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On One Approach for Modification and Expansion of the Information Interactions Models*

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1. Introduction

The Petri nets (PN) [1], are used for formal description and modeling of parallel and asynchronous interactions in complex systems. They enable the simultaneous consideration of the system structure and of the dynamics (behaviour) of the running processes.

The problem, that attracts constantly the attention to PN applications, is the synthesis of new structures with the corresponding systems parameters One of the possibilities introduced by Kron is is the use of tensor methods for its solving [3]. The last have been introduced by Kron [4] for analysis of such type of nets, as electric circuits and electric machines. The tensor analysis of sets according to Kron can be taken as a base in the modeling of complex systems and applied to their models, described with the help of PN [5]. In this case, according to Kulagin [6], the tensor approach, applied to a given PN model can be reduced to the following sequence of stages:

Analysis: a) decomposition - functionally independent PN fragments are determined; b) The model obtained is “divided” into a set of linear base fragments (LBF), which define the so called reduced system (RS); c) a primitive system (PS) is obtained from the LBF set. The last step of the analysis is the defining of the transforming tensor (TT). For this purpose the reduced and primitive systems are regarded as projections of the generalized system on different coordinate systems.

Synthesis: The synthesis of the new net of Petri (M_{new}) is done by TT and respective operations of union on PS elements.

The digital values for PS and RS are given by Kulagin in [6]. Unfortunately, when solving the equations system, determining TT, the components of the trnsforming tensor have not one solution only. A solution is proposed in [6], defined by heuristic procedure implying the constraints suggested by Kron for (+1, 0, –1) values of the components.

The subject of the present study is the finding of formal conditions for obtaining a single solution of TT components. This problem is solved introducing a weighing matrix

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for PN arcs. The alterations, that have to be introduced in the process of a new PN synthesis, are discussed.

2. A weighting matrix – basic definitions and properties

Let the Petri net be defined as 5 ranked elements:

(1) $$N = (P, T, I, O, \mu_0),$$

where $P = \{p_1, p_2, ..., p_n\}$ is a nonempty set of positions; $T = \{t_1, t_2, ..., t_m\}$ is a nonempty set of transitions, $P \cap T = \varnothing$, I and O are an input and output function, describing the set of transitions in the set of positions and μ_0 is the initial marking [1]. The denotations pre(t) and post(t) are sets of input and output positions of the transition t. An elementary set is defined as a PN, consisting of a transition with one input and one output position – $P = \{p', p''\}$, $T = (t)$ and $p' \in$ pre (t), $p'' \in$ post (t). Let $*d$ and $d*$ are vectors describing the set of input and the set of output positions of the transition t. The elements of these vectors get positive integer values. The vector $d = d* - *d$ gives the connection of transition t with PN positions, and at that $d(i)>0$, if $p_i \in$ post (t); $d(i)<0$ if $p_i \in$ pre(t); $d(i)=0$ if ($p_i \in$ pre(t) U post (t)) where $i = 1, 2, ..., n$. The equation of alteration of PN marking at transition t firing will be:

(2) $$\mu' = \mu_0 + d,$$

where μ is the old vector of PN marking, μ' – the vector of PN marking after transition t is fired.

Let the input and output functions be given by the matrices I and O. Then D is called an incidence matrix, comprising vectors $d_1, ..., d_m$. The last ones describe the set of input and the set of output positions of transitions $t_1, ..., t_m$:

(3) $$D = O - I .$$

The matrix equation of operating (marking alteration) of PN has the form:

(4) $$\mu'' = \mu' + D f(\sigma),$$

where $f(\sigma)$ is an existing sequence of fired transitions (Parih image [1]).

The matrix D_T of the arcs weights in PN [7] is defined as follows.

A basic PN (BPN) is called an ordinary and without @ net, which satisfies the conditions of Kulagin for start and stop positions [6].

Let $l_T(p_i, t_j)$ be the weight of the arc, directed from position p_i towards transition t_j: $l_T(p_i, t_j) \geq 0$ (integer value), and $l_B(p_i, t_j)$ be the value of the incidence function of the same arc, obtained after the transformation of the output PN (when $l_T(p_i, t_j)$ is available) in the BPN [7]. The coefficients $l_T(p_i, t_j)$ of the matrix l_T of the weights of the input function I are defined by the equation:

(5) $$l(p_i, t_j) = l_T(p_i, t_j) + l_B(p_i, t_j).$$

Hence $l_B(p_i, t_j)$ accepts values of zero or one; $l_T(p_i, t_j) = 0$ when $l(p_i, t_j) = 1$ or $l_T(p_i, t_j) = k-1$ when $l(p_i, t_j) = k$ ($k>1$). In case there does not exist an arc between p_i and t_j, the three values are zeroes.

In a similar way the weight of the output arcs between p_i and t_j is determined as:

(6) $$O(p_i, t_j) = O_T(p_i, t_j) + O_B(p_i, t_j).$$

Passing towards the matrix form of PN functioning according to (5) and (6), it is written:

(7) $$\mu'' = \mu' - [\, l_T + l_B \,] f(\sigma) + [\, O_T + O_B \,] f(\sigma),$$

where l_T and O_T are matrices of the weights of the input and output arcs towards PN transitions, and I_B and O_B are matrices of the input and output function of the corresponding BPN.

After some transformations described in detail in [7], the index form ($|\gamma| = n$, $|\beta| = m$) is obtained:

(8) $$\mu''^{\gamma} = \mu'^{\gamma} + (D_T{}^{\gamma}{}_{\beta} + D_{B\beta}{}^{\gamma})\, f(\sigma)^{\beta},$$

Here $D_T{}^{\gamma}{}_{\beta} = O_T{}^{\gamma}{}_{\beta} - I_T{}^{\gamma}{}_{\beta}$ is the common matrix of the weights, and $D_B{}^{\gamma}{}_{\beta} = O_B{}^{\gamma}{}_{\beta} - I_B{}^{\gamma}{}_{\beta}$ is the incidence matrix of BPN. The incidence matrix of the output PN is $D^{\gamma}{}_{\beta}$ and it satisfies equation (4):

(9) $$\mu''^{\gamma} = \mu'^{\gamma} + (D^{\gamma}{}_{\beta})\, f(\sigma)^{\beta},$$

(10) $$D^{\gamma}{}_{\beta} = D_T{}^{\gamma}{}_{\beta} + D_B{}^{\gamma}{}_{\beta}.$$

The check of equation (9) is trivial: if $D_T{}^{\gamma}{}_{\beta} = 0$, i.e. the output PN is ordinary, then the matrix equation of BPN functioning will coincide with that of the output net.

According to the tensor approach a tensor transformation is applied on equation (10) and in the transition from the system of coordinates ${}^{\gamma}{}_{\beta}$ towards the new system of coordinates ${}^{\gamma'}{}_{\beta'}$, the following incidence matrices are obtained:

(11) $$D^{\gamma'}{}_{\beta'} = D_T{}^{\gamma'}{}_{\beta'} + D_B{}^{\gamma'}{}_{\beta'}.$$

The tensor form of PN equations leads to the equations, from which the components of the transformed tensor C can be obtained [5]:

(12) $$D' = C\, D,$$

(13) $$\mu' = C\, \mu_0,$$

where there are two projections of PN in two coordinate systems (CS) respectively – N and N', with incidence matrix D and initial marking μ_0 (for N); and D' and μ_0' (for N') respectively. According to the postulates, the transition from the first towards the second CS is determined by the transforming tensor.

As a result of introducing the weights matrix, a new possibility is achieved thanks to (11) – the transformation of the weighting matrix can be added as another equation instead of equation (13) (the BPN transformation is used as equation (12)), which formally means solving the system of equations:

(14) $$D_T' = C\, D_T,$$

(15) $$D_B' = C\, D_B,$$

The system has only one solution if the arcs of each transition have different weights. Then the TT coefficients are uniquely defined.

In case PN has no multi-arcs, and all the arcs are singular (as in BPN), this corresponds to the degeneration of the system of equations (14) and (15) and the appearance of a set of solutions.

3. Determining of TT coefficients with the help of D_r

Determining of TT coefficients with the help of equations (11)–(15) is illustrated below solving a system of LBF with two transitions. According to tensor methodology, the solutions of LBF with more transitions (of higher order) have a similar form.

The components of the incidence matrix ($D_B{}^{\gamma'}{}_{\beta'}$) are shown in the first two columns in Fig.1 for a LBF of second order, and the components of the weighting matrix $D_B{}^{\gamma'}{}_{\beta'}$ – in the third and forth column (for a RS).

The components of the incidence matrix $D_{B}{}^{\gamma}{}_{\beta}$. are shown in the first two columns in Fig. 2 for the respective primitive system, and the components of weighting matrix $D_{B}{}^{\gamma}{}_{\beta}$ – in column 3 and 4.

The arcs weights are *a, b, c, d* (integer numbers). A solution of the matrix equation *R*=*X***T* is searched for. The problem has only one solution subject to condition

(16) $$a + b \neq 0, \; c + d \neq 0.$$

It can be proved that the condition each transition to have different weights for the input and output arcs is reproduced for LBF of a higher order, which is not shown here for convenience.

Fig. 3a shows the solution of the coefficients of transformation for LBF of second order, while the coefficients *a, b* and *c, d* satisfy condition (16).

$R(R_B \text{II}\, R_T \text{II})$

$$\begin{matrix} -1 & 0 & a & 0 \\ 1 & -1 & b & c \\ 1 & -1 & b & c \\ 0 & 1 & 0 & d \end{matrix}$$

Fig. 1.
RS components

$P(P_B \text{II}\, P_T \text{II})$

$$\begin{matrix} -1 & 0 & a & 0 \\ 1 & 0 & b & 0 \\ 1 & 0 & b & 0 \\ 0 & 1 & 0 & d \end{matrix}$$

Fig. 2.
RS components

X

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$

Fig. 3.
Solution of the system for LBF of second and third order

Fig. 3b shows a solution of LBF of third order. The components of LBF of higher order are with a similar structure.

It is interesting that the solution obtained with the help of the weighting matrix has the same coefficients values as the one suggested in [6] by an heuristic procedure.

4. Weighting matrix in synthesis

The analysis stage is accomplished on the incidence matrix of BPN, i.e. in the absence of multi-arcs. The weights matrix is used in the stage of synthesis of the new PN, the main part of which is the use of union operations on the arcs of the primitive system obtained in analysis.

The union operations and their features have been investigated in [6]. In order to validify one synthesis algorithm, we shall add the following constraints:

– union operations of a node with itself are not allowed;

– union operations of nodes, belonging to one LBF are not allowed;

– repeating of an operation already executed is not allowed.

The input data of the synthesis algorithm are: an incidence matrix of PS – D_{pr} with *r* transitions and 2*r* positions, the number of LBF of PN *h*; the numbers of the transitions in every LBF (together with their coincidence with the numbers of the output model transitions); finite number of union operations $C_{op}(q, s)$ – together with the numbers *q* and *s* of the nodes they include. The results of the operations $C_{op}(q, s)$ are stored in a matrix D_{new}, which is initially equalled to D_{pr}.

In order to satisfy constraint 1, it is always required the indices to be different at $C_{op}(q, s)$: $q \neq s$.

In order to satisfy constraint 2, the belonging of every transition to the corresponding LBF should be determined. The procedure for this will be based on the following prerequisites.

The matrix D_{pr}. contains h in number LBF and $k_1, k_2, \ldots, k_i, \ldots, k_h$ – the number of transitions in each fragment is known. The total number of transitions is r. They will have numbers $t_1, t_2 \ldots, t_r$ which are different from the numbers 1, 2, ..., m of transitions $t_1, t_2 \ldots, t_m$ in the incidence matrix of the initial BPN, since the last ones are not subjected to division in the general case. But the correspondence is known, because it is a result of the division.

The transitions in RS are ranked as follows:

$t_1, t_2 \ldots, t_{k1}$ – belonging to the first LBF;

$t_{(k1)+1}, t_{(k1)+2}, \ldots, t_{k1+k2}$ – belonging to the second LBF;

$t_{k1+k2+k(i-1)+1}, t_{k1+\ldots+k(i-1)+2}, \ldots, t_{k1+\ldots+ki}$ – belonging to the i-th LBF;

$t_{k1+k2+\ldots+k(h-1)+1}, t_{k1+\ldots+k(h-1)+2}, \ldots, t_{k1+\ldots+kh} = t_r$ – belonging to the h-th LBF.

The elementary transitions in D_{pr} are ranked in the same way.

The positions in D_{pr} are also ranked. The transition t_i is assigned a starting position with number $2i-1$ ($p(2i-1)$) and a stop position with number $2i$. If transition t_j belongs to the i-th LBF, it is accepted that its positions in D_{pr} also belong to this fragment respectively.

Hence for the positions and the transitions in the primitive system the record $k_1, k_2, \ldots, k_i, \ldots, k_h$ can be used. The belonging record – which transition belongs to a given fragment, has the form:

1, k_1 – number of the first and the last transition in the first LBF;

k_1+1, k_1+k_2 – number of ... in the second LBF;

$1+\sum_{i=1}^{w-1} k_i$, $\sum_{i=1}^{w} k_i$ – number of ... in the w-th LBF;

$1+\sum_{i=1}^{h-1} k_i$, $\sum_{i=1}^{h} k_i$ – number of ... in the h-th (last) LBF;

If A_w is the sum of the transitions of the first w LBF:

$$A_w = \sum_{i=1}^{w} k_i,$$

k_i – number of transitions in k-th LBF.

Let us write the row $A_1, A_2, \ldots, A_{i-1}, A_i, \ldots, A_h$.

Consequence 1. The necessary and sufficient condition to find the transition t_j from D_{pr} in the i-th LBF is to satisfy the condition:

$$j > A_{i-1} \text{ AND } j \leq A_i .$$

Let us write the row $2{*}A_1, 2{*}A_2, \ldots, 2{*}A_{w-1}, 2{*}A_w, \ldots, 2{*}A_h$.

Consequence 2. The necessary and sufficient condition to find the position p_i from D_{pr} in the w-th LBF is to satisfy the condition:

$$i > 2{*}A_{w-1} \text{ AND } i \leq 2{*}A_w.$$

With the help of these two consequences the presence of constraint 2 is checked. The check of constraint 3 consists of finding an element in D_{new}, which has a value different from +1, 0 or –1.

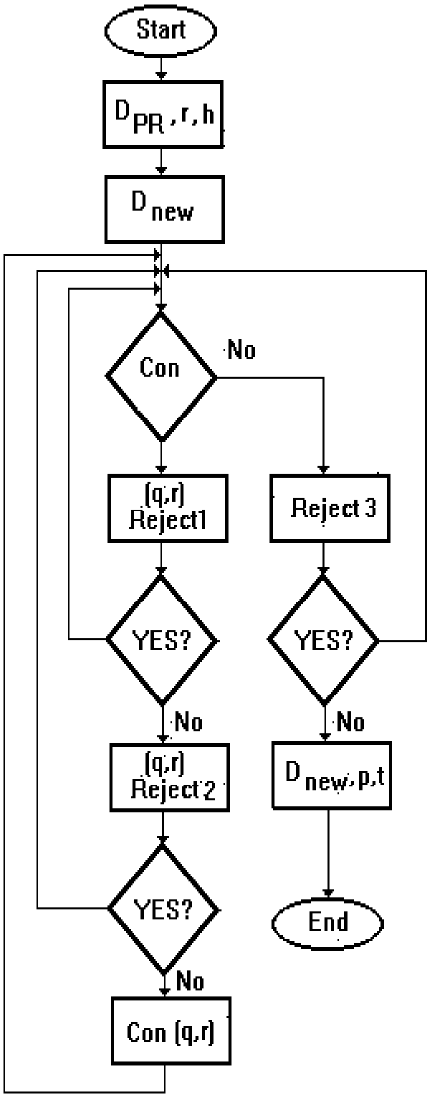

Fig. 4

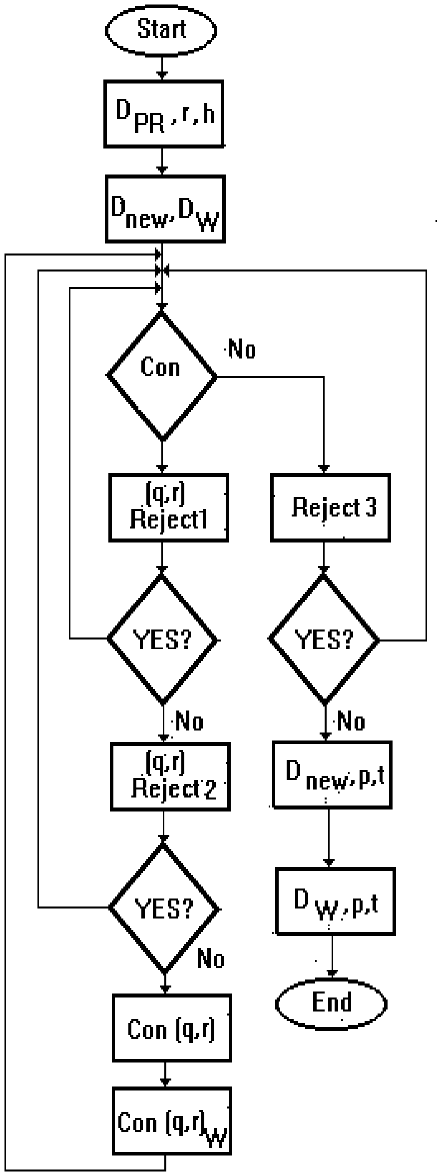

Fig. 5

The block-diagram of the synthesis algorithm is shown in Fig. 4.

Modification of this algorithm is necessary in order to use the weigth matrix as well. The last one will be entered in the input data as D_w. After that each permitted operation $C_{op}(q, s)$ will be executed on the elements of D_{pr} as well as on the elements of D_w, i.e. no changes are necessary in the algorithm, just an addition, as shown in Fig. 5. At last there follows summing of D_w and D_{new}, multiplying of the new matrix by TT coefficients and removing of the coinciding rows or columns.

As a result the incidence matrix of the new PN is obtained, which will not be a BPN, but a generalized PN (with multiarcs), which has been the purpose of weights matrix introducing.

5. An example of weighting matrix application

The approach discussed concerning a weighting matrix use enables the extending of the modeling capacities of the tensor methods applied in Petri nets. After the synthesis a new model is obtained that formally covers the possibilities from the class of ordinary to the class of generalized PN without примки. It is appropriate to apply it when the interactions being modeled are adequately represented by multi-arcs.

The possibilities of weighting matrix application are shown modeling by PN telecommunication interferences – the phase of connection establishing. According to OSI philosophy, the model for data exchange which is recommended, is by connection establishing. In Recommendation X.120 [9] the interaction between two protocol objects of N-layer is described with the help of four primitives – request (Req), indication (Ind), response (Res), confirm (Con) that are exchanged by the N–1 layer service.

The graphical PN model of connection establishing between two protocol objects is shown in Fig. 6. This is asymmetrical interaction since the first object wants connection, while the second one responds. Table 1 gives an interpretation of the positions and transitions from PN in Fig. 6, where SDU is a service data unit, and PDU – a protocol data unit.

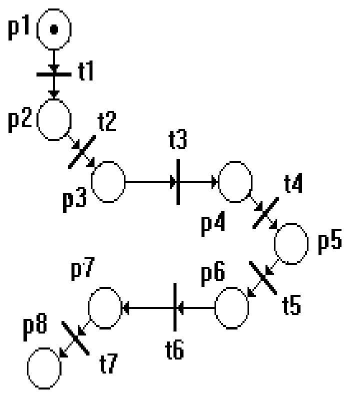

Fig. 6. PN model of asymetric connection

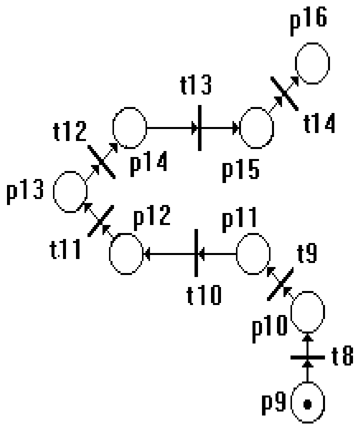

Fig. 7. Second PN model of asymetric connection

In Recommendation X.210 [10] it is noted for the primitive confirm that it is not in direct relation with the rest. That is why it is detached in a separate LBF, as well as t1 (which does not reflect a primitive).

In case, when a balance scheme (symmetrical procedure) is searched for to establish connection between two equivalent protocol objects, i.e. any of them can request connection establishing, there will be a second PN respectively (shown in Fig. 7). The second PN has positions p9-p16 and transitions t8-t14 (p9 being equivalent to p1, t8-to t1 and so on).

Table 1. Correspondence of the conditions and events in the output PN

p1	procedure start	t1	transmission of a request for connection
p2	request for connection	t2	transmission of the primitive Req
p3	transformation SDU → PDU	t3	transmission of the protocol block PDU
p4	transformation PDU → SDU	t4	transmission of the primitive Ind
p5	resource separation	t5	transmission of the primitive Res
p7	transformation PDU → SDU	t6	transmission of the primitive block PDU
p8	answer received	t7	transmission of the primitive Con

If the nets from the two figures are regarded as one common interaction, i.e., PS is a sum of LBF of the two PN and the matrix of PS will have dimension [14 columns, 28 rows]. The respective operations of union are executed on the incidence matrix of PS. The incidence matrix of PS obtained, is multiplied by the coefficients of the transforming tensor and after the equivalent elements are cancelled the final variant of the new PN is obtained, i.e. the matrix of incidence M_{new} has the form:

tNEW	1	2	3	4	5	6	7	8	9	10	11	12
pNEW1	-1	0	0	0	0	0	0	0	0	0	0	0
2	1	-1	0	0	0	0	0	0	0	0	0	0
3	0	-1	0	0	0	0	1	0	0	-1	1	0
4	0	1	-1	0	0	0	0	0	0	0	1	0
5	0	0	1	-1	0	0	0	0	0	0	0	-1
6	0	0	0	1	-1	0	0	0	0	0	0	0
7	0	0	0	0	1	-1	0	0	1	0	0	0
8	0	0	0	0	0	1	-1	0	0	-1	0	0
9	0	1	0	0	0	0	-1	0	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0	1	0
11	0	0	0	0	0	0	0	-1	0	0	0	0
12	0	0	0	0	0	0	0	1	-1	0	0	0
13	0	0	0	-1	1	0	0	0	-1	0	0	1
14	0	0	0	0	0	0	0	0	0	1	-1	0
15	0	0	0	0	0	0	0	0	1	0	0	-1
16	0	0	0	0	0	0	0	0	0	0	0	1

Its analysis can show that the net obtained is active and safe.

This is a new structure of interactions. The weights matrix enables the modeling of messages duplication in the communication channel.

For the purpose the transition t3 and the primary interaction from Fig. 6 is assigned weight 1 of the initial arc. This weight is entered in the weights matrix. The same operations are accomplished on it as in the obtaining of Mnew and the matrix obtained is added to the last.

The graphical type of PN, corresponding to messages duplication, is shown in Fig. 7. This PN is also active but not safe, it is 2-limited which corresponds to the interaction entered (duplication-doubling). Unserviced request for connection will remain at position p4, but this is the doubled request, i.e., the interaction scheme obtained reacts adequately to the action introduced. The mechanism of resources control has the task to remove it, but this is the subject of future investigations.

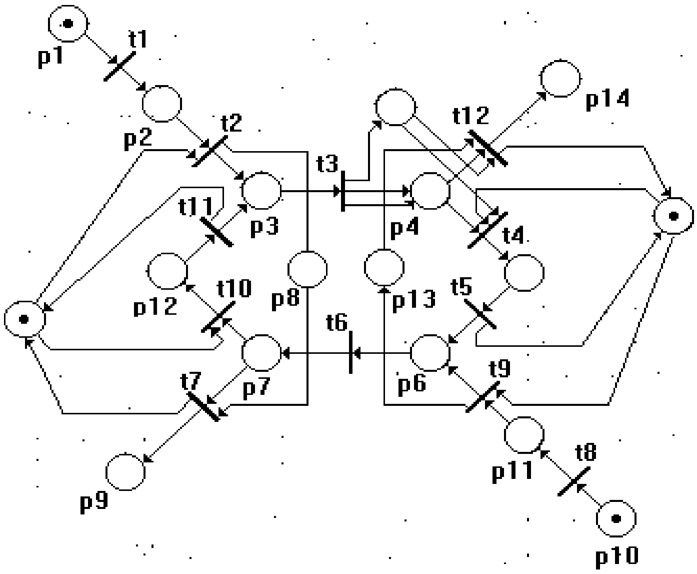

Fig. 7. Graphical representation of PN processing messages duplication

6. Conclusion

The application of tensor methodology in the analysis and synthesis of net models of the information interactions gives a general approach for the obtaining of nontrivial mechanisms in a formal way. The introduction of the matrix of arcs weights requires minimal supplements to the algorithm of synthesis and expands the modeling possibilities.

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Один подход к модификации и расширению моделей информационных взаимодействий

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(Р е з ю м е)

Рассматриваются вопросы моделирования информационных взаимодействий сетями Петри. Для получения новых структур взаимодействий используется тензорный подход. Дефинирована матрица весов дуг сети Петри в его контексте. Выведены условия еднозначного определения коефициентов трансформирующего тензора. Показано, что в алгоритме синтеза новых моделей нужно внести минимальные дополнения. Возможности модификации иллюстрованны на примере моделирования телекоммуникационной процедуры. Получена симметричная схема взаимодействия для канала с дублированием сообщений в одном направлении.