos\theta = C\ .$$

On the other side, as the connection $h=l.\cos\theta$ is ideal and stationary and the active forces have potential $\Pi=Mgh$, the integral of power occurs

(4.7) $$T+\Pi=h^*,$$

where h^* is an integrated constant. Here T is the kinetic energy of the part, for which based on the Koenig's theorem we have

(4.8) $$T = \frac{1}{2}Mv_C^2 + \frac{1}{2}J_{x'}(\omega_{x'}^2 + \omega_{y'}^2) + \frac{1}{2}J_{z'}\omega_{z'}^2,$$

where $v_C = \dot{h} = -l\dot{\theta}\sin\theta$ is the mass center velocity. As we use the kinematic Euler's equations (4.5), the integral of energy (4.7) is written as

(4.9) $$(J_{x'} + Ml^2\sin^2\theta).\dot{\theta}^2 + J_{x'}\sin^2\theta.\dot{\psi}^2 + 2Mgl\cos\theta = H,$$

where $H = 2h^* - J_{z'}r_0^2 = \text{const.}$

The integrals (4.2), (4.6) and (4.9) allows the full solution of the problem. We are going to observe the special case when the part is rotated around its axis of symmetry and is placed on the horizontal plane without initial velocity of its mass center *C*. Its axis is inclined according to the vertical of the angle θ_0. Therefore the initial conditions for motion when $t = 0$ are

(4.10) $$\dot{\psi} = 0\ ,\ \dot{\theta} = 0\ ,\ \theta = \theta_0\ ,\ \dot{\phi} = r_0\ ,$$

as it is assumed that the projection of the velocity $\vec{v}_C$ on the horizontal plane is also equal to zero.

After these initial conditions we can write the integrals (4.6) and (4.9) as follows:

(4.11) $$J_{x'}\dot{\psi}\sin^2\theta = J_{z'}r_0(\cos\theta_0 - \cos\theta),$$

(4.12) $$(J_{x'} + Ml^2\sin^2\theta)\dot{\theta}^2 + J_{x'}\sin^2\theta\dot{\psi}^2 = 2Mgl(\cos\theta_0 - \cos\theta).$$

Defining $\dot{\psi}$ from (4.11) and replacing the expression in (4.12) we have the equation

(4.13) $$J_{x'}\sin^2\theta(J_{x'}+Ml^2\sin^2\theta)\dot{\theta}^2 = f(\theta),$$
where

(4.14) $$f(\theta)=(\cos\theta_0-\cos\theta)[2J_{x'}+Ml^2\sin^2\theta-J_{z'}^2r_0^2(\cos\theta_0-\cos\theta)],$$

in which only the parameter θ takes place.

We will consider this equation. The left side of the equation (4.13) is not negative. Therefore the angle θ can be in that range of values for which $f(\theta)\geq 0$. It follows that $\theta\geq\theta_0$, because when $\theta\leq\theta_0$, the function $f(\theta)$ from (4.14) will be a product of two multipliers with opposite signs. The angle θ will alter in the interval defined by θ_0 and the value θ_1, which is the nearest root to θ_0 of the equation $f(\theta)$=0. It is clear that $\theta_1 \leq\pi$ because $f(\pi)=-(1+\cos\theta)^2 J_{z'}^2 r_0^2 < 0$. Consequently during motion of the part, the angle θ should remain in the interval

(4.15) $$\theta_0\leq\theta\leq\theta_1\leq\pi,$$

and the length of the segment *OD* (Fig. 2) will change in the interval

(4.16) $$l\sin\theta_0 \leq OD\leq l\sin\theta_1.$$

This means that the trajectory of point *D* over the support plane will be between two concentric circles with radiuses $l\sin\theta$ and $l\sin\theta_1$, with center point *O* (Fig. 2).

From (4.11) follows that when θ takes its initial value θ_0 during motion then $\dot{\varphi}=0$ .Therefore the trajectory of point *D* will have on its inner circle with radius $l\sin\theta_0$ point of reversion (Fig. 2).

If the initial velocity of the part r_0 is very big , the angle θ will slightly differ fro its initial value θ_0 . Actually, if the expression from the middle brackets in (4.14) is equal to zero, we will have for angle θ_1 (with accuracy in order to $\frac{1}{r_0}$)

$$\theta_1=\theta_0+\frac{2J_{x'}Mgl\sin\theta_0}{J_{z'}^2r_0^2}.$$

From here it follows that θ_1, and also θ are close enough to θ_0, if the value of r_0 is big enough.

The analyses made on the motion of the part shows that when point *D* reach the inner circle with radius $l\sin\theta_0$ and it coincides with the outer circle of the chamfer of the assembly part (nut) a sliding of the part to the hole of the nut occurs. A contact among the threads of the nut and the screw exists and because of the high angular velocity the fit is made. The experience shows that when we have an automatic assembly of such junctions, to reach a better interaction between the parts and higher efficiency it is advisable ,for example, the trunk to be with diameter 6 mm, the chamfer of the nut to be 2×30° and the screw – 3×30°.

5. Friction influence over the motion of the part

Till now we assume that the supporting plane, which the part is touching is absolutely flat. Actually it is real and causes forces of friction. Besides that the part don't end with a sharp apex (point) but with some rotational surface, more or less sharp-pointed so the contact point *L* along with the supporting plane don't lie on the axis of symmetry (Fig. 3). Because of that, the motion of the part will slightly differ from the previously described.

One of the most interesting *effects, caused by the friction force in the contact point L, is that the force tries to approximate the axis of symmetry of the part to the vertical axis, i.e. to set it up straight.* We will consider this effect from its qualitative side, using the theorem for changing the kinematic moment.

Let the part rotates very fast around its own axis of symmetry and without initial velocity of its mass center. It is placed on the supporting plane such as the axis of symmetry and the vertical axis make a small acute angle θ_0 (Fig. 3).

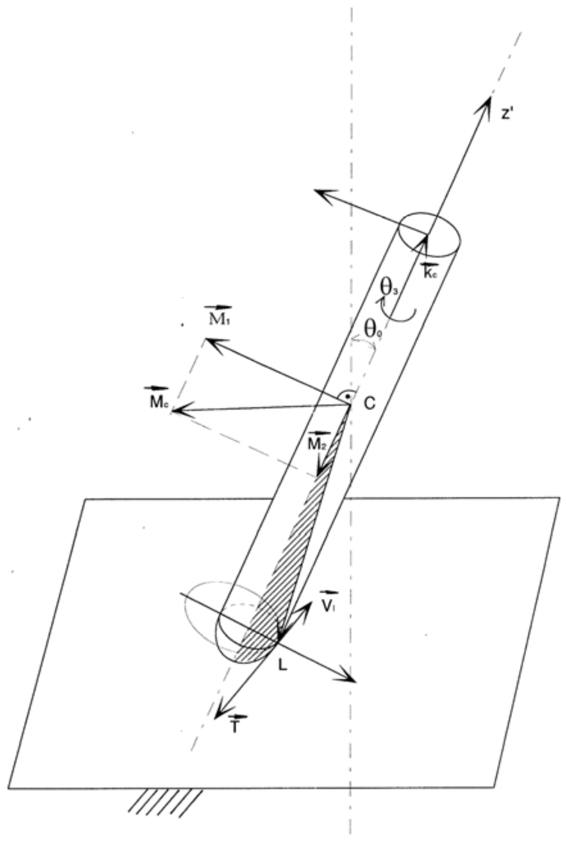

Fig. 3. Influence of the roundness of the piece in the contact zone over its motion

The kinematic moment $\vec{K}_C$ of the part according to its mass center C in the initial moment is pointed towards the axis $C_{z'}$, as it is shown on Fig. 3. Let *L* be the contact point of the part with the supporting plane, which now is considered not to be sharp-pointed. In that point a force of friction $\vec{T}$ occurs, which is pointed opposite to the velocity. The moment $\vec{M}_C$ of the friction force $\vec{T}$ according to the inertia mass center *C* is directed perpendicularly to the plane, which is defined by the center *C* and the vector $\vec{T}$. The moment $\vec{M}_C$ can be expanded into two components $\vec{M}_1$ and $\vec{M}_2$, i.e. $\vec{M}_C = \vec{M}_1 + \vec{M}_2$, where $\vec{M}_1$ is a vector perpendicular to $\vec{K}_C$, and $\vec{M}_2$ is a vector collinear $\vec{K}_C$ but in an opposite direction. Based on the Rezal's theorem the velocity of the kinematic moment $\vec{K}_C$ at its point is equal to $\vec{M}_C$. Therefore the vector $\vec{K}_C$, decreasing by its magnitude because of the vector components $\vec{M}_1$ and $\vec{M}_2$, tries to reach a vertical position because of the component $\vec{M}_1$. In such a way the vector $\vec{K}_C$ and also the axis of symmetry of the part under the influence of the force of friction will aim at the vertical axis. If the friction force is acting long enough the axis of the part will take a vertical position and remain steady. In this case the part is said to be asleep. The examined *effect, caused by the force of friction also contributes for the better positioning and compatibility of the assembly parts.*

6. Pseudo regulating precession of parts

Let assume that after separation of the part from the edge point *A* it has gained a great angular velocity ω_3 of its own rotation and the fulcrum *O* with the horizontal plane remains steady, i.e. there is no slip. In this case the Euler's angles can be expressed in time function through simple functions.

It is known that Euler's dynamic equations contains three first integrals. They are as follows: the integral for preserving the mechanic energy, the integral for preserving the kinematic moment according to the vertical axis *Oz* and the integral for preserving the own angular velocity $\omega_{z'} = \omega_3$ towards the axis of symmetry *Oz'*. According to the movable coordinate system *Ox'y'z'*, with a beginning at the fulcrum *O* and fixed to the part, it follows:

(6.1) $$J_{z'}(\dot{\psi}^2 \sin^2\theta + \dot{\theta}^2) + J_{z'} r_0^2 = -2Mgl\cos\theta + 2h^*,$$

(6.2) $$J_{x'}\dot{\psi}\sin^2\theta + J_{z'} r_0 \cos\theta = C,$$

(6.3) $$\dot{\psi}\cos\theta + \dot{\varphi} = r_0\,,$$

where h^*, *C* and r_0 are integrating constants. For simplicity and better clearness we will consider the following initial conditions for motion: when $t = 0$, let $\omega_{x'} = \omega_{y'} = 0$, $\omega_{z'} = \omega_3 = r_0$, $\theta = \theta_0$, $\psi_0 = 0$, $\varphi_0 = 0$. From $\omega_{x'} = \omega_{y'} = 0$ it also follows that $\dot{\varphi}_0 = 0$, $\dot{\theta}_0 = 0$. It means that in the initial moment the part has gained an angular velocity ω_3 around its axis of symmetry, inclined at an angle θ_0 according to the vertical axis.

Based on this initial conditions for the integrating constants we find

$$2h^* = 2Mgl\cos\theta_0 - J_z r_0^2\ ,$$

(6.4) $$C = J_z r_0 \cos\theta_0\ ,$$

$$r_0 = \dot\varphi_0 = \omega_3\ ,$$

where, because the assumption $\dot\varphi_0 = \omega_3$ is great so r_0 is quite big at its absolute value.

Using (6.4), the equations (6.1)-(6.3) become

(6.5) $$\dot\psi^2 \sin^2\theta + \dot\theta^2 = \frac{2Mgl}{J_{x'}}(\cos\theta_0 - \cos\theta),$$

(6.6) $$\dot\psi \sin^2\theta = \frac{J_{z'} r_0}{J_{x'}}(\cos\theta_0 - \cos\theta),$$

(6.7) $$\dot\psi\cos\theta + \dot\varphi = r_0\,.$$

If from the first two equations (6.5) and (6.6) remove $\dot\psi$, we have

(6.8) $$\sin^2\theta.\dot\theta^2 = (\cos\theta_0 - \cos\theta)\left[\frac{2Mgl}{J_{x'}}(1-\cos^2\theta) - \frac{J_{z'}^2 r_0^r}{J_{x'}^2}(\cos\theta_0 - \cos\theta)\right].$$

We replace cosθ = u and the equation turns to

(6.9) $$\left(\frac{du}{dt}\right)^2 = (u_0 - u)\left[\frac{2Mgl}{J_{x'}}(1-u^2) - \frac{J_{z'}^2 r_0^2}{J_{x'}^2}(u_0 - u)\right],$$

where cos θ = u_0. It is a deferential equation from first order with respect to u, from where we determine the alteration law for θ. Therefore the equation (6.9) to have a real meaning the function $f(u)$ from its right side, which is a polynomial from third order with respect to u, should be positive at some interval of values between −1 and +1. It is known that the equation $f(u)$=0 has three real roots, one of which is $u_0 = \cos\theta_0$ determined by the initial condition $\theta = \theta_0$.The other two are the roots of the quadratic equation which we have after nullify the expression in the middle bracts in (6.9) and (6.8). It is known that, the equation $f(u)$=0 has three real roots. The one of them is $u_0 = \cos\theta_0$, which is determined from initial conditions $\theta = \theta_0$. The other two roots are the roots of the quadratic equation, which is obtained, as we make equal to zero the expression in the middle parentheses of the equation (6.9) or (6.8). We write it as follows

(6.10) $$\cos^2\theta - 2\lambda\cos\theta + 2\lambda\cos\theta_0 - 1 = 0,$$

where we replace $2\lambda = \dfrac{J_{z'}^2 r_0^2}{2MgJ_{x'}}$ and for the roots find

(6.11) $$u_{1,2} = \cos\theta_{1,2} = \lambda \pm \sqrt{1 - 2\lambda\cos\theta_0 + \lambda^2}\,.$$

Since cosθ_0 < 1 and $1 - 2\lambda\cos\theta_0 + \lambda^2 > (1-\lambda)^2$, the root which is less than 1 is obtained when we have sign "–", i.e.

(6.12) $$u_1 = \cos\theta_1 = \lambda \pm \sqrt{1 - 2\lambda\cos\theta \ + \lambda^2}\,.$$

From (6.8) follows that $\cos\theta_1 < \cos\theta_0$ and therefore $\theta_1 > \theta_0$, since $1 - \cos^2\theta = \sin^2\theta_1 > 0$. The meaning is that θ_0 is the minimum angle and θ_1 is the maximum between which the angle of notation θ will change, i.e. $\theta_0 \le \theta \le \theta_1$. From the equality $f(u_1)=0$, we obtain

(6.13) $$\cos\theta_0 - \cos\theta_1 = \frac{2MglJ_{x'}(1-\cos^2\theta_1)}{J_{z'}^2 r_0^2}\,.$$

It is clear that if the own angular velocity $\omega_3 = r_0$ expands, the subtraction $\cos\theta_0 - \cos\theta_1$ diminish fast and θ_1 will aim to θ_0. In such way the axis of symmetry *Oz′* of the part will move in one quite confined zone defined by the cones with orifices $2\theta_0$ and $2\theta_1$ and point at the fulcrum *O*.

Because θ is changing in a very confined interval between θ_0 and θ_1 during the motion, lets replace

(6.14) $$\theta = \theta_0 + \alpha\,,$$

where $\alpha = \alpha(t)$ is function under determination. We obtain

(6.15) $$\cos\theta = \cos(\theta_0 + \alpha) = \cos\theta_0\cos\alpha - \sin\theta_0\sin\alpha = \cos\theta_0 - \alpha\sin\theta_0\,,$$

where it is taken in mind that $\cos\alpha \approx 1$, and $\sin\alpha \approx \alpha$. Based on (6.15) and considering that $\sin\theta \approx \sin\theta_0$, the basic equation (6.8) is turning into

(6.16) $$\dot{\alpha}^2 = \frac{2Mgl}{J_{x'}}\sin\theta_0.\alpha - \frac{J_{z'}^2 r_0^2}{J_{x'}^2}\alpha^2\,.$$

Replacing

(6.17) $$\frac{2MglJ_{x'}}{J_{z'}^2 r_0^2} = 2\rho\,,$$

this equation is reduced to

(6.18) $$\dot{\alpha} = \frac{Mgl\sin\theta_0}{J_{x'}}\sqrt{1 - \frac{(\rho-\alpha)^2}{\rho}}\,,$$

whence through integrating taking into account that when t = 0 it follows that α = 0, and we obtain

(6.19) $$\alpha = \frac{MglJ_{x'}\sin\theta_0}{J_{z'}^2 r_0^2}\left(1 - \cos\frac{J_{z'}r_0}{J_{x'}}t\right).$$

Considering (6.14), the angle θ will change according to the law

(6.20) $$\theta = \theta_0 + \frac{MglJ_{x'}\sin\theta_0}{J_{z'}^2 r_0^2}\left(1 - \cos\frac{J_{z'}r_0}{J_{x'}}t\right),$$

which is a periodic function with a period $T = \dfrac{2\pi J_{x'}}{J_{z'} r_0}$. This period will decrease when r_0 is increased, and the frequency of the oscillations $\dfrac{2\pi}{T}$ will increase along with r_0.

We can get the precise motion of the part from (6.6) taking into account that $\cos\theta_0 - \cos\theta \approx \alpha \sin\theta_0$ and $\sin\theta \approx \sin\theta_0$. We have

(6.21) $$\dot{\psi} = \frac{J_{z'} r_0}{J_{x'} \sin\theta_0}\alpha = \frac{Mgl}{J_{z'} r_0}\left(1 - \cos\frac{J_{z'} r_0}{J_{x'}} t\right),$$

from where, if t=0,ψ=0, we find

(6.22) $$\psi = \frac{Mgl}{J_{z'} r_0}\left(t - \frac{J_{x'}}{J_{z'} r_0}\sin\frac{J_{z'} r_0}{J_{x'}} t\right).$$

Finally from (6.3) we find α with precision to the second degree:

(6.23) $$\dot{\phi} = r_0 - \dot{\phi}\cos\theta = r_0 - \frac{J_{z'} r_0 \cos\theta_0}{J_{x'} \sin\theta_0}\alpha,$$

where (6.21) is taken into account. From this equation, after integrating with initial conditions $t = 0$ we have $\varphi_0 = 0$, as we take into account (3.18), we get

(6.24) $$\phi = r_0 t - \frac{Mgl}{J_{z'} r_0}\cos\theta\left(t - \frac{J_{x'}}{J_{z'} r_0}\sin\frac{J_{z'} r_0}{J_{x'}} t\right).$$

Following this way we found the law of the nutation (6.20), of the precision (6.22) and of the own rotation (6.24). The undetermined precision (3.19), when θ slightly differs from θ_0, is called *pseudoregular precision.* The precision and the nutation define the motion of the own axis of rotation of the part. If we neglect the periodic addends, they define the motion of this axis around the vertical, *deflected on angle* θ_0 and rotation with a very small angular velocity of the precision. The deflections of the angle θ, especially the bigger values, also helps *for the getting over the force of friction in the support, and the part to slide to the axis of the assembly part. Because of the high angular velocity it can self-center.*

7. Conclusion

In the two parts offered above are examined some dynamic problems of the automated assembly process of threaded and cylindrical joints through a pneumowhirl head with an intermediate sleeve, which puts into practice the pneumowhirl method of Rank–Hill–Levchuk [6, 7, 8]. The principle structure of the pneumowhirl head with an intermediate sleeve and its action, caused by a pneumowhirl air stream, is introduced. According to the constructed dynamic examinations four main conclusions can be made.

1. The dynamic interaction of the system *assembly part–intermediate sleeve–air pneumostream* is examined when there is a contact without sliding between the part and the sleeve. According to the law of conservation of the kinetic moment

with respect to the vertical axis, the angular velocities of the intermediate sleeve and the part are defined (3.17) and (3.18) according to the angular velocity of the stream and the geometric-mass characteristics of the sleeve and the part.

2. According to the kinetic-static equation for the motion of the part as an approximately regular precision, the angular velocities of the intermediate sleeve and the part are defined (4.7) and (4.8). Thus, the part separates from the sleeve.

3. In the second part, the motion of the part after its separation from the edge point with the intermediate sleeve is examined. Three possible cases are introduced:

- *regular precision*, for which the angular velocities for slow and high precision are defined (1.6) and (1.7);
- *motion in the support plane*, when the support point D of the part with the plane is moving in a band, closed of two concentric circles (3.16) around the hole of the assembly part.
- *pseudo regular precision*, for which the law of the motion is found (6.20), (6.22) and (6.24).

4. The influence of the friction, through the support plane, on the motion of the part is analyzed. *The interval* (*2.16*), *in which the own angular velocity* $\vec{\omega}_3$ *must change, for the support edge point O to be stable, is defined.* When that angular velocity is not in this interval, the support point O begins to move on the support plane. And when unavoidably the tip (the edge) rounds off, *the friction causes effect of straightening of the axis of symmetry of the part and helps for the better positioning and joining with the assembly part.*

The caused motions of the assembly part are objectively along with some chaotically behavior, and that is because of the possible additional contacts with the intermediate sleeve. But after all, along with the useful effects of the friction, they lead to automated behavior and assembly process of the desired joint.

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Теоретические аспекты автоматизированной сборки цилиндрических и резбовых соединений пневмовихровым методом (Часть II)

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(Р е з ю м е)

В работе рассматривается поведение собираемой детали после ей сепарирования от междинной втулки в собираемой головке. Исследованы три возможные варианта движения детали.