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A Lexicographic Algorithm Solving a Problem of a Multiobjective Flow in a Network*

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1. Introduction

Let for a given oriented connected network $G\{N, U\}$ with a set of nodes $N=\{1, \ldots, n\}$, $1=s$ and $n=t$, and a set of arcs $A=\{(i, j): i, j\in N\}$, x_{ij} denotes the flow along the arc (i, j) and x – the flow vector. The problem for a multiproduct flow (MF) in G could be formulated as follows:

$$\text{MF:}\quad \min f_1(X) = \sum_{(i,j)\in U} a^1_{ij} x_{ij},$$

$$\ldots$$

$$\min f_k(X) = \sum a^k_{ij} x_{ij}$$

under the constraints

$$(1)\qquad \sum_{j\in N} x_{ij} - \sum_{j\in N} x_{ji} = \begin{cases} v, & \text{if } i=s, \\ 0, & \text{if } i\neq s, t, \\ -v, & \text{if } i=t; \end{cases}$$

$$(2)\qquad 0 \le x_{ij} \le c_{ij}, \quad (i, j)\in U,$$

where c_{ij} is the upper limit of the flow along the arc (i, j).

The constraints (1) are called constraints for flow conservation, and constraints (2) – capacity constraints. The problem is a partial case of the multicriteria linear problem.

It is obvious that the optimal solution of the problem, which is multicriteria, cannot be found by the algorithms designed to find a minimal flow in the single criterion case. These algorithms are efficient in a computing aspect mainly for large dimension problems. This efficiency designates the necessity for neglecting the general methods of multicriteria linear problems solution and the search for specialized algorithms using the flow structure of the constraints matrix of the problem being set.

The problem of such algorithms design is not at all easy, since the solution of

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the multicriteria problems is connected with the addition of supplementary constraints to the constraints of the problem considered, which destroys the special structure of the constraints matrix.

Some algorithms, adaptations of simplex-methods have been proposed, that use the presence of flow constraints in the constraints matrix [1, 2]. A specialized multicriteria simplex-algorithm has been proposed in [3] for generation of all non-dominated solutions of a MF. A methodology is developed in [4] for the generation of a set of non-dominated solutions of the bicriteria flow problem. The properties of the non-dominated solutions of a MF have been investigated in [5].

An approach solving a MF is recommended in the present paper, characterized by apriori criteria ranking in conformance with the decision maker's (DM's) preferences. In this case a sequence of single criterion flow problems is solved, the network structure being altered in each one in such a way, that the value of the criterion with higher priority is not deteriorated. An analogue of the out-of kilter method is represented in [6], satisfying the conditions for flow conservation, while those for capacity and duality may be broken. The structure of the network for a subsequent single-citerion problem is defined in [7] depending on the dual conditions being satisfied.

2. A lexicographic algorithm

When defining the MF problem, the value of the variable v should be specified – the flow size. Three cases are possible: a maximal flow is searched for; a flow with a defined size is searched for; an optimal size flow is seeked for. In the last case an arc (t, s) is added to the network arcs set and the condition that the balance of the flow should be 0 at nodes s and t also, is implied.

It is assumed that the DM has ranked the objective functions according to their absolute value (priority). In general the algorithm consists of the following: a single-criterion optimization problem for a minimal flow in a network is solved with the highest priority criterion, and a network – the network being set. Let the flow – a solution of this problem, be denoted by x_1. A constraint is set that this criterion remains equal to its minimum and under this condition the second in significance criterion is minimized, after that – the third, under the condition that the first two will preserve their values and so on, until all the criteria are minimized.

The direct addition of these constraints would deconstruct the flow structure of the problem constraints. Hence this property of the optimal flow is used that in an appropriately built network $G(x_1)$, the structure of which depends on the values of the flow x_1 along the original network, there do not exist cycles of a negative value. The omitting of an additional flow of cycles with a positive value would worsen the flow value, that is why all the arcs forming cycles with a positive value are extracted from the given network G, i.e. the network G is reduced to the network G_i so that in the network $G_i(x_1)$ there are cycles with a zero value only. On the network thus obtained the problem for a minimal flow is solved with a criterion – the second preferred function, a new network is constructed, depending on its optimal solution, the arcs forming positive cycles are isolated and so on, till all the criteria are minimized or a graph not connected with respect to the node s and t is constructed.

The general description of the algorithm is the following:

Iteration 0

p=1 and U^p=U is set.

Iteration p

the problem for a minimal flow

$$\text{MinF: } \min_{(i,j)\in U} f_1(X) = \sum a_{ij} x_{ij}$$

is solved, where $a_{ij} = a_{ij}^{p}$

subject to constraints

$$\sum_{j\in N} x_{ij} - \sum_{j\in N} x_{ji} = \begin{cases} v, & \text{if } i = s, \\ 0, & \text{if } i \neq s, t, \\ -v, & \text{if } i = t, \end{cases}$$

$$0 \le x_{ij} \le c_{ij}, \quad (i, j) \in U^p.$$

Let the optimal solution of this problem be denoted by x^p.

If $p=k$, stop, the flow x^p is the solution searched for.

Otherwise:

A network $G^p(x)=\{n, U^p\}$ is constructed as described below.

Let $A=U^p(x)$ be the set of isolated arcs,

$$p = p+1.$$

$$U^p = U^{p-1} \setminus U^{p-1}(x)$$

is determined.

If $U^p = \varnothing$, stop, the flow x^{p-1} is the solution searched for.

Isolating of arcs

A network $G(x^p)=\{N, U^p\}$ is constructed in the following way.

For each arc $(i, j)\in N$, if

1. $0 \le x^p_{ij} \le 0$, the arc $(i, j) \in U_p$ with a value $-a_{ij}$, and the arc $(j, i) \in U_p$ with a value a_{ij} is constructed;

2. $0 = x^p_{ij}$, the arc $(i, j) \in U_p$ with a value $-a_{ij}$ is built;

3. $x^p_{ij} = c_{ij}$, the arc $(j, i) \in U_p$ with a value a_{ij} is constructed.

Negative cycles are searched for in the network $G(x^p)$ with the help of any known flow programming algorithm and the arc, forming it, is isolated for each cycle. The set of all these arcs forms the set $U^p(x)$. The sense of network $G(x^p)$ constitution is the determination of the direction of cycle tracing, i.e., in what direction the flow would be let so that the value of the objective function would be increased. It is obvious that for cycles with a null value, this value is preserved independently on the direction of flow tracing.

Theorem 1. The algorithm finds an efficient solution for MF problem.

The algorithm suggested is in its essence lexicographic. The problem MinF being solved optimizes at each iteration the subsequent criterion on the network, constructed so that the optimal value of the criterion with higher priority is preserved.

The successive iterations of the algorithm suggested are illustrated on the following test example.

The network $G(N, U)$ is given, the number of the nodes being 8, and of the arcs – 15. Four criteria are defined $f_1(X)$, $f_2(X)$, $f_3(X)$, $f_4(X)$, ranked according to their priority. An optimal in size efficient flow is looked for.

The network G is shown in Fig. 1. The numbers around the arcs denote their capacities.

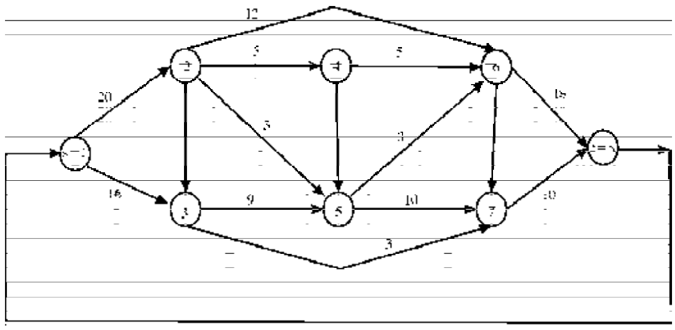

Fig. 1.

The criteria satisfy the following:

$$f_1(X)= x_{23} + 3\,x_{35} + 3\,x_{45} + 2\,x_{56} + 2\,x_{68}\,,$$

$$f_2(X)= 4\,x_{13} + 4\,x_{25} + 2\,x_{56} + 7\,x_{68} + 3\,x_{76}\,,$$

$$f_3(X)= 2\,x_{23} + 4\,x_{25} + x_{35} + 7\,x_{37} + 4\,x_{45} + 8\,x_{46} + 6\,x_{57} + 6\,x_{76}\,,$$

$$f_4(X)= 2\,x_{12} + 4\,x_{13} + 7\,x_{25} + 2\,x_{35} + 2\,x_{37} + 4\,x_{45} + 10\,x_{56} + 2\,x_{57} + 9\,x_{76} + 2.\,x_{78}$$

The optimal solution of the problem with respect to the criterion $f_1(X)$ is found at the first iteration. The values of the flow are given below, s=1 and t=8:

$$x_{12} = 9,\ x_{13}=10,\ x_{23}=0,\ x_{24}=3,\ x_{25}=0,\ x_{26}=6,\ x_{35}=7,\ x_{37}=3,\ x_{45}=1,\ x_{46}=2,$$
$$x_{56}=0,\ x_{57}=8,\ x_{68}=15,\ x_{76}=7,\ x_{78}=4,\ x_{81}=19.$$

The arcs (1, 3), (2, 3), (3, 7), (4, 5), (5, 6), (6, 8) are isolated and on the network obtained the second criterion is optimized. The values of the flow are as follows:

$$x_{12} = 11,\ x_{13}=10,\ x_{23}=0,\ x_{24}=5,\ x_{25}=0,\ x_{26}=6,\ x_{35}=7,\ x_{37}=3,\ x_{45}=4,\ x_{46}=2,$$
$$x_{56}=0,\ x_{57}=8,\ x_{68}=15,\ x_{76}=5,\ x_{78}=6,\ x_{81}=21.$$

The arcs (2, 6), (2, 4) are isolated and the third criterion is optimized on the newly constructed network. The flow values are:

$$x_{12} = 11,\ x_{13}=10,\ x_{23}=0,\ x_{24}=5,\ x_{25}=0,\ x_{26}=6,\ x_{35}=7,\ x_{37}=3,\ x_{45}=4,\ x_{46}=2,$$
$$x_{56}=0,\ x_{57}=8,\ x_{68}=15,\ x_{76}=5,\ x_{78}=6,\ x_{81}=21.$$

The arc (2, 5) is isolated. A network not connected to the source and the sink is obtained. The solution searched for is attained.

3. Conclusion

An algorithm is suggested that implements a lexicographic approach in the solution of the problem for a multiobjective flow in a network, when the criteria are ranked by priority. At each subsequent iteration the next criterion with a lower priority in the ranking order is minimized on a network obtained from the initial one after isolation of the arcs defined at the previous iteration. In this way the possibility to apply efficient flow algorithms is conserved.

The algorithm correctness is proved and it is applied on a test example.

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Лексикографический алгоритм решения задачи многокритериального потока в сети

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Исследована задача многокритериального потока в сети в случае, когда критерии даны в приоритетном ряду. Для решения задачи предлагается алгоритм, реализирующий лексикографический подход, при чем на каждой итерации решается однокритериальная задача минимального потока в сети.