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Benchmarking of Process Control Performance

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1. Introduction

Process control benchmarking is a technique for comparing actual loop behaviour and tuning against the theoretical or optimal possible. A key research step in this direction was taken by Harris [3] who used minimum variance control (MVC) as a benchmark for controller loop assessment. In 1967 Astrom first proposed the MVC principle and the use of autocorrelation to indicate how close the existing controller is to MV controller. Harris contribution was the idea to take into account the process time delay. Further work by Desborough and Harris [2] proposed the use of performance index and the use of analysis of variance for feedforward (FF)/feedback (FB) control loops to indicate whether a poor performance is due to the FF controller or the FB controller. The technique was applied for multivariate processes [5].

The Generalised Minimum Variance (GMV) benchmarking method extends the MV method. It takes into account actuator movements and control power limitations [6]. This reduces the control activity.

Nowadays a wide variety of commercial performance monitoring tools are available on the software market. Among the most often used are the Process doctor™ from Matrikon, ABB's Loop Optimizer Suite, Honeywell's Loop Scout™, Invensys's Performance Watch, Emerson's EnTech Toolkit, Control Arts' Control Assessment Tools. All of them use MVC based indices, autocorrelation function and various deterministic indices.

In the recent years the efforts of many industrial engineers, process control engineers and the academic control community are directed to the problem of defining deterministic and stochastic indices for loop tuning, detecting sluggish control, oscillations, valve stiction, etc. [4]. At the same time the idea of using MVC as a benchmark for control problems is now well established and has been proven useful in the practice. The reason for the wide spread use of the Harris index is the low data requirement – only time delay and process output are needed. But statistical analysis of the index accuracy is not made. The aim of the presented work is to investigate the Harris index dependence on the number of lags for correlation coefficients and whitening filter and to propose methodology for preliminary tests for obtaining adequate length of data set needed for index estimation.

2. Harris index

The controller cannot influence the system output before the dead time has run out. Therefore, if a stable process output y_t is modelled by an infinite moving-average (MA) model, then its first d terms constitute an estimate of the minimum variance term $\tilde{R}_1 v(t)$. An infinite-order MA model is defined as:

(1) $$y(t) = (f_0 + f_1 q^{-1} + ... + f_{d-1} q^{-(d-1)} + f_d q^{-d} + ...)v(t).$$

Multiplying equation (1) by $v(t), v(t-1), ..., v(t-d+1)$ respectively and then taking the expectation of both sides of the equation yields

(2) $$\begin{aligned} r_{yv}(0) &= E[y(t)v(t)] = f_0 \sigma_v^2, \\ r_{yv}(1) &= E[y(t)v(t-1)] = f_1 \sigma_v^2, \\ r_{yv}(2) &= E[y(t)v(t-2)] = f_2 \sigma_v^2, \\ &\vdots \\ r_{yv}(d-1) &= E[y(t)v(t-d+1)] = f_{d-1} \sigma_v^2. \end{aligned}$$

Therefore the minimum achievable variance or invariant portion of output variance can be calculated as:

(3) $$\begin{aligned} \sigma_{mv}^2 &= (f_0^2 + f_1^2 + ... + f_{d-1}^2)\sigma_v^2 = \\ &= \left[\left(\frac{r_{yv}(0)}{\sigma_v^2}\right)^2 + \left(\frac{r_{yv}(1)}{\sigma_v^2}\right)^2 + \left(\frac{r_{yv}(2)}{\sigma_v^2}\right)^2 + ... + \left(\frac{r_{yv}(d-1)}{\sigma_v^2}\right)^2\right]\sigma_v^2 = \\ &= \left[r_{yv}^2(0) + r_{yv}^2(1) + r_{yv}^2(2) + ... + r_{yv}^2(d-1)\right]\sigma_v^2, \end{aligned}$$

where f_i are coefficients of the model and σ_v^2 is the noise variance.

The performance index (Harris index) will then be obtained as the ratio of the minimum achievable variance and the actual output variance σ_y^2:

(4) $$\eta(d) = \frac{\sigma_{mv}^2}{\sigma_y^2}.$$

Index varies between [0, 1] where 1 is MVC and values close to 0 show poor control performance.

Substituting Equation (3) into Equation (4) yields:

(5)
$$\eta(d) = \left[r^2_{yv}(0) + r^2_{yv}(1) + r^2_{yv}(2) + \ldots + r^2_{yv}(d-1)\right] / \sigma^2_y \sigma^2_v =$$
$$= \rho^2_{yv}(0) + \rho^2_{yv}(1) + \rho^2_{yv}(2) + \ldots + \rho^2_{yv}(d-1) = ZZ^T,$$

where Z is the cross-correlation coefficient vector between $y(t)$ and $v(t)$ for lags 0 to d–1.

3. Harris index estimation

The proposed above method for Harris index estimation is known as FCOR algorithm [5] .

The estimate of the performance index is written as:

(6)
$$\hat{\eta}(d) = \hat{\rho}^2_{yv}(0) + \hat{\rho}^2_{yv}(1) + \hat{\rho}^2_{yv}(2) + \ldots + \hat{\rho}^2_{yv}(d-1).$$

Although $v(t)$ is unknown, it can be replaced by the estimated innovations sequence $\hat{v}(t)$. The estimate $\hat{v}(t)$ is obtained by pre-whitening the process output variable $y(t)$ via time series-analysis. AR or ARMA models can be used for estimating $v(t)$.

This technique is based on filtering and correlation analysis of routine close-loop data and is named as FCOR algorithm. That algorithm is schematically shown in Fig. 1.

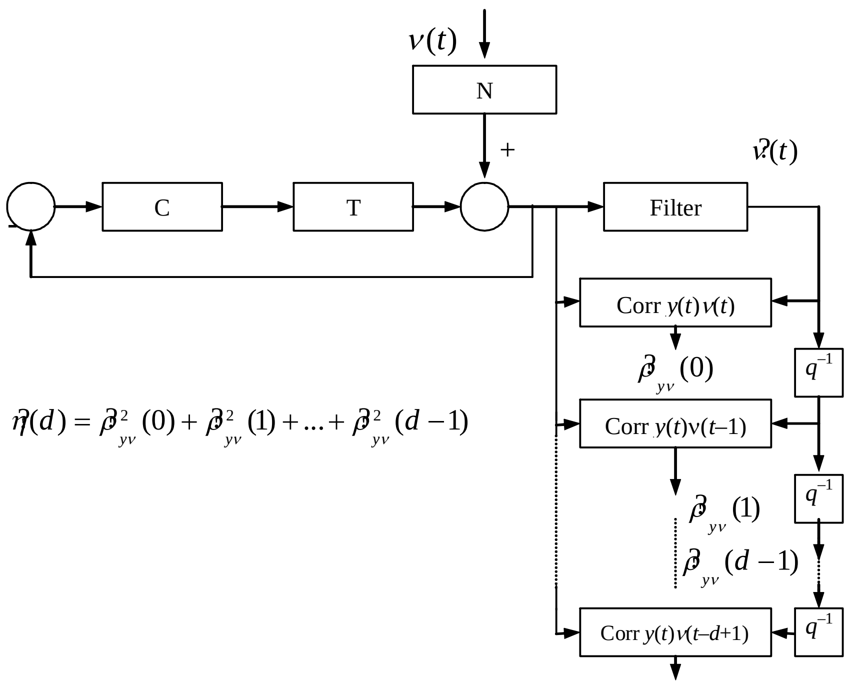

Fig. 1. FCOR algorithm

3. Statistical assessment of the estimated index

The aim of this work is to study the main statistical properties of the Harris index in order to propose justified procedure to assess it with preliminary given accuracy.

The mathematical expectation of the index $\eta(d)$ is sum of the mathematical expectations of the squares of cross-correlation coefficients for lags from 0 to d–1:

(7) $$M[\eta(d)] = M\left[\hat{\rho}^2_{y\nu}(0)\right] + M\left[\hat{\rho}^2_{y\nu}(2)\right] + \ldots + M\left[\hat{\rho}^2_{y\nu}(d-1)\right].$$

The estimate of cross-correlation function of two signals y and γ for finite time of analysis T_a is defined as

(8) $$\hat{R}^2_{y\nu}(\tau, T) = \frac{1}{T}\int_0^T y(t)\gamma(t+\tau)dt,$$

where $T=T_a-\tau$.

The mathematical expectation of the square of the estimate can be then obtained as:

(9) $$M\left[\hat{R}^2_{y\nu}(\tau,T)\right] = \frac{1}{T^2}\int_0^T\int_0^T M\left[y(t)\nu(t+\tau)y(u)\nu(u+\tau)\right]dtdu = \frac{1}{T^2}\int_0^T\int_0^T P^2_{y\nu}(\tau,t,u)dtdu.$$

For stationary processes equation (9) can be transformed into a single definite integral. Let replace $\theta = u-t, d\theta = -dt$ and after several transformations (9) results in [8]:

(10) $$M\left[\hat{R}^2_{y\nu}(\tau,T)\right] = \frac{2}{T}\int_0^T\left(1-\frac{\theta}{T}\right)P^2_{y\nu}(\tau,\theta)d\theta,$$

where $P^2_{y\gamma}(\tau,\theta)$ denotes the forth mixed central moment.

Taking into account that the process output is a random signal consisting of additive signal $y1(t)$ and noise $n1(t)$, and $\gamma(t)$ is a noise $n2$ ($y(t) = y1(t) + n1(t), \gamma(t) = n2(t)$), the forth moment can be presented as a sum of lower (second) moments (for stationary process):

(11) $$P^2_{y\nu}(\tau,\theta) = R_{y1}(\theta)R_{n2}(\theta) + R_{n1}(\theta)R_{n2}(\theta) + R_{n1n2}(\theta+\tau)R_{n1n2}(\tau-\theta) + R^2_{n1n2}(\tau).$$

Then the analytical expression for the mathematical expectation of the square of the cross-correlation function is

(12) $$M\left[\hat{R}^2_{y\nu}(\tau,T)\right] =$$

$$= \frac{2}{T}\int_0^T\left(1-\frac{\theta}{T}\right)\left[R_{y1}(\theta)R_{n2}(\theta) + R_{n1}(\theta)R_{n2}(\theta) + R_{n1n2}(\theta+\tau)R_{n1n2}(\tau-\theta) + R^2_{n1n2}(\tau)\right]d\theta.$$

By replacing auto- and cross-correlation functions R with the normalised correlation function – ρ, the expression for the mathematical expectation will be:

(13) $$M[\hat{\eta}(d,T)] =$$

$$= \sum_{i=0}^{d}\frac{2}{T}\int_0^T\left(1-\frac{\theta}{T}\right)\left[\rho_{y1}(\theta)\rho_{n2}(\theta) + \rho_{n1}(\theta)\rho_{n2}(\theta) + \rho_{n1n2}(\theta+d)\rho_{n1n2}(d-\theta) + \rho^2_{n1n2}(d)\right]d\theta.$$

In similar way an expression for the second moment the variance $D[\hat{\eta}(d,T)]$ of the index estimate could be obtained. The problem is that it would contain the eight moment and hence would be quite long sum of estimates of statistical quantities and will be very complicated, unpractical and quite inaccurate. The main conformity observed is that $D[\hat{\eta}(d,T)]$ depends on the $\frac{1}{T}$, i.e. on the time of the data analysis. Thus for the examined control system and given operational conditions one has to determine a reasonable value of T in order to obtain correct estimate for the system behaviour.

4. Laboratory experimental results

Harris index was tested using laboratory data from a pilot flotation tank, located in the "Laboratory of Process Control and Automation" in Helsinki University of Technology. Flotation plants are very susceptible to disturbances of two types: variations in the quality of the ore supplied to the plant and plant upsets resulting from any of a large number of operating problems in the plant. There are a lot of researches concerning the importance of level control in a flotation plant. The successful stabilises the levels in the flotation circuits can result in improvements in recovery of the order of 1% [7]. Therefore in this work special attention is paid to the level circuit.

Three data sets were received from the pilot flotation cell. They present the system behaviour in cases of steady-state conditions (lack of set point changes or load disturbance occurrence) with different controller parameters. The results for Harris index estimated on-line using FCOR algorithm are presented in Fig. 2. First the PI controller of the outflow valve was tuned by the Ziegler-Nichols method. The optimal parameters obtained in this way are: gain K=3 and integration time constant T_i=15 s (section I in the Fig. 2).

Index values for fast control (overtuned PI controller) was received with controller parameters K=4.5 and T_i=10 s (section II in Fig. 2, and slow control – parameters K=1 and T_i=40 s (section III in Fig. 2.)

Clearly during the first period of normal operation conditions and optimally tuned controller the index value is the highest. During the first some iterations the index value should not be taken into account because the recursive algorithm needs time for

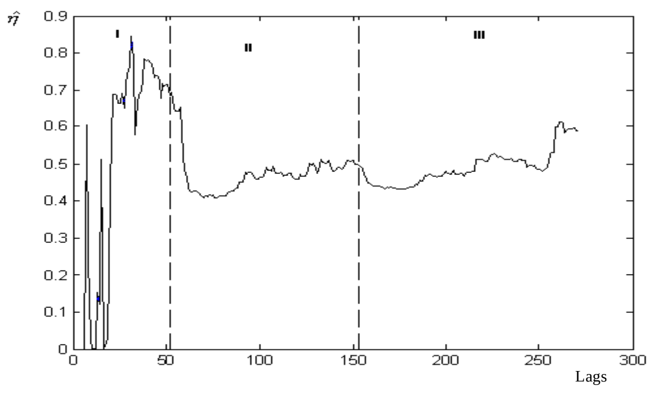

Fig. 2. On-line Harris index estimation

establishing. It can be also concluded that during the fast and slow control periods the index value is quite similar. I.e. one cannot give advice for the direction of the changes of the controller parameters. It can be only concluded that the process performance is not satisfactory and the controller needs retuning.

5. Statistical Estimates

Data from steady state regime with optimal tuning parameters of the same flotation cell are further used to define the minimum length of the data sets enough for adequate obtaining Harris index.

If the index dependency on number of lags used for estimation of correlation coefficients *M* was studied Fig. 3 shows that for the first 30 lags the values for Harris index cannot be used for process assessment. The filter – ARMA model ($\gamma = \frac{A(q^{-1})}{C(q^{-1})} y$ with n_a=4 and n_c=4) is estimated from the whole data set.

The influence of accuracy of the filter (Fig.1) has been considered. Values for Harris index were estimated for *M*=30 but with filter models received using different number of data and recursive algorithm for ARMA model estimation. Again n_a=4 and n_c=4. Results are presented in Fig. 4.

Finally the recursive on-line algorithm was tested, i.e. the index dependence on model estimated using recursive algorithm plus *M* varied from 1 up to 30 according to the number of data available (Fig. 5).

The index value becomes stable after the first 45 lags.

Mathematical expectation received by moving average filter (parameter l=5) from the index results for the last case is presented in Fig.6. Used filter cannot give enough smooth data for $M[\hat{\eta}]$ in the initial lags. After 40-45 lags $M[\hat{\eta}]$ is stable.

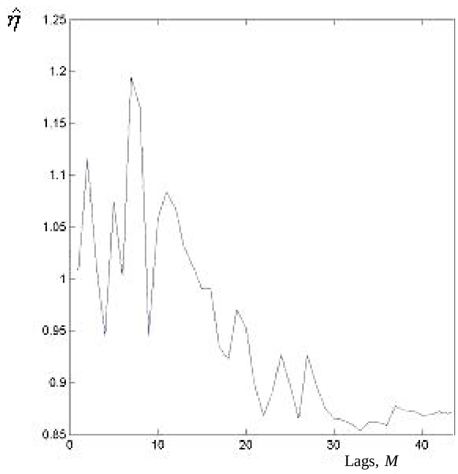

Fig. 3. Dependence of $\hat{\eta}$ on *M*

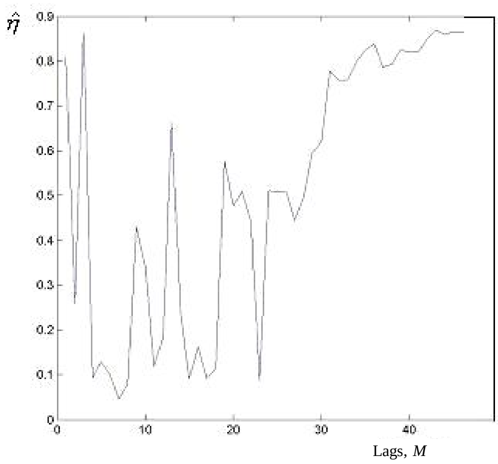

Fig. 4. Dependence of $\hat{\eta}$ on filter model (on-line estimation)

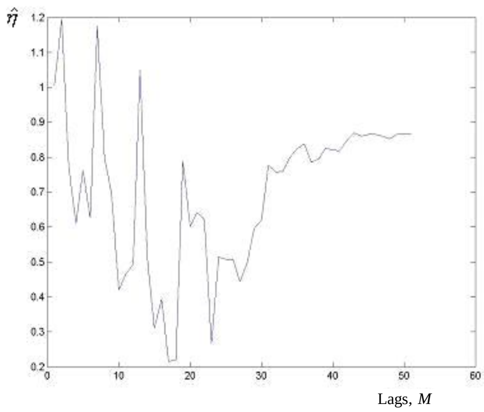

Fig. 5. Dependency of $\hat{\eta}$ on M and filter model

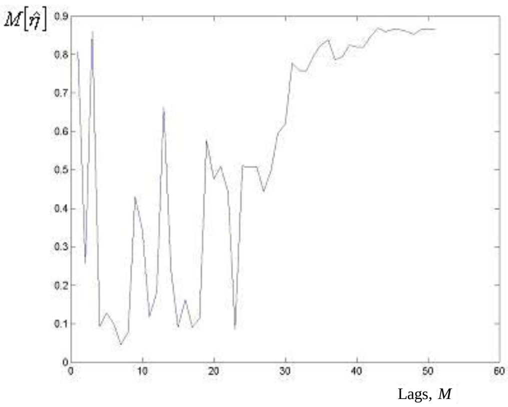

Fig. 6. Mathematical expectation of $\hat{\eta}$ – MA filter

More important for the accuracy of Harris index estimate is its variance (Fig.7).

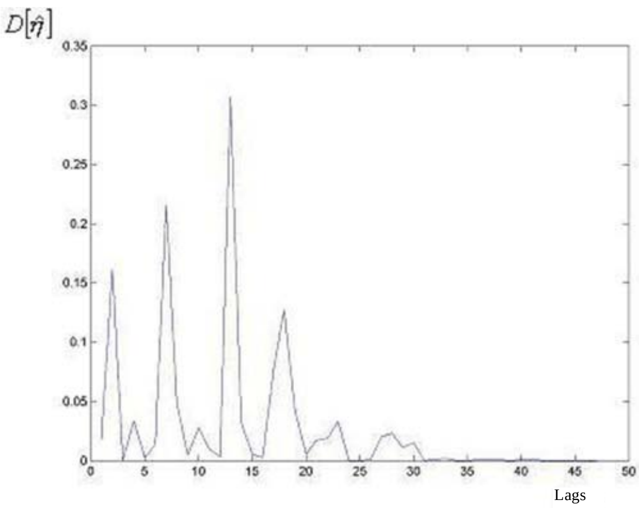

Fig. 7. Variance of the Harris index

By using MA filter the index variance tends to 0 faster then its mathematical expectation. After 33-35 lags index value is adequate enough to be used for process assessment.

Conclusions

Harris index is a simple and effective benchmark measure for assessment of control loop performance. Statistical analysis and simulation data however clearly show that the index value becomes adequate and could be used reasonably for decision-making only after a relevant data collection. The minimum length of data sets is process dependant. Therefore, preliminary tests have to be carried out in order to define this length. The data for determining dead-time could be used also for this purpose.

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Испитания поведения процесса управления

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(Р е з ю м е)

Испитания поведения управления связаны с обнаружением количественных индикаторов, показывающих как работают контролеры в сравнении с лучшей теоретической или оптимальной методологией управления. Сделана статистическая оценка индекса Хариса при помощи FCOR алгоритма. Проведены симмуляции на основе лабораторных данных и они анализированы, чтобы определить зависимость индекса от длины данных, от числа опозданий при оценке корреляционных коэффициентов и от модели фильтра. Результаты показывают, что предварительные испитания нужны, чтобы определить подходящую длину последовательности для вычисления индекса Хариса.