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On One Approach in Justapositions Application in Nondeductive Inference*

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1. Introduction

When several selected notions – investigation objects are discussed, the use of justapositions is characterized by its simplicity. Denote $A_1, A_2, \ldots, A_n \in \{A\}$ as features (or properties), characterizing the set of all the objects justaposed $C_1, C_2, \ldots, C_k$. As a result of the justapositions a matrix Z is formed with dimensions $k \times n$, such that each of its elements $z_{ij} \in Z$ is assigned the value 1 in case the feature A_i belongs to the object C_j or 0 (in case it does not belong). It is also not difficult to introduce a value for z_{ij}: ("yes" or "no": indefinite truth value), leading to the application of ternary logic in justapositions formalization. Truth values of the type "neither yes, nor no" with elements of four-valued or reduced to it logic are beyond the investigation area. Regardless of the simplicity in the description and operation with justapositions, their use in different inference types can be quite efficient. Let the notion nondeductive inference be used, that unites the notions inference by analogy and inductive inference. Examples of the interaction between these two types of inference are found in research papers on inductive and probabilistic logic and on analogy also [1–7]. For example D.M.Keynes [8] has shown that Mill's (Eliminating) induction can be represented as recomputing induction plus analogy. When the schemes of nondeductive inference methods are compared, some repeating elements that play an important role in every one of them can be mentioned.

The justaposition tools enter all these schemes and this is one of the reasons for their study. There are two more purposes for the investigation:

I. The development and analysis of justapositions helps finding the connection between different nondeductive approaches.

II. Since the justapositions are the most simple units in any nondeductive approach, the study of the constraints implied on the approach as a whole can start with them. But the complexity of the inference in the transition from justapositions towards the approach

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as a whole can increase considerably and in some cases the specifics of a part of the approach cannot be transferred to the approach as a whole.

The paper discusses the justapositions tools referring to elements of sets, realized in inference by analogy and in inductive inference also. Though this apparatus has been used in the first attempts to use nondeductive inference in artificial intelligence (AI) [9], it still remains in these applications. The justapositions are weakly represented even in encyclopedia literature about AI [10, 11] or in decision making systems [12]. Anyway, the omission of justapositions (and counter positions) leads to incorrect transformation of knowledge and inference. The next three chapters represent three types of inference and the use of justapositions in them.

2. Justapositions in analogy by features

The analogy by features (which is also called Epicurean analogy [5]) is easy for formalization and combines all the characteristic features of the inference by analogy with its simple description. In order to simplify the inference diagram represented, it is conditionally divided into two variants.

a. Let the object B has features from the set $\{A_i \mid i \in D_0\}$ where D_0 is the set of indices of patterns describing B, for example $D_0 = \{2, 5, 11, 12\}$. Let also the object C_i be characterized by a part of these patterns $\{A_i \mid i \in D_1\}$, at that $D_0 \subset D_1$ and in this way the sets of features of n similar objects are given $\{C_k \mid k = 1, 2, \ldots, n;\ D_k \subset D_0\}$, that are compared to B. Then the rule for analogy by features [13] can be interpreted in the following way: "If the object B has features $\{A_i \mid i \in D_k\}$ (coinciding features), it is possible that it possesses sufficient patterns $\{A_i \mid i \in D_0 \backslash D_k\}$ as well, which is formalized as:

$$(1)\qquad \frac{B \leftarrow \bigcap_{i \in D_0} A_i, \quad C_k \leftarrow \bigcap_{i \in D_k} A_i}{C_k \leftarrow \bigcap_{i \in D_0 \backslash D_k} A_i}\ (\psi_k, B, C_k)$$

where $\cap$ denotes conjunction between the logical objects, ψ_k – partial similarity between them, which is straight proportional to the proximity between the objects B and C_k, estimated in the hierarchical network of objects, based on the use of relations of "predecessor - ancestor" type.

b. Let the objects $B_1, B_2, \ldots, B_m$, have patterns from the respective sets $D_{01}, D_{02}, \ldots, D_{0m}$ and $D_0 \subset D_{01}, D_0 \subset D_{02}, \ldots, D_0 \subset D_{0m}$. Let also the object compared – C_k, possesses such patterns $\{A_i \mid i \in D_k\}$ that $D_k \subset D_0$. Then in the comparisons between B_{0j}, $j = 1, 2, \ldots, m$, and C_k, the inference diagram accepts the form:

$$(2)\qquad \frac{B_{0j} \leftarrow \bigcap_{i \in D_{0j}} A_i, \quad C_k \leftarrow \bigcap_{i \in D_k} A_i}{C_k \leftarrow \bigcap_{i \in D_{0j} \backslash D_k} A_i}\ (\psi_k, B_{0j}, C_k).$$

In pairwise comparisons between objects, diagrams (1) and (2) coincide. ψ_k has an important role in the diagram since on the basis of this estimate many of the objects are eliminated as being not close enough to knowledge transformation (in the case considered – features) by analogy. There is a possibility to control the process of hypotheses generation defining the minimal threshold value for a set of coinciding features – card (D_x), where $D_x \equiv D_0$ or $D_1 \equiv D_x$, card is usually the power of a set and in this case it is reduced to the counting of elements D_x. The alterations ψ_k do not hamper the formation of incorrect hypotheses and incorrect knowledge, that is why all the hypotheses are passed through two "***filters***", where many of them are screened as invalid. First the noninconsistency of the hypotheses formed is checked with the help of their **justaposition** with all the knowledge available for the object, to which the hypothesis given is referred to (this object is called **goal of the transformation** by analogy). The consistent hypotheses are compared with knowledge from the goal of transformation. The whole complex system of hypotheses check and rejection and of the objects compared does not guarantee even satisfactory validity of the hypotheses obtained by analogy (this refers not only to the analogy by features).

Let $z_{ip} \in Z$ for the patterns justaposed $A_1, A_2, \ldots, A_n \in \{A\}$ and the objects B_{0j}, C_k form a matrix of dimension $(k+j) \times n$, and each element from z_{ip} be assigned the value 0 or 1.

The realization of justapositions enables the separation of $\{A\}$ into three nonintersecting subsets: $\{A^{(0)}\}$, $\{A^{(1)}\}$ and $\{A^{(2)}\}$, where $\{A^{(0)}\}$ includes all the features, possessed by all the objects justaposed, $\{A^{(1)}\}$–patterns A_i, for which there exists at least one object p, for which $z_{ip}=0$. By definition, $\{A^{(0)}\}$ and $\{A^{(1)}\}$ can be empty sets, but if they are simultaneously empty, this indicates shortcomings in subject area modelling, i.e. the objects are not well enough described. In a contrast to them, $\{A^{(2)}\}$ is a set of *distinguishing features* for the object–it cannot contain less than $k+j$ elements (not less than one per each column); otherwise the model of the object area is not sufficiently complete. When increasing the number of the objects justaposed (k=2, 3, 4, ..., x), $\{A^{(0)}\}$ is decreased on the account of the increase of $\{A^{(2)}\}$ and $\{A^{(1)}\}$. In further investigations such minimal quantity of the terms $\{A^{(0)}\}$ is searched for, which assures the obtaining of nondeductive inference and also the dependence of this number on k and on the type of the object area. The transformation of knowledge by analogy is more suitably done by pairwise comparison of the objects, i.e. the basis for justaposition (card($\{A^{(0)}\}$)) being the largest at that. With the increase of k, the justaposition provides more favourable conditions for operation with an expert in the process of knowledge acquisition. For example, if only two objects are considered, then $\{A^{(2)}\}$ has to be apriori given, but if the objects exceed five and in a row i just one z_{ip} accepts the value l, then a hypothesis $A_i \in \{A^{(2)}\}$ may be suggested. The simple justapositions enable the formation of questions of "HOW" and "WHY" type, put to the expert or (when some "experience" is available) for automatic operation. For example, why in the row l almost all z_{ij}=0 with the exception of one z_{ip}=1? In this way the attention is concentrated on the study of the object p explaning the reason for $z_{ip} \neq 0$. It is not excluded that z_{ip}=0 (incorrect knowledge) or that interconnection with another pattern or another reason is found, and finally an object area with new knowledge is generated. If cases are considered where z_{ij} can accept the value ?, new possibilities appear for operation with the expert. For example, if one of the rows (i) from Z contains elements with values "?" or "l" only, then a hypothesis can be formed: $A_i \in \{A^{(0)}\}$ and so on. Let the following inconsistency coefficients be introduced: $0 \le k_0 \le 1$, $0 < k_1 \le 1$, $0 < k_2 \le 1$, and "attached" to $\{A^{(0)}\}$, $\{A^{(1)}\}$ and $\{A^{(2)}\}$ where $k_0 < k_1 < k_2$. Any analogies with the coefficients $0 \le \alpha \le 1$ introduced in heuristic or probabilistic way and attached to each pattern of **every** object would be incorrect due to the deep difference in the semantics of the operations accomplished on the object area model.

An example is given below that provides an idea about the advantages of the approach. Without justapositions, the analogy by features (1) or (2) may lead to the formation of a hypothesis that an object from the goal of transformation possesses a distinguishing feature of another object (from the transformation basis), which is incorrect in the general case. These results cannot be avoided by the complex introduction and alteration of the weight coefficients α_i, of the significance factor or similar estimates of subjective character and the inconsistent hypotheses of this type may pass through all the stages of screening. Further on investigation of more complex diagrams of justapositions will be assumed, in which the interconnection between the separate patterns or groups of features is traced, and $\{A^{(1)}\}$ is separated into nonintersecting subsets and constraints on the patterns transfer are implied.

3. Other analogies

In more elaborate types of inference, there are more advantages found in justapositions realization than in operation with analogy by features. In every one of the early investigations on inference by analogy in formal systems [14], each of the objects is assigned its model, consisting of facts ("pattern A belongs to object B" – partial case of the fact) and rules of the type $G \leftarrow \bigcap_{i \in D_0} H_i$, where $\bigcap H_i$ is a reason (rule body) and G – consequence (head of the rule, consisting of one statement only in Horn's rules), D_0 – final set of the conjuncts in the rule body. As a result of the operation of the inference formal mechanism, from the model M_1, connected with the base of transformation, a rule of the type $G \leftarrow \bigcap_{i \in D_0} H_i$, is added in model M_2 connected with the goal of transformation as a fact, i.e. new knowledge is added to M_2 under the condition satisfying the partial coincidence ψ between the objects (EPIC rule from [14]). The apparatus of inference by analogy in Haragushi is more complex than the analogy by features. But the comments above given about the impossibility of the distinguishing features transfer remain in power (in a modified form): if G is false outside M_1, then its transfer to M_2 may lead to incorrect knowledge.

In the diagram represented some shortcomings are added to the ones expressed in chapter 2, that can be avoided only by a justaposition between the objects models. For example, let the object area considered, is ornithology and the object from the transformation base is flamingo. Independently on the selection of the object from the goal of transformation (for example neighbouring class, goose), the following example of incorrect knowledge transformation is represented. It is known that the pink colour of flamingo feathers may become quite pale and under some circumstances even disappear during their long stay in zoos. The reason for that is the absence of small crabs in the food. If this interconnection of reason-sequence character (crabs food – pink colour) is applied to another object, including birds living under natural conditions (salty lakes or sea bays, warm climate, pink colour of the feathers), then the transfer of this reason-sequence connection would be incorrect. This connection is characteristic (it is defining) for the flamingo, which is easily demonstrated with the help of justapositions.

In the literature on information technologies it is accepted to discuss the methodological role of the approaches considered. However the limits of justapositions application in logical inference are hardly overcome. This is so complex and sometimes even meaningless as the question: why is the inference by analogy necessary? In some well developed and already "classical" object areas, where the knowledge incompleteness is diminished to minimum, the role of analogy is reduced mainly to the description of notions or of the inference process and the purpose of its application is the description compression only. On the other hand, in the "not developed" scope, when there is less knowledge about the object available, the analogy is often used, but it can lead to inconsistent hypotheses. In a similar way, the wide use of justapositions weakens the role of nondeductive technologies. Nevertheless we must not counterset the justapositions and nondeductive methods. A common feature of all these methods and approaches is the avoiding of the incompleteness in the object area model, which enables the connection of justapositions tools with analogy and induction.

4. Justapositions in inductive inference

Usually the basis of inductive reasoning contains information about some investigated terms in the class, confirming or making a probable conclusion, which refers to the whole class or only to a part of the terms investigated. The problematic character of the connection between the basis and inductive inference gives the opportunity to use methods of probability theory in the construction of induction logic and nondeductive methods as a whole. All the attempts to explain logic probability by a degree of faith, even reasonable (as Keynes [8] does this), do not achieve the purpose. The approach of the authors who base on the notion degree of confirming one statement by another in inductive logic is much more efficient. This approach itself is grounded on semantic analysis of the statements. Let the degree of confirmation be denoted by τ. The essential declaration which establishes the probabilistic relation between statements, for example between the hypothesis H and its empirical evidences E, will be symbolically expressed as: $\tau(H/E)=p$, where p is any number within the segment $1 \geq p \geq 0$.

In such an approach the subjectivity of the confirmation degree $\tau(H/E)$ is the weakest place. In the situation shown the justapositions earlier described help the formation of the confirmation degree. Thus since in the second chapter some justapositions were given which can be expressed by frequency tools in probability theory and at the same time $\tau(H/E)$ is done with the help of logic probabilistic tools, it is assumed that the justapositions in induction are a territory, where the supporters of the frequency, as well as of logic interpretation will find a common language.

Let the semantics of the sets $\{A\}$, $\{A^{(0)}\}$, $\{A^{(1)}\}$ and $\{A^{(2)}\}$ be altered in the following way: $\{E\}$ consists of all the evidences for the set of justaposed hypotheses $\{H^*\}$ where $H \in \{H^*\}$; $\{E^{(0)}\}$ contains evidences coinciding for all $H \in \{H^*\}$; $\{E^{(2)}\}$ - evidences confirming only one of the hypotheses $H \in \{H^*\}$ and at the same time being defining for H; $\{E^{(1)}\}$ - the rest of the evidences in $\{E\}$. One coefficient $k_0 < k_1 < k_2$ is entered for each of the sets. Their value may alter in definite limits depending on the selection of the object area. In the ornithologic example above given the relation k_2/k_1 and k_1/k_0 is quite larger than in medicine. However if a parallel is made with the inductive degree of confirmation, then the difference between the formation of k_j and $\tau(H/E)$ becomes obvious: k_j allows oscillations in definite bounds, but it does not change when fixing the object area, and $\tau(H/E)$ accepts values from the whole interval [0,1]. The justaposition coefficients are just three, and the degree of confirmation – continuum. In order to define the degree of confirmation in inductive inference, formula (3) is offered:

$$\tau(H/E) = \sum_{i=1}^{p} \sum_{j=0}^{2} E_j^H k_j \tag{3}$$

where $\{E_j^H\}$ are all the evidences known in connection with H, $k_j \in \{k_0, k_1, k_2\}$ and the value k_j are determined depending on that, to which one of the sets $\{E^{(0)}\}$, $\{E^{(1)}\}$ or $\{E^{(2)}\}$, E_j^H belongs.

In connection with the investigations presented there remains the problem that in order to obtain a valid hypothesis it is not necessary to satisfy equation $\tau(H/E)=1$.

Really, in order to confirm H, it is necessary to confirm **all** the defining evidences for H from $\{E^{(2)}\}$ and only **some** of the evidences from $\{E^{(0)}\}$ and $\{E^{(1)}\}$. In other words, when forming $\tau(H/E)$ using formula (3), and if $\tau(H/E)=1$, the hypothesis is overconfirmed which does not invoke supplemental difficulties.

5. Conclusion

Some properties of nondeductive inference are presented, that decrease the consistency of the hypotheses formed. One of the examples is when the knowledge transformation (or the connections between knowledge) from one object of research to another is forbidden since the knowledge is characteristic for one of the objects only.

An approach is suggested for the case discussed based on justapositions of the available facts. Two modifications of its realization are considered – inference by analogy and inductive inference.

References

1. Cyburg, G. Probability and Inductive Logic (in Russian). Moscow, Progress, 1978.
2. Carnap, R. Value and Necessity (in Russian). Moscow, Inostrannaya literatura, 1959.
3. Markov, Z. Inductive Methods for Machine Learning (in Bulgarian). TEMPUS JEN 1497, 1996.
4. Ruzavin, G. Semantical conception of inductive logic (in Russian). –In: A. Tavanetz (Ed.) Logical Semantics and Modal Logic (in Russian). Moscow, Nauka, 1967, 84-100.
5. Uemov, A. Analogy in Practice of Scientific Investigation (in Russian). Moscow, Nauka, 1970.
6. Machine Learning. R. Michalski, J. Carbonell, T. Mitchell. (Eds.). Vol. II. Los Altos, CA, Morgan Kaufmann, 1986.
7. Rescher, N. Induction. An Essay on the Justification of Inductive Reasoning. Oxford, Basil Blackwell, 1983.
8. Keynes, D. M. A Treatise on Probability. London, 1952.
9. Nilsson, N. Artificial Intelligence (in Russian). Moscow, Mir, 1985.
10. The Handbook of Artificial Intelligence. Eds. P. Cohen, E. Feigenbaum. Vol. I-III. Los Altos, CA, William Kaufmann, 1982.
11. Encyclopedia of Artificial Intell. Ed. E. Shapiro. Vol. I-II. New York, John Wiley and Sons, 1987.
12. Saaty, T., L. Vargas. Decision Making (with the *Analytic Hierarchy Process*). Vol. VII. Pittsburg, RWS Publ., 1994.
13. Polya, A. Mathematics and Plausible Reasoning (in Russian). Moscow, Inostrannaya Literatura, 1964.
14. Haraguchi, M., S. Arikava. Reasoning by analogy as a partial identity between models/ Lecture Notes in Computer.

Об одном подходе к использованию сопоставлений при недедуктивном выводе

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(Резюме)

Рассматривается роль сопоставлений при различных типах вывода по аналогии и индуктивного вывода. Предложены постановка и подход, приводящие к повышению достоверности выводимых гипотез и упрощению оперирования с формальным аппаратом в целом, и повышения эффективности недедуктивного вывода.