БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ . BULGARIAN ACADEMY OF SCIENCES

ПРОБЛЕМИ НА ТЕХНИЧЕСКАТА КИБЕРНЕТИКА И РОБОТИКАТА, 51 PROBLEMS OF ENGINEERING CYBERNETICS AND ROBOTICS, 51

София . 2001 . Sofia

Power Factor Correctors for Rectifiers

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1. Introduction

The DC electric energy consumption grows tremendously. As a rule the primary source for this is the industrial mains. In its turn this is related with the rectification and filtering of the mains voltage mainly with big condensers. These conventional-method operations lead to a consumption of a definitely disharmonic current with intensive high harmonic components and with big peak values. The power factor K is defined as a ratio between the really consumed power and the product of the effective values of the current and the voltage on the rectifier input. The factor K may also be defined as a ratio between the basic harmonic of the input current and its effective value. In both cases K is always less than unit and very often it is in the range 0.4–0.7. The nearer K is to unit, the smaller are the high harmonics of the current and the corresponding energy losses; besides this the quality of the consumed energy improves. New standards come to life (e.g. IEC 555–2) which strongly raise the requirements for the consumed harmonics from the mains and which the conventional rectifiers fail to satisfy.

During the last years new methods and structures are being intensively developed. They make the consumed from the rectifier current purely harmonic. Also they are combined with the simultaneous stabilization of the rectified current regardless of the charges in the input voltage or in the load. Such devices are popular as correctors of the power factor K. They begin to enjoy a wide distribution with the evident tendency to be built in all consumers with powers greater than 500 W.

2. Basic parameters of the converter-corrector

The basic idea of operation for the converters-correctors is described below. Biroutely rectified and unfiltered the primary voltage feeds a current source which in turn charges a filtering condenser with a big capacity and also the consumer of the DC electric energy. The value of the current in the current source changes in accordance with the





absolute value of a sinusoid which coincides with the frequency and the phase of the input voltage. Its amplitude is set in such a way that the voltage of the load must have a predefined value. Its structure has the appearance illustrated in Fig. 1.

The real output voltage *E* is compared with the predefined E_{an} and the difference ΔE is multiplied by the absolute value of the sinusoid e. The obtained signal is used as a task for the output current of the current source U/I. *H* is a regulator which realizes the pawned in it current-controlling law. The usual regulator is of a PI- or a PID-type [1, 7]; the regulator itself contains additional functions for a soft start and for an overload protection. The signal *e* may be proportional to the rectified primary voltage |u|; also it may be normalized in its amplitude. It is possible to extract the primary harmonic from $|U_1|$ and to normalize it. In this case the consumed current is purely harmonic. The filtering process must not introduce phase shifts. It is possible to produce the sinusoid symphased with the mains. The true hardware design of this device for generation of *e* can look like the shown in Fig. 2.

The voltage-driven generator VCO activates the address counter CA in the ROM memory containing the record with the discrete representation of a sinusoid. The obtained from the DAC signal e is compared with the mains voltage in PLL. The PLL



Fig. 2



output controls the VCO frequency in such a way so as to obtain $|U_1|$ and e synphased. The conventional corrector of the power factor is a biroute rectifier which is followed by an amplifying converter of a boost type allowing to change the input

followed by an amplifying converter of a boost type allowing to change the input current in a sinusoid manner ($|\sin\theta|$) and to keep the output voltage constant [1, 2, 3, 5, 6]. Its principal scheme is illustrated in Fig. 3.

It operates with a constant frequency of commutations $w = 2\pi/T$. The current intensity is controlled by the pulse ratio $x = \tau/T$ of the commutator switching SW. Here t is the time interval in which SW is switched on.

The development of this type of converters is very intensive. New structures constantly appear and they are in the scope of the explorers. The main efforts are the increase in the operational frequency (of commutation), the decrease of the commutational and conductive losses in the active elements and also the more precise control. The basic exploitation parameters can be derived from the basic type with little changes (Fig. 3). The principle parameters and dependencies of the converter-corrector are introduced with the following denotions and assumptions:

 $u=U_{m}\sin w_{0}t$ - the primary feeding voltage with a frequency of $w_{0}=2\pi f_{0}=2\pi /T_{0}$;

 $\theta = w_0 t$ - the input voltage phase;

 $w = \lambda w_0 = 2\pi / T_0$ - the commutation frequency;

 $R = U_m/I_m$ - the equivalent primary source load;

 I_m - the amplitude of the consumed from the primary source current;

 R_r - the load of the stabilised output voltage E;

 $x(\theta)=\tau(\theta)/T-$ the pulse ratio where $\tau(\theta)$ is the interval in which SW is in the "on" state;

$$y(\theta) = U(\theta) / U_m = \sin \theta;$$

$$z(\theta) = [w_n Li (\theta)] / U_m;$$

L- the integrating inductivity; $a = w_{0}L/R$; $\phi=tg^{-1}a$;

 $\gamma_{_1}(\theta)=\!\!\Delta\,i(\theta)$ /I - the relative increase of the input current when the commutator SW is switched on within a commutation cycle;

 $\gamma_{_2}(\theta)=\!\!\Delta i(\theta)/I-$ the relative decrease of the input current when the commutator is switched off in a commutation cycle;

 $A=E/U_m$ the amplifier coefficient;

 η -the converter effectiveness;

 $\lambda >> 1; \ R_{\rm \scriptscriptstyle T} \, C_{\rm \scriptscriptstyle 0} >> T_{\rm \scriptscriptstyle 0} \ /2; \ T < 2L /R.$

The reactive elements, the commutator and the diodes are free of losses. The

losses in them can be traced to some extent by the effectiveness η .

Taking under consideration the limitations above and the assumptions for the end of the active phase of the commutator SW, the following dependency is valid:

(1)
$$z \left[\theta + (2\pi/\lambda) x\right] = z(\theta) + (2\pi/\lambda) x \sin \theta$$

and the beginning of the next active phase is according to

$$z(\theta + 2\pi/\lambda) = z(\theta + 2\pi/\lambda) - (A - \sin \theta)(2\pi/\lambda)(1-x).$$

The change of z for a cycle of the commutator is:

(2)
$$\Delta z(\theta) = z(\theta + 2\pi/\lambda) - z(\theta) = (2\pi/\lambda) (Ax - A + \sin \theta).$$

From the condition that z must coincide with the law $z = a \sin \theta$, it follows that

(3)
$$\Delta z(\theta) = (dz / d \theta) \Delta \theta = a \cos\theta (2\pi / \lambda)$$

If we make equal (2) and (3) then we shall obtain for the law of changing the pulse ratio the following:

$$x(\theta) = 1 + (a/A) \cos\theta - (\sqrt{1+a^2}/A) \sin\theta,$$

(4)

$$x(\theta) = 1 - (\sqrt{1 + a^2} / A) \sin(\theta - \phi).$$

In a real system $x(\theta)$ may not exceed unit. Therefore if $\theta < \varphi$ then the harmonic change of the current is impossible. This area may be limited only to a commutational cycle iff $w_0 T \ge tg^1 a$. This limitation is very hard and practically impossible. It restricts the inductivity L to too small values when the current pulsations within the commutation range become too big.

It is possible to admit that in the beginning of every cycle the commutator SW shall remain in the "on" state for the whole interval θ_1 when z grows up to the desired value asin θ :

(5)
$$dz / d\theta = \sin \theta$$
, $z = 1 - \cos \theta$.

For $\theta = \theta_1$, $1 - \cos \theta_1 = a \sin \theta$, $\theta_1 = 2 \operatorname{tg}^{-1} a = 2\varphi$. The fill-up coefficient is constantly unit in the interval $0 \div \theta_1$. In this way

(6)
$$x(\theta) = \begin{cases} 1; \ 0 \le \theta \le 2 \ tg^{-1}a = 2\varphi, \\ 1 - (\sqrt{1+a^2}/A) \ \sin(\theta - \varphi); 2\varphi \le \theta \le \pi \end{cases}$$

Fig. 4 shows the change in the pulse ratio $x(\theta)$ for two semi-cycles of the mains for A=1.5 and for different values of $a = w_0 L/R$. For comparison of the phase relations the same scheme illustrates also the change of the input voltage.

The impulse ratio $x(\theta)$ may not be negative. This leads to the restriction that $\sqrt{1+(w_{\rm L}/R)^2} \leq A$ which defines a minimal transfer coefficient of the converter for a defined ratio between the reactive and active energies in the converter. The same restriction in the form of $R \geq w_0 L / \sqrt{A-1}$ defines the minimal load resistance and respectively the maximal useful power which is available from the converter for a defined voltage transfer. The maximal power $P_{\rm m} = [U_{\rm m}^2/\sqrt{A-1}]/2w_0 L$ is restricted by the integrating inductivity L if the input and the output voltages and the frequency of



the primary mains are defined. Such dependencies for the industrial mains 220 V, 50 Hz are shown in Fig. 5.

If the consumed power is greater than the one defined in this way, then the input current cannot follow the prescripted harmonic law. The pulse ratio becomes zero if $\theta = \theta_2$; then SW is constantly switched off and the converter scheme becomes like the drawn in Fig. 6a.

The phase θ_2 and $z(\theta_2)$ are derived from the dependencies

(7)

$$\begin{aligned} \sin(\theta_{2} - \varphi) &= \frac{A}{\sqrt{1 + a^{2}}}, \\ \theta_{2} &= \sin^{-1}(A/\sqrt{1 + a^{2}}) + \varphi, \\ z(\theta_{2}) &= a \sin \theta_{2} = a \frac{A + \sqrt{1 + a^{2} - A^{2}}}{1 + a^{2}}. \end{aligned}$$

When $\theta = \theta_3$ then $z(\theta)$ decreases to the value $z(\theta_3) = a \sin \theta_3$ and the system becomes controllable again in the interval $\theta_2 \le \theta \le \theta_3$ (Fig. 6b)







(8)
$$z(\theta) = \cos\theta_2 - \cos\theta - A(\theta - \theta_2) + z(\theta_2).$$

When $\theta = \theta_3$, $\cos\theta_2 - \cos\theta_3 - A(\theta_3 - \theta_2) + z(\theta_2) = a\sin\theta_3$ and the converter enters the mode of a harmonic current. The mode and the converter parameters define that $\theta_3 < \pi$ or $\Box \theta_3 > \pi$. In the first case for the interval $\theta_3 \le \theta \le \pi$, $z(\theta) = a \sin\theta$. In this mode the following condition must be valid:

(9)
$$A(\pi - \theta_{2}) - \sqrt{1 + a^{2} - A^{2}} > 1,$$
$$\theta_{2} = tg^{-1} a + \sin^{-1} (A / \sqrt{1 + a^{2}}).$$

In the second case $\theta_3 > \pi$ and the current changes for the interval $\theta_3 \le \theta \le \pi$, according to (8) and for $\theta_3 > \theta > \pi$, -according to the dependency

(10)
$$z(\theta) = 1 - \cos \theta - A(\pi + \theta - \theta_2) + z(\theta_2); \quad 0 \le \theta \le \theta_1.$$

Here $\theta = \theta \mod \pi$ and $\theta_3 = \theta_1 + \pi$. When for $\theta = \theta_3$ the system becomes controllable again then a new cycle of the process starts.

In the case of controllable rectifiers it is possible to feed the equivalent boost converter not only with positive, but also with negative voltages. Choosing a proper polarity it is possible to decrease significantly the intervals when the real current differs from the desired harmonic.

3. Pulsations and spectrum of the input current

The analysis shows that the consumed from the mains current will change in the following below way provided a perfect control in accordance with the real mode:

I
$$A/\sqrt{1+a^2} \leq 1$$
.

(11)
$$z(\theta) = \begin{array}{l} |1 - \cos \theta; \quad 0 \le \theta \le 2\rho = 2 \operatorname{arctg} a, \\ |a \sin \theta; \quad 2\rho \le \theta \le \pi. \end{array}$$

I.
$$A/\sqrt{1+a^2} < 1; \theta_3 \le \pi.$$

 $|1 - \cos\theta; 0 \le \theta \le 2\varphi;$
 $|a \sin \theta; 2\varphi \le \theta \le \theta_2; \theta_2 = tg^{-1} a + sin^{-1} (A / \sqrt{1+a^2});$
(12) $z(\theta) = |$
 $|a \sin \theta_2 + \cos\theta_2 - \cos\theta - A(\theta - \theta_2); \theta_3 \le \theta \le \pi;$
 $|a \sin \theta, \theta_2 \le \theta \le 7.$

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III.
$$A/\sqrt{1+a^2} < 1; \theta_3 > \pi.$$

(13) $|1 - \cos\theta - A(\theta + \pi - \theta_2); \theta_4 \le \theta \le \theta_2;$
 $|a \sin\theta, \theta_4 \le \theta \le \theta_2;$
 $|z - (\theta_2) + \cos\theta_2 - \cos\theta - A(\theta - \theta_2); \theta_2 \le \theta \le \pi.$

The case when the converter is fed with a bipolar voltage is omitted. Fig. 6 demonstrates the evolution of $z(\theta)$ for a=0.8 and A=1.2. The control is regained for $\theta_3 < \pi$.

Besides in the beginning of every cycle the control may be lost also for very special correctors of the power factor which have too low comutation frequency or too big integrating inductivity. Practically for A>1.2 and usually for A>1.3 the control may be lost if $a^2>0.69$ and $a\geq0.83$. This means that the inductive resistance is comparable with the one of the load for frequencies of the primary voltage and that the inductivity has big values. For the practical comutation frequencies (>20 kHz) the current pulsations due to the comutation process are not big. It is possible to assume that practically in the power-factor correctors the only deviations in the input current from the harmonic law are in the beginning of every new cycle.

The spectrum with frequencies less than the commutation frequency of the consumed current for $A > \sqrt{1+a^2}$ is defined unambiguously by a. For $A < \sqrt{1+a^2}$ it depends also on A. It was already presented that the second case is rather specific and it shall be out of the scope in the present research. In the first case the distortions are only in the beginning in every cycle and the spectrum is defined by the following dependencies:

(14)
$$\begin{aligned} |1 - \cos\theta; & 0 \le \theta \le 2\varphi; \\ |a\sin\theta; & 2\varphi \le \theta \le \pi; \\ |a\sin\theta; & \pi \le \theta \le \pi + 2\varphi; \\ |a\sin\theta; & \pi + 2\varphi \le \theta \le 2\pi; \\ \sqrt{2} & \pi \end{aligned}$$

(15)
$$S_{k}(a) = -- \operatorname{mod}_{0} z(\theta) e^{-jk\theta} d\theta,$$
$$\pi = - \operatorname{mod}_{0} z(\theta) e^{-jk\theta} d\theta,$$

(16)
$$H(a)=D(a)/S_1(a)$$

where

(17)
$$D(a) = \int_{-\pi}^{2\rho} (a \sin \theta - 1 + a \cos \theta)^2 d\theta.$$



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The current contains only odd harmonics because $z(\theta) = -z(\theta + \pi)$. The computed spectral characteristics $S_k(a)$ and H(a) $S_k(a)$ and H(a) are shown in Fig. 7. Fig. 7a demonstrates the total harmonic distortions H(a) which are defined as the ratio between the energy of all the harmonics and the basic harmonic. Fig. 7b depicts the dependency of the first five odd harmonics (3, 5, 7, 9, 11) related also with the basic harmonic. Fig. 7c illustrates the amplitude of the basic harmonic related with the one for a=0 (no distortions).

The graphs show that for a<0.2 (which can be very easily obtained for commutation frequencies above 20 kHz) the share of the harmonics is below 5 %.

The input current pulsations depend on the commutation frequency f_q and on the filtering inductivity L and also on the phase of the process within the semiperiod bounds of the primary voltage.

The basic dependencies

(18)
$$x = 1 - [\sin(\theta - \phi)] / A\cos\phi,$$
$$i = I \sin\theta,$$
$$u = U_m \sin\theta$$

for the pulsation with the increase of the phase $\boldsymbol{\theta}$ give that

(19)
$$U_{m} \sin \theta = -T x (\theta),$$
$$L$$

$$\gamma_{1}(\theta) = \frac{\Delta i(\theta)}{I_{m}} = \frac{2\pi R}{\lambda w_{0}L} \begin{pmatrix} \sin(\theta - \phi) \\ 1 - - - - - \\ A \cos\theta \end{pmatrix} \sin \theta = \frac{2\pi}{\lambda a} \begin{pmatrix} \sin(\theta - \phi) \\ 1 - - - - - \\ \lambda a \end{pmatrix} \sin \theta.$$

If the phase is $\theta = w_0 t$, then the relative pulsations have the maximal value

(20)

$$\frac{d\gamma_{1}(\theta)}{d\theta} = 0; \quad A = \frac{\sin(2\theta_{\tau}-\phi)}{\cos\phi\cos\phi_{\tau}};$$

$$\gamma_{1\mu} = \frac{\Delta i}{I_{m}} = \frac{2\pi}{\lambda tg\phi} \left(\frac{\sin(\theta-\phi)}{1-\frac{\sin(\theta-\phi)}{A\cos\phi}} \right) \sin\theta_{\tau}.$$

The two tables below present the pulsations for different values of A and λ =400. This corresponds to a frequency commutation 20 kHz and a primary mains frequency $f_0=50$ Hz. The first table is built for $\varphi=5^\circ$ or L=278 R μ H and the second table – for $\varphi=15^\circ$ or L=853R μ H which corresponds to a=0.087 and a=0.2C.

 $\phi=5^{\circ}$, $f_{0}=20$ Hz, f=20 kHz

А	1	1.2	1.5	2	2.5	3	4	>>1
θ _r , °	32	38	48	67	81	85.1	87.5	90
∆I _{max} / I,%	5.22	6.03	7.25	9.2	10.8	12	13.5	18

 ϕ =15°, f_{o} =50 Hz, f=20 kHz

А	1	1.2	1.5	2	2.5	3	4	>>1
θ _r , °	34.1	39.35	47.2	60	70.4	76.8	82.6	90
$\Delta I_{ m max}/$ I,%	2.17	2.48	2.73	3.2	3.64	6.97	4.75	5.86



The increase of the inductivity L strongly diminishes the pulsations. This is accompanied by an expansion of the interval of φ where the current is uncontrollable. The increase of the transfer coefficient A shifts the maximum of the pulsations towards the input voltage maximum. The increase of the commutation frequency unambiguously leads to a decrease of the pulsations and this is quite natural.

Fig. 8a introduces a quasi-3D graph for the dependency of the pulsation maximums γ_{im} from a semiperiod of the input frequency from *a* and *A* for λ =10³.

Fig. 8b shows a family of curves for $\gamma_{lm} = f(A)$ for different values of a.

For intervals which are longer than the comutation cycle the current pulsations in the energy accumulation process and during the energy release time are different. This is due to the energy accumulation in the inductivity. Also the moments when the current pulsations reach their highest values are different.

The pulsation during the energy release time interval is defined from the following below equation:

(21)

$$\begin{array}{c}
\Delta i = -\underbrace{U_{m} \sin \theta}{\Delta i = -\underbrace{U_{m} \sin \theta}{L}} T (1-x), \\
\Gamma \\
\gamma_{2}(\theta) = \underbrace{\Delta i}_{-1} = 2\pi \\
I_{m} \quad \lambda \sin \varphi \\
\end{array} \left(\begin{array}{c}
1 \\
1 - -i \sin \theta \\
A \\
\end{array} \right); \quad 2\gamma < \theta \le \pi
\end{array}$$

The condition $d\gamma/d\theta = 0$ defines the phases θ_s when the pulsations have their greatest extremums $A= [\sin(2\theta_s - \phi)]/[\cos(\theta_s - \phi)]$. The maximums of the pulsations are defined by

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Fig. 9a and Fig. 9b show the changes of the pulsations within a semiperiod of the input voltage for a=0.05 and A=1.2 and 4. The pulsations are distributed more evenly for small values of A when the input- and the output voltages have near values.

4. Conclusions

The rectifiers with a power factor near to unit have simple structures. Their applications especially for greater powers form a separate domain. It is shown that they can operate for quite wide load modes when they keep the load current very close to the harmonic form. The intervals when the desired form of the current is out of control are very narrow and the increase of the operating frequency narrows these intervals further more. The structure of the devices allows the introduction of nodes which enable the commutations in the moments with zero currents or with zero voltages. In its turn this strongly reduces the dynamic losses. Also it is possible to combine the commutation with the rectification thus leading to a decrease of the static losses [2–5]. From a purely energetic point of view the rectifiers in the scope consuming harmonic currents may have a high effectiveness which is better than 90 %. The presented in the paper research in the form of formulas, tables and graphs gives an ample idea for the design and the evaluation of the basic parameters of such converters.

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Корректоры мощностного фактора токовыпрямителей

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(Резюме)

Функциональная структура выпрямителей с фактором мощности, близким к единицу, вылагается системным образом. Основные протекающие процессы представлены вместе с описывающими их зависимостями. Выводятся основные рабочие и эксплуатационные параметры, которые допускают приложения инженерного проектирования. Рассмотрены разные рабочие режимы в зависимости от товара. Дефинируются спектральный состав и пульсации потребляемого тока. Результаты исследования представлены в форме уравнений, таблиц и график.