БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ . BULGARIAN ACADEMY OF SCIENCES

ПРОБЛЕМИ НА ТЕХНИЧЕСКАТА КИБЕРНЕТИКА И РОБОТИКАТА, 51
PROBLEMS OF ENGINEERING CYBERNETICS AND ROBOTICS, 51

София . 2001 . Sofia

A User-Friendly Interactive Algorithm of the Multicriteria Linear Integer Programming

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1. Introduction

The interactive algorithms are often used [2] to solve multicriteria linear integer programming problems (MCIP). These algorithms [3, 6, 7, 11, 13] are modifications of interactive approaches solving multicriteria linear problems that include the integrality constraints. Linear integer programming problems are used as scalarizing problems in these interactive algorithms. These problems are NP-difficult problems [4]. Moreover, finding a feasible integer solution can be as difficult as finding an optimal solution. That is why in the interactive algorithms solving MCIP the time to solve the scalarizing problem plays a significant role. For this reason an effort is made to reduce the number of the integer problems solved; continuous problems (instead of integer problems) are solved and continuous (weak) nondominated solutions obtained are presented to the DM for evaluation (especially in the DM's learning phase). Some of the interactive algorithms work with the aspiration levels of the criteria, others use weight to denote the relative significance of the criteria. Many show one while others show several (weak) nondominated solutions to the DM for evaluation at each iteration.

In the paper a learning-oriented [5] interactive algorithm are suggested. The main features of the algorithm proposed, which improves the dialogue with the DM, are:

– they reduce the number of the integer problems solved because in most of the iterations the solutions of single criterion linear problems with continuous variables (which are easy to solve) are presented to the DM for evaluation. This is used under the assumption [10, 11] that the criteria values for the scalarizing problems with continuous variables differ relatively little from the solutions with integer variables and under the assumption that the DM prefers to work in the criteria rather than in the variable space.;

– at every iteration the DM provides his/her local preferences in terms of the desired changes in the criteria values of some of the criteria, the desired directions of change of the other criteria and permitted deterioration with or without set limiting value of the remaining criteria, instead of aspiration levels of the criteria. The current preferred solution and the local preferences of the DM define a reference neighborhood in which the next preferred solution is searched for;

– at every iteration in a reference neighbourhood a set of continuous (weak) nondominated solutions or integer (weak) nondominated solution is searched for solving continuous or integer scalarizing problems ;

2. Problem formulation

The multicriteria linear integer programming (I) can be formulated as:

(1) $$\text{"max"}\{f_k(x),\ k \in K\}$$

subject to:

(2) $$\sum_{j\in N} a_{ij}x_j \leq b_i,\ i \in M,$$

(3) $$0 \leq x_j \leq d_j,\ j \in N,$$

(4) $$x_j - \text{integer},\ j \in N,$$

where symbol "max" means that all the objective functions are to be simultaneously maximized; $K = \{1, 2, \ldots, p\}$, $M = (1, 2, \ldots, m\}$, $N = \{1, 2, \ldots, n\}$ denote the index sets of the objective functions (criteria), the linear constraints, and the decision variables, respectively: $f_k(x)$, $k \in K$ are linear criteria (objective functions); $f_k(x) = \sum_{j\in N} c_j^k x_j$ and $x = (x_1, x_2, \ldots, x_j, \ldots, x_n)^T$ is the vector of the decision variables.

The constraints (2)–(4) define the feasible region X_1 for the integer variables.

The problem (1)–(3) is a multicriteria linear programming problem (P). The feasible region for the continuous variables is denoted by X_2. Problem (P) is a relaxation of (I).

For clarity of exposition, we introduce a few definitions:

Definition 1. A current preferred solution is a near (weak) nondominated solution (a feasible solution located comparatively close to the (weak) nondominated solutions) or (weak) nondominated solution chosen by the DM at the current iteration. The most preferred solution is a preferred solution that satisfies the DM to the greatest degree.

Definition 2. Desired changes of the criteria values are the amounts by which the DM wishes to increase or to be worsened the criteria in comparison with their value in the current preferred solution. The desired directions of change of the criteria are the directions, in which the DM wishes to improve or to deteriorate the criteria in comparison with their values at the current preferred solution.

Definition 3. Reference neighbourhood is defined by the current preferred solution; the desired changes in the values of some of the criteria, the desired directions of change of the other criteria and permitted deterioration with or without set limiting value of the remaining criteria as specified by the DM.

Problems (I) and (P) do not possess a mathematically well-defined optimal solution. Hence it is necessary to select one of the (weak) nondominated solutions, which is most appropriate for the global DM's preferences. This choice is subjective and depends entirely on the DM.

3. Scalarizing problems

We formulate the scalarazing problems [1, 16] under the assumption that the set of criteria K can be divided into three subsets – K_1, K_2 and K_3. The set K_1 contains the indices $k \in K$ of the criteria for which the DM wants to improve their values compared to the values in the current preferred solution. The set K_2 includes the indices $k \in K$ of the criteria for which the DM agrees to worsen their values not setting the exact values of deterioration. The set K_3 contains the indices $k \in K$ of the criteria whose values the DM wants to preserve or agrees to be worsened by the value δ_k. The set K_1 is divided into two subsets – K'_1 and K''_1; K'_1 contains indices of the criteria $k \in K_1$ that the DM wants to improve by desired values δ_k, and K''_1 consists of indices of the criteria $k \in K_1$, that the DM wants to improve and for which he/she is not able to set the exact values of improving.

The following scalarizing problem, named E_1, is proposed to obtain a (weak) nondominated solution of the multicriteria integer problem (I) in the reference neighbourhood of the current preferred solution.

Minimize

(5) $$S(x) = \max[\max_{k\in K'_1}(\bar{f}_k - f_k(x))/|f'_k|, \max_{k\in K_2}(f_k - f_k(x))/|f'_k|] + \max_{k\in K''_1}(f_k - f_k(x))/|f'_k|,$$

subject to:

(6) $$f_k(x) \geq \tilde{f}_k,\ k \in K_3 \cup K''_1,$$

(7) $$x \in X_1,$$

where

f_k – the value of the criterion with an index $k \in K$ in the current preferred solution, $\bar{f}_k = f_k + \Delta_k$ is the desired level of the criterion with an index $k \in K'_1$;

$$\tilde{f}_k = \begin{cases} f_k, \text{ if } k \in K''_1, \\ f_k, \text{ if } k \in K_3 \text{ and the DM wants to preserve the current value of the criteria with index } k, \\ f_k - \delta_k, \text{ if } k \in K_3 \text{ and the DM is agree to be worsen with value } \delta_k \text{ the current value of the criteria with index } k, \end{cases}$$

f'_k – a scaling coefficient,

$$f'_k = \begin{cases} f_k, \text{ if } f_k \neq 0, \\ 1, \text{ if } f_k = 0. \end{cases}$$

Theorem 1. The optimal solution of the scalarizing problem E_1 is a weak efficient solution of the multicriteria integer programming problem (I).

Proof.

Let K'_1 and $K''_1 \neq \varnothing$.

Let x^* be an optimal solution of problem E_1. Then the following condition is satisfied:

$$S(x^*) \leq S(x),\ x \in X,$$

$$f_k(x^*) \geq \tilde{f}_k,\ k \in K''_1 \cup K_3.$$

Let us assume that x^* is not a weak Pareto optimal solution of the initial multiple criteria integer problem (I). In this case there must exist, $x' \in X$ for which:

(8) $$f_k(x') > f_k(x^*) \text{ for } k \in K \text{ and } f_k(x^*) \geq \tilde{f}_k,\ k \in K''_1 \cup K_3.$$

After transformation of the objective function $S(x)$ of the scalarizing problem, using the inequalities (8), the following relation is obtained:

$$(9)\ S(x') = \max[\max_{k\in K'_1}(\bar{f}_k - f_k(x'))/|f'_k|,\ \max_{k\in K_2}(f_k - f_k(x'))/|f'_k|] + \max_{k\in K''_1}(f_k - f_k(x'))/|f'_k| =$$

$$= \max[\max_{k\in K'_1}(\bar{f}_k - f_k(x^*)) + (f_k(x^*) - f_k(x')))/|f'_k|,$$

$$\max_{k\in K_2}(f_k - f_k(x^*)) + (f_k(x^*) - f_k(x')))/|f'_k|] +$$

$$+ \max_{k\in K''_1}(f_k - f_k(x^*)) + (f_k(x^*) - f_k(x')))/|f'_k| <$$

$$< \max[\max_{k\in K'_1}(\bar{f}_k - f_k(x^*))/|f'_k|,\ \max_{k\in K_2}(f_k - f_k(x^*))/|f'_k|] + \max_{k\in K''_1}(f_k - f_k(x^*))/|f'_k| = S(x^*).$$

It follows from (9) that $S(x') < S(x^*)$ and $f_k(x^*) \geq \tilde{f}_k$, $k \in K''_1 \cup K_3$, which contradicts to (8). Hence x^* is a weak efficient solution of the multiple criteria integer problem (I).

Consequence. Theorem 1 is true for arbitrary values of f_k, $k \in K$.

The proof of this consequence follows from the fact that the proof of Theorem 1 does not assume any constraints on the values of the criteria f_k, $k \in K$.

To obtain a (weak) nondominated solution for the problem (P) in the reference neighbourhood of the current preferred solution, we may use the scalarizing problem E_2, which is obtained from E_1 replacing constraint (7) by constraint:

$$(10)\qquad x \in X_2.$$

Theorem 2. The optimal solution of the scalarizing problem E_2 is a weak efficient solution of the multiple criteria linear problem (P).

The proof of Theorem 2 is analogous to the proof of Theorem 1 because nature of the variables x_i^*, $i=\overline{1,n}$, is not used explicitly.

Because the objective function of the scalarizing problem E_1 is nondifferentiable, one may solve the following equivalent mixed integer programming

$$(11)\qquad \min(\alpha + \beta)$$

subject to:

$$(12)\qquad \alpha \geq (\bar{f}_k - f(x))/|f'_k|,\ k \in K'_1,$$

$$(13)\qquad \alpha \geq (f_k - f_k(x))/|f'_k|,\ k \in K_2,$$

$$(14)\qquad \beta \geq (f_k - f_k(x))/|f'_k|,\ k \in K''_1,$$

$$(15)\qquad f_k(x) \geq \tilde{f}_k,\ k \in K''_1 \cup K_3,$$

$$(16)\qquad x \in X_1,$$

$$(17)\qquad \alpha, \beta - \text{arbitrary}.$$

Problems E_1 and E'_1 have the same feasible sets of the variables. The value of the objective functions of problems E_1 and E'_1 are equal which can be easily proved.

The scalarizing problem E'_1 has two properties, that help to improve the dialogue with the DM, as with respect to the required from him/her information and with respect to the reducing of the waiting time for evaluation of new solutions also. The first property is connected with the required information from the DM. Instead of the aspiration levels of every criteria for the defining of the reference point [7, 9, 11], the DM has to provide only changes in the criteria values of some of the criteria and the directions of change of the remaining criteria to specify the reference neighbourhood. The second property of the problem E'_1 is that with it the DM can realize the search strategy "no great benefit – little loss". The solutions obtained in the reference neighbourhood are comparatively close, which makes it easier for the DM to compare several solutions and choose the next preferred solution

The scalarizing problem (E_2) is equivalent to the following linear programming problem E_2'

(18) $$\min(\alpha + \beta)$$

subject to:

(19) $$\alpha \geq (\bar{f}_k - f(x))/|f_k'|,\ k \in K_1',$$

(20) $$\alpha \geq (f_k - f_k(x))/|f_k'|,\ k \in K_2,$$

(21) $$\beta \geq (f_k - f_k(x))/|f_k'|,\ k \in K_1'',$$

(22) $$f_k(x) \geq \tilde{f}_k,\ k \in K_1'' \cup K_3,$$

(23) $$x \in X_2,$$

(24) $$\alpha, \beta - \text{arbitrary}.$$

The parametric extension of the scalarizing problem E_2' (denoted by $\overset{|}{E}_2$) has the fallowing form (similar to the one in [13]):

(25) $$\min(\alpha + \beta)$$

subject to:

(26) $$f(x) + |f_k'|\alpha \geq f_k + (\bar{f}_k - f_k)^t,\ k \in K_1',$$

(27) $$f(x) + |f_k'|\alpha \geq f_k - t,\ k \in K_2,$$

(28) $$f(x) + |f_k'|\beta \geq f_k + t,\ k \in K_1'',$$

(29) $$f_k(x) \geq \tilde{f}_k,\ k \in K_1'' \cup K_3,$$

(30) $$x \in X_2,$$

(31) $$t \geq 0,$$

(32) $$\alpha, \beta - \text{arbitrary}.$$

Problems E_2' and $\overset{|}{E}_2$ have the same properties as problem E_1', but they give continuous solutions.

Let us assume that we have found a (weak) nondominated solution of problem (P) with the help of the scalarizing problems E_2' and $\overset{|}{E}_2$ and wish to find a (weak) nondominated solution of problem (I), which is near the (weak) nondominated solution of problem (P). Let us denote by $\hat{f} = (\hat{f}_1, \ldots, \hat{f}_p)^T$ a (weak) nondominated solution of problem (P).

To find a (weak) nondominated solution of problem (I), close to the (weak) nondominated solution $\hat{f}_k$ of problem (P), the following Chebychev's problem E_3 may be used [26]:

Minimize

(33) $$S(x) = \max_{k \in K}(\hat{f}_k - f_k(x))/|f_k'|,$$

subject to

(34) $$x \in X_1,$$

where

$$\hat{f}_k = \begin{cases} \hat{f}_k, \text{ if } \hat{f}_k \neq 0, \\ 1, \text{ if } \hat{f}_k = 0. \end{cases}$$

This problem is equivalent to the following mixed integer programming problem E_3':

(35) $\min \alpha$

under the constraints

(36) $\alpha \geq (\hat{f}_k - f_k(x))/|\hat{f}_k|,$

(37) $x \in X_1,$

(38) α – arbitrary.

4. A user-friendly interactive algorithm of multicriteria linear integer programming

A user-friendly interactive algorithm solving multicriteria linear integer problems can be suggested on the basis of the scalarizing problems E_1', E_2', E_2 and E_3'. The dialogue with the DM has been improved with respect to the information required from him/her and to the learning possibilities of the specifics of the problem solved.

The basic steps of the algorithm are the following:

Step 1. An initial (weak) nondominated solution of the multicriteria problem (P) is defined, setting $f_k = 1$, $k \in K$, $\bar{f}_k = 2$, $k \in K$, and solving problem E_2'.

Step 2. Ask the DM to specify the reference neighbourhood of the current preferred solution defining desired changes in the values of some criteria, desired directions of change of other criteria and permitted deterioration with or without set limiting value of the remaining criteria.

Step 3. Ask the DM to define whether to search for a (weak) nondominated solution of the multicriteria problem (*P*) or (weak) nondominated solutions of the multicriteria problem (*I*). In the first case, *Step 4* is executed, in the second case go to *Step 6*.

Step 4. Ask the DM to specify parameter *s* – maximal number of (weak) nondominated solutions of the multicriteria problem (*P*) which can be saved in the set M_1. Solve the scalarizing problem E_2 with the help of an algorithm of linear parametric programming. Present the set M_1 to the DM for evaluation and selection. In case the DM wants to see a (weak) nondominated solution of the multicriteria problem (*I*), close to the current preferred solution of the multicriteria problem (*P*), *Step 5* is executed, otherwise – *Step 2*.

Step 5. Solve problem E_3'. Show the (weak) nondominated solution of multicriteria problem (*I*) obtained by the exact integer algorithm chosen for solving problem E_3'. If the DM approves this solution as current preferred solution of the multicriteria problem (*I*) go to *Step 7*. If this solution is the last preferred solution – go to *Step 8*.

Step 6. Solve problem E_1'. Show the (weak) nondominated solution of the multicriteria problem (*I*) to the DM. In case the DM approves this solution as a current preferred solution of the multicriteria problem (*I*) go to *Step 7*. If the solution is the last preferred solution – go to *Step 8.*

Step 7. If the DM wants to store the current preferred solution of the multicriteria problem (*I*) – check if it has been saved before, if not, add it to LIST – a set of stored preferred solutions – Go to *Step 2*.

Step 8. Does the DM want to compare the last preferred solutions of the multicriteria problem (*I*) with the solutions selected and stored in LIST – go to *Step 9*. If no – *Stop*. That is, the last preferred solution is the most preferred solution of the multicriteria problem (*I*).

Step 9. Show the final set of solutions, saved in LIST and the last preferred solution to the DM for comparison and selection of the most preferred solution of the multicriteria problem (*I*). *Stop*.

The proposed algorithm for solving multicriteria linear integer problems is a learning oriented [5] interactive algorithm and the DM controls the dialogue, the computations and the stopping conditions.

Problems of linear parametric programming (scalarizing problems E_2') are solved in the interactive algorithm. The linear parametric programming problems are easily solved problems and the DM must not wait long for the obtaining and estimation of new solutions. Problems of mixed integer linear programming (scalarizing problems E_1' and E_3') are also solved. The number of the integer problems solved can be very small. They are solved only in the cases when the DM feels uncomfortable to operate with continuous variables or when he is searching for an integer solution near to the current preferred continuous solution.

The DM operates mainly in the criteria space, because in most of the cases the criteria have physical or economic interpretation and this enables the more realistic estimation and choice. The information required from the DM refers only to the defining of a reference neighbourhood of the current preferred solution and sometimes, if he/she wants, to the presenting of inter- and intra-criteria information.

5. Illustrative example

With the purpose to illustrate the interactive algorithm proposed the following multicriteria problem is solved. It is solved with the help of a developed small research software system and the DM is supposed to use all the possibilities that the algorithm provides.

$$\max f_1(x) = 5x_1 - x_2 + 2x_4,$$
$$\max f_2(x) = -x_1 + 2x_2 + x_3,$$
$$\max f_3(x) = 4x_2 - 8x_3 + 2x_4$$

under the constraints

$$-x_1 + 2x_2 + x_3 + 2x_4 \leq 34,$$
$$2x_1 + x_2 - 3x_3 - x_4 \leq 16,$$
$$3x_1 + 2x_2 + 4x_3 - x_4 \leq 28,$$
$$x_1 + 6x_2 - x_3 + 4x_4 \leq 43,$$
$$x_1, x_2, x_3, x_4 - \text{integer}.$$

Let us denote the feasible region for the integer variables by X_1 and the feasible region for the continuous variables by X_2.

In order to find an initial non-dominated solution, a scalarizing problem of E_2' type is solved, at apriori set $f_k = 1$, $k \in K$, $\bar{f}_k = 2$, $k \in K$ and $K_1' = \{1, 2, 3\}$;

$$\min \alpha,$$
$$\alpha \geq 2 - 5x_1 + x_2 - 2x_4,$$
$$\alpha \geq 2 + x_1 - 2x_2 - x_3,$$
$$\alpha \geq 2 - 4x_2 + 8x_3 - 2x_4,$$
$$x \in X_2,$$
$$\alpha - \text{arbitrary}.$$

The solution found is: $x_1 = 3$, $x_2 = 6.17$, $x_3 = 1.98$ and $x_4 = 1.25$. The values of the criteria for this solution are: $f_1 = 11.3$, $f_2 = 11.3$, $f_3 = 11.3$. Let us assume that the DM would like to improve the first criterion and sets only the reference point $\bar{f}_1 = 15$, $K_1' = \{1\}$ for it; he agrees to deteriorate the second criterion, not defining by what value, $K_2 = \{2\}$, and to improve the third criterion, not defining any certain values, $K_1'' = \{3\}$ and $K_3 = \varnothing$. The DM would like to see integer solutions at this iteration, but not more than 3, i. e., $s = 3$. A problem of E_1' type is formed:

$$\min(\alpha + \beta),$$
$$\alpha \text{ i } (15 - 5x_1 + x_2 - 2x_4)/11.3,$$
$$\alpha \text{ i } (10 + x_1 - 2x_2 - x_3)/11.3,$$
$$\alpha \text{ i } (10 - 4x_2 + 8x_3 - 2x_4)/11.3,$$
$$4x_2 - 8x_3 + 2x_4 \text{ i } 11.3,$$
$$x \in X_1,$$
$$\alpha, \beta - \text{arbitrary}.$$

With the help of an algorithm of mixed integer programming the following solution is found: $f_1 = 14$, $f_2 = 8$ and $f_3 = 24$. But the DM decides that the value of the second criterion does not satisfy him/her and chooses to continue with one iteration more, in order to search for a better solution. He defines aspiration level of the second criteria – $\bar{f}_2 = 12$, $K_1' = \{2\}$. He agrees to deteriorate the first and third criteria, but he wants the first criterion to be worsened by the value $\delta_1 = 4$, i. e. $K_2 = \{3\}$ and $K_3 = \{1\}$. A following scalarizing problem of E_2' type is solved and 3 new continuous solutions are found.

$$\min \alpha,$$
$$-x_1 + 2x_2 + x_3 + 8\alpha \text{ i } 8 + 4t,$$
$$4x_2 - 8x_3 + 2x_4 + 24\alpha \text{ i } 24 - t,$$
$$5x_1 - x_2 + 2x_4 \text{ i } 8,$$
$$x \in X_2,$$
$$t \text{ i } 0,$$
$$\alpha - \text{arbitrary}.$$

f_i	f_1	f_2	f_3	T
1	8	10,42	26,7	0
2	8	11,4	21,17	1
3	8	12,5	11,5	2

The DM selects the third solution as the current preferred solution of the multicriteria continuous problem. A problem of type is used in order to consider which is the nearest integer solution.

$$\min \alpha,$$
$$\alpha \text{ i } (8 - 5x_1 + x_2 - 2x_4)/8,$$
$$\alpha \text{ i } (12.5 + x_1 - 2x_2 - x_3)/12.5,$$

$\alpha \geq (11{,}5 - 4x_2 + 8x_3 - 2x_4)/11.5,$

$x \in X_1,$

α – arbitrary.

The values of the criteria for the solution obtained are $f_1 = 11$, $f_2 = 10$ and $f_3 = 10$. This solution does not satisfy the DM either and he sets new conditions for the criteria: to improve the first and the third criteria, $K_1'' = \{1, 3\}$, the second one may be deteriorated by 1 unit, $\tilde{f}_2 = 9$, $K_3 = \{2\}$. A following scalarizing problem of E_1' type is solved with the help of an exact algorithm of mixed integer programming.

$$\min \beta,$$
$$\beta \geq (11 - 5x_1 + x_2 - 2x_4)/11,$$
$$\beta \geq (10 - 4x_2 + 8x_3 - 2x_4)/10,$$
$$5x_1 - x_2 + 2x_4 \geq 11,$$
$$4x_2 - 8x_3 + 2x_4 \geq 10,$$
$$-x_1 + 2x_2 + x_3 \geq 9,$$
$$x \in X_1,$$
$$\beta \text{ – arbitrary.}$$

The solution found is: $f_1 = 14$, $f_2 = 9$, $f_3 = 18$.

The DM selects this solution as the most satisfactory for him/her. With this the operation of algorithm is brought to an end.

7. Conclusion

A user-friendly interactive algorithm is proposed based on the reference neighbourhood approach to solve multicriteria linear integer programming problems. The scalarizing problems, E_1', E_2', E_2 and E_3' provide the opportunity to improve the dialogue with the DM with respect to several features:

– according to DM's wish, he/she may set different type and different quantity of information at each iteration;

– the time during which the DM is expecting solutions for evaluation and choice is reduced, because in most of the time he/she works with the continuous solutions;

– his/her possibilities for learning the specifics of the multiple criteria integer problems being solved can be increased.

These features of interactive algorithm proposed characterise it as an appropriate and user-friendly algorithm solving multicriteria linear integer programming problems.

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Потребительски ориентированный алгоритм для решения задач многокритериального линейного целочисленного программирования

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(Р е з ю м е)

Предложен потребительски ориентированный интеррактивный алгоритм, предназначеннный для решения линейных целочисленнных задач многокритериального программирования. Этот алгоритм основан на формированной оттправной области при помощи заданных предпочитаний лицом принимающем решения (ЛПР) для изменений стоимостей критерий. Формулированные скаляризирующие задачи открывают сравнительно близкие недоминированные непрерывные или целочисленные решения в этой отправной области. Текущее использование непрерывных недоминированнных решений редуцирует значительное время поиска и позволяет ЛПР быстрее понимание спесифику многокритериальной задачи.