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Theoretical Aspects of Automated Assembly of Cylindrical and Threaded Joints Using the Pneumowhirl Method

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Part III. Vertical Oscillations of the Intermediate Sleeve in the Mounting Head

1. Introduction

The assembly process of cylindrical and threaded joints in the mounting head using the pneumowhirl method [1, 2] is accompanied by vertical oscillations caused by the air stream in the whirl tube (Fig. 1). Here the task is *to examine these oscillations by finding the laws of their alternation and to analyze their effect on the assembly process.*

When the air stream enter in the space between the whirl tube and the intermediate sleeve it separates in two flows: one is streaming downward and exert pressure on the support plain and the other is streaming upward and exerts pressure on the flange of the intermediate sleeve (Fig. 1), trying to lift it and flows out. It can be assumed that the upper part or this space is a chamber, composed of cylinder and piston with contact area

(1.1) $$A = \pi(r_1^2 - r_2^2),$$

where r_1 is the internal radius of the whirl tube and r_2 is the external radius of the intermediate sleeve.

Let's note p_0 and V_0 be the pressure and the volume of this chamber when the intermediate sleeve is in equilibrium state, i.e. when the force p_0A, caused by the pressure and the weight of the sleeve, mutually equalize each other. Also denote p_1 and V_1 as the pressure and the volume in the chamber at an instant position of the moving sleeve. Assuming that the thermodynamic process of changing the air in the space between the whirl tube and the intermediate sleeve is adiabatic, i.e.

(1.2) $$p_1V_1^{\gamma} = p_0V_0^{\gamma},$$

where $\gamma = 1.4$ is the coefficient of the adiabat in the air.

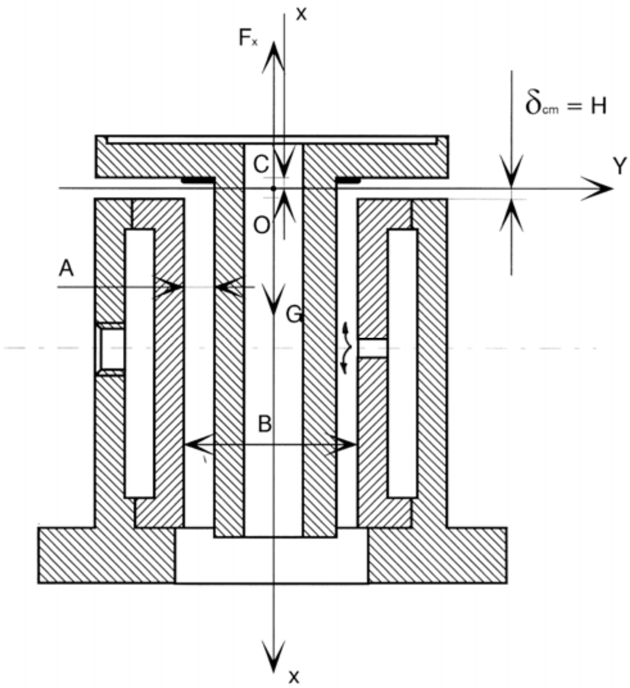

Fig. 1. Vertical vibrations of the intermediate sleeve

After differentiate equation (1.2) in respect to the shift from the equilibrium state *x*, we have

(1.3) $$\frac{dp_1}{dx}V_1^{\gamma} + p_1\gamma V_1^{\gamma-1}\frac{dV_1}{dx} = 0,$$

from where, taking into account (1.2), we find

(1.4) $$\frac{dp_1}{dx} = -\gamma p_0 V_0^{\gamma} V_1^{-(\gamma+1)}\frac{dV_1}{dx}.$$

Due to the fact the volume of this closed space is changing linearly with respect to *x*,

(1.5) $$V_1 = V_0 - Ax.$$

For the derivative $\frac{dV_1}{dx}$ we have

(1.6) $$\frac{dV_1}{dx} = -A.$$

The closed air chamber can be considered as spring (air cushion), for which elastic coefficient (defined as the ratio of the force to a shift unit) we will have

(1.7) $$c = A\frac{dp_1}{dx} = \frac{\gamma p_0 A^2}{V_0}\left(1 - \frac{Ax}{V_0}\right)^{-(\gamma+1)}.$$

The result for the coefficient c shows that, it together with the produced by it elastic force F_x, will have linear dependence with respect to the shifting of the intermediate sleeve. This force causes the vertical oscillations of the sleeve.

We will examine two cases: linear and nonlinear cases of oscillation.

2. Linear oscillations of the intermediate sleeve

Taking into account that shifting x of the sleeve is small, the expression in the brackets in (1.7) can be taken so that in first approximation is almost equal to one. Then the equation (1.7) can be approximated with the following expression

$$c = \frac{\gamma p_0 A^2}{V_0} = \frac{\gamma G A}{V_0}, \tag{2.1}$$

which is a constant. In this case G is the weight of the sleeve measured in the condition of $p_0 A = G$, expressing that in the equilibrium state the force of weight of the sleeve is counterweighted by the force caused by the pressure of the air stream.

In this case the resultant force, which exerts on the flange of the intermediate sleeve, will be

$$F_x = -cx, \tag{2.2}$$

where the coefficient of proportionality is the constant (2.1). It is caused by the compressed air, which going into the whirl tube forms an air cushion between the flange of the sleeve and the joint of the tube. In this air cushion the pressure is changing due to the flow out of the air. The resultant force is the one, which cause the oscillations of the sleeve. These oscillations will be free continuous oscillations if we not consider the resistances.

The differential equation that describes the vertical translational motion of the sleeve is

$$\ddot{x} + k^2 x = 0, \tag{2.3}$$

where

$$k = \sqrt{\frac{c}{m}} = \sqrt{\frac{\gamma G A}{m V_0}} = \sqrt{\frac{\gamma g A}{V_0}} \tag{2.4}$$

is its natural frequency. Here m is the mass of the intermediate sleeve, and $g = \frac{G}{m}$ – acceleration of the gravity.

Taking into account the initial conditions $t = 0$, $x_0 = h$ and $\dot{x}_0 = 0$, the solution of the differential equation is

$$x = h \cos \sqrt{\frac{\gamma g A}{V_0}} t, \tag{2.5}$$

which describes a pure harmonic oscillation with amplitude h and frequency k. The amplitude h specifies the height to which the flange of the intermediate sleeve can be lifted or lowered in respect to its equilibrium position (assumed as begin of the coordinate system). The amplitude ensures the adequate surrounding cylindrical area, through which the fluid flows out pulsating. The frequency k of these oscillations is defined using the formula (2.4) and depends mostly on the volume V_0 of the closed chamber in equilibrium state and the contact surface A between the flange of the sleeve and the whirl tube.

Vibrographic pictures of the intermediate sleeve's movements are taken during experiments done using this pneumowhirl method. Based on the accomplished harmonic analysis of the velocity law, it is established that basic harmonic has frequency k from 18 up to 19 s^{-1}. Calculated using formula (2.4), the theoretically established frequency is k from 20 up to 52 s^{-1}. These results show that experimental and theoretical results are almost equal.

The begin O of the coordinate system, from which we orientate the Ox axis downward, is determined by the height $\delta_{st} = H$ of the air cushion between the flange of the sleeve and the whirl tube in equilibrium state. From the equilibrium condition, it follows

$$G = c\ \delta_{st}\ ,$$

Taking into account (2.1), we find

$$\delta_{st} = H = \frac{G}{c} = \frac{V_0}{\gamma A}. \tag{2.6}$$

This is the distance from the joint of the whirl tube, where we set the begin O of the coordinate system.

3. Nonlinear oscillations of the intermediate sleeve

In practice the coefficient of elasticity (1.7) is not constant, but is changing nonlinearly with respect to the displacement x. This led to the nonlinear alteration in the elastic force aroused by the air cushion between the flange of the sleeve and the whirl tube. The differential equation, which describes the movement of the sleeve, is nonlinear and is

$$m\ddot{x} = -\frac{\gamma p_0 A^2}{V_0}\left(1 - \frac{A}{V_0}x\right)^{-(\gamma+1)} x. \tag{3.1}$$

Obviously this equation is independent and nonlinear. In most cases $\frac{A}{V_0} < 1$, so the coefficient c in front of x can be developed as binomial theorem in infinite series for the powers of x. We have

$$c = \frac{\gamma p_0 A^2}{V_0}(1 - \frac{A}{V_0}x)^{-(\gamma+1)} = \frac{\gamma p_0 A^2}{V_0}[1 + \frac{(\gamma+1)A}{V_0}x + \frac{(\gamma+1)(\gamma+2)A^2}{2!V_0^2}x^2 + \dots]. \tag{3.2}$$

We enter as small parameter the ratio $\mu = \frac{A^2}{B^2} < 1$, where B is the cross-sectional surface of the whirl tube and replacing (3.2) in (3.1), we rewrite the equation (3.1) in the following quasi-linear form

$$\ddot{x} + k^2 x = -\mu\alpha x^2 - \mu^2 \beta x^3 - \dots, \tag{3.3}$$

where the right side is brought under the small parameter μ. Here k is the frequency of the originating solution and it takes the value (2.4), and about the coefficients α, β, ... we have

$$\alpha = \frac{\gamma(\gamma+1)gB^2}{V_0^2}, \quad \beta = \frac{\gamma(\gamma+1)(\gamma+2)gB^4}{2V_0^3 A}, \ldots \tag{3.4}$$

The further problem is to find the periodic solution of (3.3) at least foe the second power of the small parameter μ and the relation between the aroused free oscillations and their amplitude. The Liapunov–Lindsted–A. N. Krilov method is suitable for the solution of (3.3).

The equation (3.3) satisfies the conditions of the Poincaré theorem for the existence of periodical solution for independent systems. In conformity with the adopted method we will find $x(t)$ and the unknown frequency p, taking them together into account and at the same time expanded with respect to the exponents of the small parameter μ:

$$x(t) = \varphi_0 + \mu\varphi_1 + \mu^2\varphi_2 + \ldots, \tag{3.5}$$

$$p^2 = k^2 + \mu h_1 + \mu^2 h_2 + \ldots, \tag{3.6}$$

where the functions $\varphi_0(t), \varphi_1(t), \varphi_2(t), \ldots$ will be defined as periodic functions with equal period so the (3.5) will be a periodic solution of the differential equation (3.3). The position (3.6) of the square of the frequency, which we are looking for, will be used in order to the conditions of periodicity be fulfilled. The solution will be developed under the initial conditions:

$$t = 0, \ x(0) = H, \ \dot{x}(0) = 0. \tag{3.7}$$

To find the needed periodic solution of (3.3) with accuracy to the second power of μ inclusive, we replace (3.5) and (3.6) in it and after equalize the coefficients of the equal exponents of μ in the both side of the equation we get

$$\ddot{\varphi}_0 + p^2\varphi_0 = 0, \tag{3.8}$$

$$\ddot{\varphi}_1 + p^2\varphi_1 = h_1\varphi_0 - \alpha\varphi_0^2, \tag{3.9}$$

$$\ddot{\varphi}_2 + p^2\varphi_2 = h_1\varphi_1 + h_2\varphi_0 - 2\alpha\varphi_0\varphi_1 - \beta\varphi_0^3, \tag{3.10}$$

…

which leads that we have to define the functions $\varphi_0(t), \varphi_1(t), \varphi_2(t), \ldots$ and the constants $h_1, h_2, \ldots$ consecutively.

The initial conditions (3.7) about $x(t)$ will be satisfied if the functions $\varphi_0(t), \varphi_1(t), \varphi_2(t), \ldots$ will be defined so that they fulfilled the initial conditions

$$\begin{aligned} &\varphi_0(0) = H, \ \dot{\varphi}_0(0) = 0, \\ &\varphi_1(0) = 0, \ \dot{\varphi}_1(0) = 0, \\ &\varphi_2(0) = 0, \ \dot{\varphi}_2(0) = 0, \\ &\ldots \end{aligned} \tag{3.11}$$

From the first (*cause*) equation (3.8) we find

$$\varphi_0(t) = H \cos pt. \tag{3.12}$$

We replace this solution in the second equation (3.9), which with the help of the formula

$$\cos^2 pt = \frac{1}{2}(1+\cos 2pt)$$

we present in the following form:

(3.13) $$\ddot{\varphi}_1 + p^2\varphi_1 = -\frac{\alpha H^2}{2} + h_1 H \cos pt - \frac{\alpha H^2}{2}\cos 2pt .$$

To avoid secular term with t cos pt with a multiplier, we choose h_1 such that the coefficient in front of cospt in the ride side of (3.13) to be equal to zero. Then we get

(3.14) $$h_1 = 0.$$

After this solution of the equation (3.13) it will has the following form

$$\varphi_1 = M_1 \cos pt + N_1 \sin pt - \frac{\alpha H^2}{2p^2} + \frac{\alpha H^2}{6p^2}\cos 2pt.$$

With zero initial conditions according to (3.11) for the constants M_1 and N_1 we find

$$M_1 = -\frac{\alpha H^2}{3p^2}, \; N_1 = 0$$

and then

(3.15) $$\varphi_1 = -\frac{\alpha H^2}{2p^2} - \frac{\alpha H^2}{3p^2}\cos pt + \frac{\alpha H^2}{6p^2}\cos 2pt .$$

In this manner the solution (3.5) *in first approximation* will be

(3.16) $$x(t) = H \cos pt + \frac{\mu\alpha H^2}{6p^2}(\cos 2pt - 2\cos pt - 3),$$

where

(3.17) $$p^2 = k^2 \quad (p = k).$$

To find *the second approximation*, we replace the obtained expressions for φ_0 and φ_1 from (3.12) and (3.15) in (3.10). Using the formulas

$$\cos pt \cos 2pt = \frac{1}{2}(\cos pt + \cos 3pt),$$

$$\cos^3 pt = \frac{1}{4}(\cos 3pt + \cos pt)$$

and after some transformations we get

(3.18) $$\ddot{\varphi}_2 + p^2\varphi_2 = \left(h_2 H + \frac{5\alpha H^3}{6p^2} - \frac{\beta H^3}{4}\right)\cos pt + \frac{\alpha H^3}{3p^2} + \\ + \frac{\alpha H^3}{3p^2}\cos 2pt - \left(\frac{\alpha H^3}{6p^2} + \frac{\beta H^3}{4}\right)\cos 3pt.$$

To eliminate the secular term, we substitute the coefficient in front of cospt with zero, from where we find

(3.19) $$h_2 = \frac{\beta H^2}{4} - \frac{5\alpha H^2}{6p^2}.$$

After this, from equation

(3.20) $$\ddot{\phi}_2 + p^2\phi_2 = \frac{\alpha H^3}{3p^2} + \frac{\alpha H^3}{3p^2}\cos 2pt - \left(\frac{\alpha H^3}{6p^2} + \frac{\beta H^3}{4}\right)\cos 3pt$$

we find that

(3.21) $$\varphi_2 = M_2 \cos pt + N_2 \sin pt + \frac{\alpha H^3}{3p^4} - \frac{\alpha H^3}{9p^4}\cos 2pt + \left(\frac{\alpha H^3}{48p^4} + \frac{\beta H^3}{32p^2}\right)\cos 3pt\ .$$

With respect to the zero initial conditions (3.11) for M_2 and N_2 we obtain

(3.22) $$M_2 = -\left(\frac{35\alpha H^3}{144p^4} + \frac{\beta H^3}{32p^2}\right),\ \ N_2 = 0.$$

From where we find that

(3.23)
$$\varphi_2 = \frac{\alpha H^3}{3p^4} - \left(\frac{35\alpha H^3}{144p^4} + \frac{\beta H^3}{32p^2}\right)\cos pt - \frac{\alpha H^3}{9p^4}\cos 2pt + \left(\frac{\alpha H^3}{48p^4} + \frac{\beta H^3}{32p^2}\right)\cos 3pt.$$

The final solution of the equation (3.3) in second approximation will be

(3.24)
$$x = H\cos pt + \frac{\mu\alpha H^2}{6p^2}(\cos 2pt - 2\cos pt - 3) +$$
$$+\frac{\mu^2 H^3}{3p^2}\left[\frac{\alpha}{p^2} - \left(\frac{35\alpha}{48p^2} + \frac{3\beta}{32}\right)\cos pt - \frac{\alpha}{3p^2}\cos 2pt + \left(\frac{\alpha}{24p^2} + \frac{3\beta}{32}\right)\cos 3pt\right],$$

where

(3.25) $$p^2 = k^2 + \left(\frac{\beta}{4} - \frac{5\alpha}{6p^2}\right)\mu^2 H^2\ .$$

To calculate p^2 we have to solve equation (3.25). The needed accuracy for p^2 is to with respect to the second power of μ. For this reason the right side of the equation must be substitute with $p^2 \approx k^2$ so we find

(3.26) $$p^2 = k^2 + \left(\frac{\beta}{4} - \frac{5\alpha}{6k^2}\right)\mu^2 H^2\ .$$

In this manner the expression (3.24) gives us the oscillations principle of the intermediate sleeve in vertical direction with precision to the second power of the small parameter μ. It shows that oscillation of the sleeve is sum of harmonics, which frequencies are divisible of first, second and third order of the frequency of the main harmonic. The relation of (3.25) is defined. It shows the dependence of the frequency on the amplitude of the main harmonic. From statement (3.24) using the non-harmonic addends in the right side it is obvious that there is a displacement of the static equilibrium position around which the sleeve does it resultant oscillation.

4. Conclusion

In the propounded research work is discussed the problem of vertical oscillations of the intermediate sleeve in the mounting head, which aroused during the assembly of cylindrical or threaded joints under the influence of pneumatic stream. The physical basis of the oscillations appearance is shrinking of the air encapsulated in the space between the whirl tube and the sleeve with its flange. Examining the process of its shrinking and expanding as adiabatic, a formula for the coefficient of elasticity of the encapsulated air is deduced. Using this the air is presented as "air spring". The obtained relation is nonlinear with respect to the vertical displacement of the sleeve. The static displacement of the sleeve δ_{st} is determined from the equilibrium state condition. This displacement ensures the flow out of the air aside from the whirl tube. On the base of these the following main results are obtained:

- For first approximation the problem is solved *linearly,* where *the coefficient of elasticity is approximated to one defined constant. A pure harmonic law* of the motion of the sleeve with *amplitude and initial phase, which depend on the initial conditions* of movement, and the frequency of oscillation dependent on the air adiabatic coefficient, cross-sectional surface and the volume of the space in which the air is encapsulated, is obtained.
- On a higher level of examination the problem is observed as *nonlinear*. For this purpose the coefficient of elasticity of the "air spring" is expanded *in series with respect to the power of a small parameter* For it, the square of the ratio between the cross-sections of the space where the whirl tube's air is encapsulated, is chosen. *The differential equation of movement of the sleeve is reduced to quasi-linear and using the nonlinear mechanic's methods a solution to the second power of the small parameter is obtained.* The resultant motion is a sum of first, second and third order harmonics with respect to the main harmonic. The obtained law of motion of the sleeve is *concurred with the experimentally obtained results,* and the frequency deviations are about $5 \div 7\%$, and amplitude ones – $6 \div 8\%$.
- The so caused *oscillations of the intermediate sleeve assist in the assembly process.* They do this by trying to set up straight and self center the assembling part in respect to the main one, as also shown in the experiment.

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Теоретические аспекты автоматизированной сборки цилиндрических и резбовых соединений пневмовихровым методом (Часть III)

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(Р е з ю м е)

В работе рассматривается трептение по вертикалам междинной втулки в сборочной головке, генерированное пневмовихровым воздушным потоком, который из-за сжатия и растяжения является „воздушной пружиной“. Проблемма обсуждается в линейном и нелинейном аспекте.