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An Improved Network Optimization Model of Transport Processes Control

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The recent experience in mathematic modeling field and the availability of computer systems with large computing capacity, as well as the constant need for optimization of the transport (and similar to them) processes, determine the necessity for discussing, investigating and improving the related models and methods. The tools of flows on networks have proved in practice as one of the most appropriate directions in transport processes modeling. The present paper formulates and considers an improved network optimization model coordinated with the technological requirements of a generalized transport process and with the computing capacities of modern computers.

Let a transport process be described in terms of flows on graphs by the so called output graph $G=[Y, A]$, where Y is the set of nodes and A – the set of arcs [3, 5, 6, 7, 8]. An example of a similar graph is shown in Fig. 1 (where 1 and 9 are one and the same finite stations, where unlimited stay of the transport unit is permitted).

According to the terminology accepted in [8], the nodes in the output graph G, corresponding to the places of loading (unloading) are called basic nodes and the existence of arcs-loops is characteristic for them, which expresses possible outage of the load and the transport vehicles. The rest of the arcs, interpreted as intermediate points on the route, are called nonbasic nodes. The modeling of a real transport process requires that each of the arcs in the set A be characterized by a function of the transition time $t(x, y)$, for which:

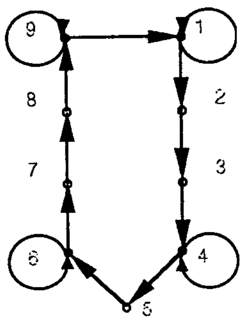

Fig. 1. Output graph $G=[Y,A]$

$$0 \le t(x, y) \le k, \ (x, y) \in A, \tag{1}$$

k is a natural number.

The presence of this function of time and of arcs-loops enables the use of known relations for stationary flows [2]. Let the construction of the output graph be realized so that for each arc $(x, y) \in A$, in power is $t(x, y)=1$ (a selected unit for time). This enables the expression of the transport process development in time, transforming the output graph G into a dynamic graph $G_t = [X, B]$ [2, 3]. For this purpose each node x, y of the output graph is assigned a set of nodes $\{x_t, y_t, t= 0, 1, ..., n\} \rightarrow X$ in the dynamic graph and each arc (x, y) – a set of arcs $\{(x_t, y_{t+1}), t= 0, 1, ..., n\}\} \rightarrow B$, where t accepts discrete values according to (1) and n is the length of the dynamic graph or the total duration of the transport process investigated. The example form of a dynamic graph with length $n=15$, corresponding to the output graph indicated in Fig. 1, is shown in Fig. 2.

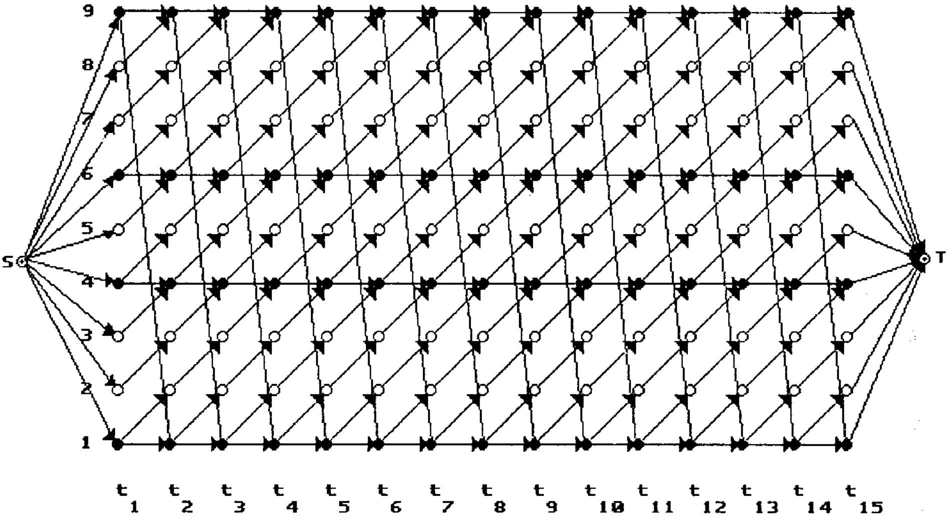

Fig.2

As seen from the figure, in order to use the mathematic apparatus of flows on graphs, the dynamic graph G_t thus constructed is completed by two additional nodes – a source S and a sink T and their corresponding arcs. The methodology of a dynamic graph construction on the basis of an output graph and the biunique correspondence between them is discussed in detail in [8].

Let on the basis of a detailed generalized dynamic graph a mathematic model of a generalized transport process be developed using the tools of flows on graphs, which could assure optimal control of this process.

For this purpose an integer function of the capacity is set on the arcs of the dynamic graph G_t:

(2) $$c_f(x, y) \geq 0, \ (x, y) \in B,$$

which expresses the real capacity of the transport route with respect to the transport vehicles. Their movement is modeled by an arc flow function f subject to:

(3) $$\sum_{y \in A(x)} f(x, y) - \sum_{y \in B(x)} f(y, x) \begin{cases} \leq a \text{ for } x= S_0, \\ = 0 \text{ for } x \neq S_0, T_0, \\ \geq a \text{ for } x= T_0, \end{cases}$$

(4) $$f(x, y) \leq c_f(x, y),$$

(5) $$f(x, y) \geq 0, \quad \text{integer},$$

for each $(x, y) \in B$, where

$$A(x)= \{y: (x, y) \in B \},$$
$$B(x)= \{y: (y, x) \in B \},$$

and a is the maximum number of the transport units.

Besides the flow of transport units each transport process is characterized by a flow of loads also, modelled by another flow function Φ, defined by the conditions:

(6) $$\sum_{y \in A(x)}\Phi(x, y) - \sum_{y \in B(x)}\Phi(y, x) \begin{cases} \leq b \text{ for } x= S_0, \\ = 0 \text{ for } x\neq S_0, T_0, \\ \geq b \text{ for } x= T_0, \end{cases}$$

(7) $$\Phi(x, y) \leq C, \text{ for } \{(x^0_t, x^0_{t+1}), t = 0, ..., n-1\},$$

(8) $$\Phi(x, y) \leq cf(x, y) \text{ for the remaining arcs,}$$

(9) $$\Phi(x,y)\geq 0, \text{ continuous,}$$

for each $(x, y) \in B$, where x^0_t denotes the basic nodes in the dynamic graph at different discrete moments of time, b is the maximum quantity of loads and c is the capacity of one vehicle. The condition (7) expresses the capacity of the loading/unloading places for the flow of loads Φ, and condition (8) – the capacity of all the other movement arcs along the transport route. It follows from (8) that no arcs capacity is defined according to the classic flow definition and it is dynamically obtained by the values of the transport units flow. This flow model can be regarded as a generalization of the multiproduct flows and it is introduced, defined and specified in [8] as an interconnected flow and the necessary and sufficient conditions for its existence have been proved.

The mathematical formulation above stated can be used to design a model of a transport process using the linear programming tools for optimal control of this process and adjust them with respect to the computing difficulties. For this purpose some assumptions are accepted which would simplify the mathematical model from the view point of the variables and constraints number and would be reasonable from a practical view point. We assume at first that the outage time of the haulage units for loading and unloading (at the main nodes of G_t) is included in the movement time. This is actually feasible for most of the transport processes and would not cause significant distortions in the model excluding the cases when the loading and unloading time is commensurable with the time of movement. The result of this assumption is defining the flow f only on one subset of arcs of the graph G_t as shown in Fig. 3.

The reduced dynamic graph G_t^1 thus obtained corresponds to an output graph in which the arcs between the finite and initial station are replaced by one arc.

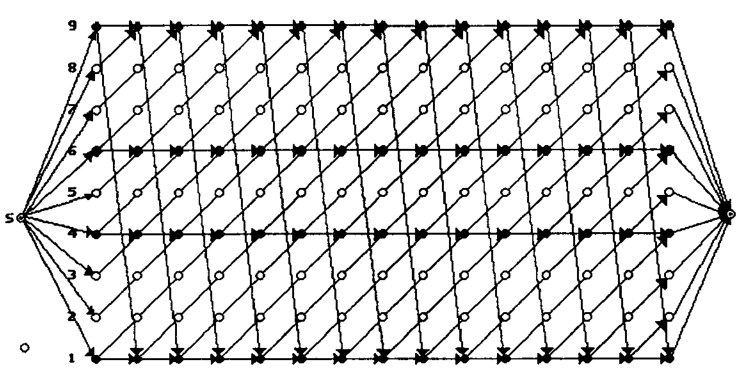

Fig.3.

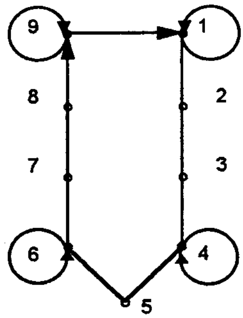

Fig. 4

This leads to the reduction of the total number of arcs in the dynamic graph (and respectively the variables for the flow of haulage vehicles f) by the number $((k-2)n)-6$, where k is the number of the last node and n – the length of G^1. In the example selected, for $k=9$ and $n=15$, the arcs in the dynamic graph are reduced from 208 upto 109, i.e. they are diminished by 48 % approximately, which is significant considering the computing difficulties decrease.

The other assumption, facilitating the computing difficulties is the removing of constraints (4) and (7). This in practice avoids the limiting of haulage units number for the movement arcs, as well as the limiting of the outage at the places of loading and unloading and is also usual in practice. The neutralization of this model inacuracy in the objective function is done introducing elements, expressing the losses from loads downtime. This assumption reduces the number of constraints by $(k-1)n+2k+k_0n$, where k is the number of nodes in the output graph and n – the length of the dynamic graph, k_0– the number of the basic nodes. For the example ($k=9, n=15, k_0=4$), the constraints are reduced by 198, or by 42 % approximately.

An optimization problem with respect to the arcs values of the flows f and φ is formulated on the basis of the model above described with the help of linear programming tools. Its solution will define the necessary number of transport units and their traffic, which is one of the main purposes in transport process control. The optimality of the solution searched for is determined with respect to an objective function including the profit from transported goods P_1, stay losses P_2 and exploitation expenses P_3.

(10) $$P= P_1 - P_2 - P_3 \rightarrow \max.$$

The profit of transported goods is expressed as:

(11) $$P_1 = \Sigma \lambda (x, y) \varphi (x, y),$$

$$(x, y) \in \{ (x^{t,k}, y^{t+1,k+1}), t = 1, ..., n, \; k = 1,..., n\},$$

where the arc estimate $\lambda (x, y)$ expresses the gain from the dispatch of goods for one unit on the movement arcs.

The losses from the outage are:

(12) $$P_2 = \Sigma \mu (x, y) \varphi (x, y),$$

$$(x, y) \in \{ (x^{t,k}, y^{t+1,k}), t = 1, ..., n, \; k = 1, 4, 6, 9\},$$

where the arc estimate $\mu (x, y)$ expresses the loss from the outage of one stock unit in the basic nodes.

The exploitation expenses are determined as

(13) $$P_3 = \Sigma \nu (x, y) f(x, y),$$

$$(x, y) \in \{ (x^{t,k}, y^{t+1,k+1}), t = 1, ..., n, \; k = 1,..., n$$

where the arc estimate $\nu (x^{t,k}, y^{t+1,k+1})$ denotes the expenses from the use of one vehicle.

Regarding the values of the flow functions f and φ as variables in the search of the maximum of P, the constraints (3) and (6) – (9) are taken into account. Passing from flow terminology towards this of linear programming, we represent the arc values of the flow functions for the example shown in the figures, like variables as follows:

u_1 upto u_9 – haulage units along the arcs (S, x^k_1), $k=1, ..., 9$;

u_{10} upto u_{30} – haulage units along the arcs (x^k_t, y^9_t), $k=1, ..., 9$, $t=1, ..., 9$, and (x^1_t, y^k_t), $k=1, ..., 15$; $t=2, ..., 15$;

u_{31} upto u_{44} – haulage units along the arcs (x^9_t, y^1_{t+1}), $t=2, ..., 15$;
u_{45} upto u_{58} – haulage units along the arcs (x^9_t, y^9_{t+1}), $t=1, ..., 15$;
u_{59} upto u_{67} – haulage units along the arcs (x^k_{15}, T), $k=9, ..., 1$;
u_{68} upto u_{76} – loads along the arcs (S, x^k_1), $k=1, ..., 8$;
u_{77} upto u_{97} – loads along the arcs (x^k_t, y^{k+1}_{t+1}), $k=1, ..., 9$, $t=1, ..., 15$;
u_{98} upto u_{139} – loads along the arcs (x^k_t, y^k_{t+1}), $k=1, 4, 6$, $t=1, ..., 15$;
u_{140} upto u_{181} – loads along the arcs (S, x^k_{15}), $k=1, 4, 6$, $t = 1, ..., 15$;
u_{182} upto u_{204} – loads along the arcs (x^9_t, T), $t=1, ..., 15$, (x^k_{15}, T), $k=9, ..., 1$.

The correspondence between the variables and the arcs is shown in Fig. 5 and Fig. 6, where the letters u are omitted for clarity and only their numbers are shown.

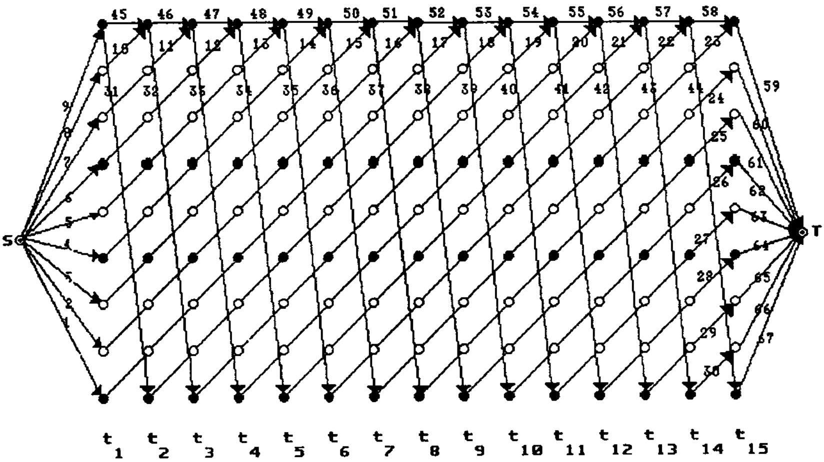

Fig. 5

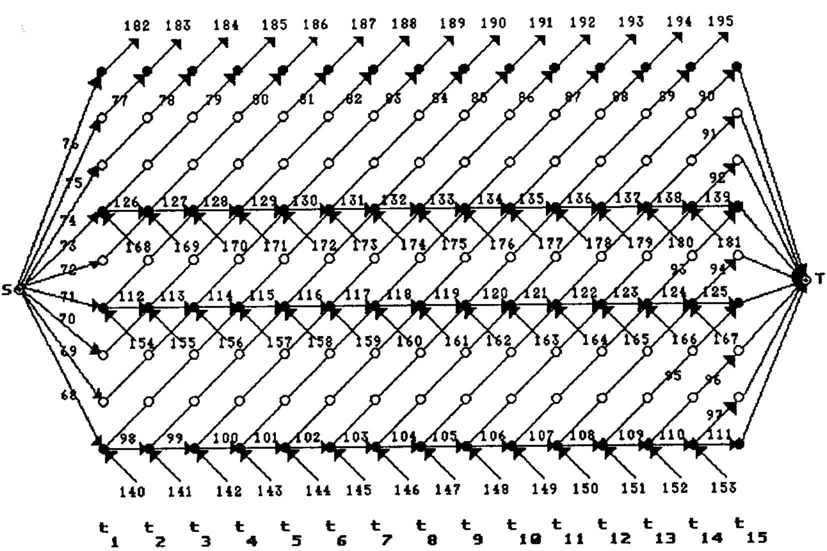

Fig. 6

As seen in Fig. 6, arcs for the flow of loads have been added towards the arcs of the example graph G', expressing the possibility for loads supplying at arbitrary discrete time moments to the basic nodes. Their corresponding variables (for the flow of loads φ) are denoted by u_{140} upto u_{181}. It is possible to use them in order to set the loads that have to be dispatched (as fixed values of the corresponding variables), i.e., the initial conditions for the dispatch process.

Let the arc estimates in (11), (12) and (13) be denoted by p_1, p_2 and p_3. Then the optimization problem corresponding to the reduced dynamic graph G^1 will have the form:

(14) $$\sum_{i=78}^{99} p_1 u_i - \sum_{i=100}^{141} p_2 u_i - \sum_{i=10}^{31} p_3 u_i \to \max,$$

under constraints corresponding to the equations for flow reservation [2]

(15) $$\sum_{i=1}^{9} u_i = a, \text{ number of available vehicles;}$$

(16) $$\sum_{i=59}^{67} u_i = a;$$

(17) $$u_i - u_j = 0;$$

for the pairs $(i, j) \in \{(1, 17), (2, 16), ..., (8, 10),$
$(31, 18), (32, 19), ..., (43, 30),$
$(24, 60), (25, 61), ..., (30, 66), (44, 67)\}.$

(18) $$u_9 - (u_{31} + u_{45}) = 0;$$

(19) $$(u_{23} + u_{58}) - u_{59} = 0;$$

(20) $$u_i + u_j - (u_m + u_n) = 0;$$

for the quarters $(i, j, m, n) \in \{(10, 45, 32, 46\}, (11, 46, 33, 47), ...,$
$(22, 57, 44, 58)\},$

(21) $$\sum_{i=68}^{76} u_i + \sum_{i=140}^{181} u_i = b, \text{ number of loads;}$$

(22) $$\sum_{i=182}^{204} u_i = b,$$

(23) $$u_i - u_j = 0;$$

for the pairs $(i, j) \in \{(69, 83), (70, 82), (72, 80), (74, 78), (75, 77),$
$(76, 182), (77, 183), ..., (92, 198), (94, 200),$
$(96, 202), (97, 203), (111, 204)\},$

(24) $$(u_i + u_j) - (u_m + u_n) = 0;$$

for the quarters $(i, j, m, n) \in \{(68, 140, 84, 98), (98, 141, 99, 85), (99, 142, 100, 86),$
$(98, 141, 99, 85), ..., (110, 153, 111, 97), (71, 154, 112, 81),$
$(112, 155, 113, 82), (113, 156, 114, 83), ..., (138, 181, 139, 92),$
$(139, 93, 199),$

(25) $$(u_{139} + u_{93}) - u_{199} = 0;$$

(26) $$(u_{125} + u_{95}) - u_{201} = 0.$$

The constraints corresponding to (5), (8) and (9) have to be added to the constrains above given

(27) $u_i \geq 0$, integer, $I = 1, 2, ..., 64$;

(28) $cu_i - u_j \geq 0$, $I = 10, 11, ..., 39, i = 77, 78, 97$;

(29) $u_i \geq 0$, noninteger, $I = 68, 69, ..., 204$.

In order to complete the model, the initial conditions for the flow of haulage units f must be given as values of the respective variables indicating the presence of moving transport vehicles on the route.

The solution of the optimization problem of linear programming thus formulated will enable the control of a transport process with some initial conditions, including the necessary number of haulage units, their loading, their optimal traffic with respect to a given objective function. The experimental investigations of the improved network transport model described, including the solution of practical examples, their comparison with similar models, some inferences and recommendations are an object of another publication.

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Улучшенная сетевая оптимизационная модель для управления транспортных процессов

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(Р е з ю м е)

Предлагается применение теории потоков в графах для целей анализа и управления транспортных процессов. С исходного графа, соответствующего реальной транспортной сети, переходится к динамическому графу, позволяющему описание транспортных процессов во времени. Введена целевая функция и условия, которые уменьшают размерность полученной задачи линейного программирования. Приводится иллюстрирующий пример.