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On the Performance Indexes for Robot Manipulators

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1. Introduction

The estimation of the performance of manipulators is important to manipulators application and design. Dexterity, manipulability, accuracy and some others determine the performance characteristics of robot manipulators. One of the performance characteristics is so called service angle, a concept first introduced by V i n o g r a d o v et al [7]. The service angle is defined as the total range of the approach angle of a manipulator around a point of the workspace. A similar approach was used by Y a n g and L a i [8]. They studied the service angle to a given point and introduced service sphere and free service regions. K u m a r and W a l d r o n [4] introduced the concept of dextrous workspace, defined as a volume within which every point can be reached by the manipulator end-effector with any desired orientation. As another measure for manipulator performance can be used the evaluation of the determinant of the Jacobian. On this base Y o s h i k a w a [9] introduced the term manipulability, which involves the Jacobian and its transpose. P a u l and S t e v e n s o n [5] estimated the kinematic performance of a spherical wrist by using the absolute value of the determinant of the Jacobian. Another performance index was proposed by S a l i s b u r y and C r a i g [6], i.e., the condition number of the Jacobian. A n g e l e s [1] discussed manipulability and conditioning number and on this base considered the isotropy in machines. K l e i n and B l a h o [3] consider four measures for dexterity: determinant, condition number, minimum singular value and joint range availability.

In the present paper, the dexterity index, manipulability, condition number and minimum singular value are considered. The indexes are applied to a SCARA type robot (Fig.1) and the results are graphically presented.

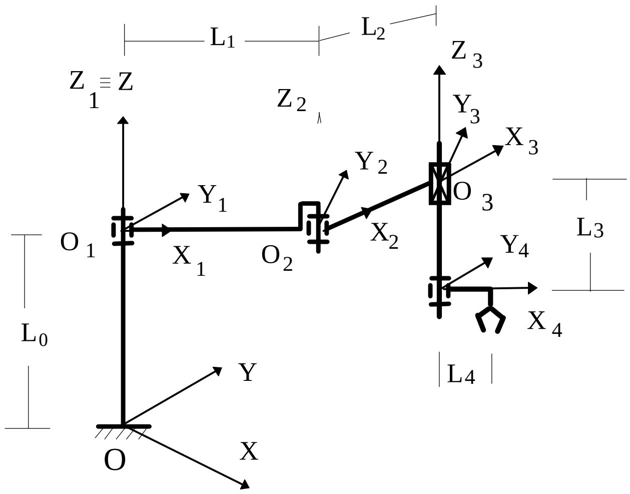

Fig.1. SCARA type robot

2 Dexterity index

Dexterity index is a measure of a manipulator to achieve different orientations for each point within the workspace. The orientation of the end-effector of a manipulator can be described by the equivalent rotation matrix $R_{rpy}(\gamma,\beta,\alpha)$ using roll, pitch and yaw angles, i.e.,

$$R_{rpy}(\gamma,\beta,\alpha) = \mathrm{ROT}(\hat{Z}, \alpha)\mathrm{ROT}(\hat{Y}, \beta)\mathrm{ROT}(\hat{X}, \gamma) \tag{1}$$

The angles α,β and γ can vary within the range of $(0\div2\pi)$. We introduce *X, Y* and *Z* dexterity indexes, which can be written as follows:

$$d_x = \frac{\Delta\gamma}{2\pi} \tag{2}$$

$$d_y = \frac{\Delta\beta}{2\pi} \tag{3}$$

$$d_z = \frac{\Delta\alpha}{2\pi} \tag{4}$$

where $\Delta\alpha$, $\Delta\beta$ and $\Delta\gamma$ are the possible range of variation of the roll, pitch and yaw angles for each point of the workspace. Thus the dexterity index can be defined as:

$$D = \frac{1}{3}(d_x + d_y + d_z). \tag{5}$$

The dexterity index (D) can vary within the range of $(0\div1)$. If the dexterity index is equal to unity we will say that the manipulator has full dexterity at a particular point or an area. A full *X*-, *Y*- or *Z*- dexterity can exist as well, i.e., if $dx = 1$ we can call it full

X-dexterity. For example *X*- and *Y*- dexterity are zero (dx=0, dy=0) in case of SCARA type robot.

The following algorithm for determination of the dexterity index is proposed:

- for a point of the workspace of the manipulator vary the roll, pitch and yaw angles (γ,β,α);
- solve the inverse position problem;
- check whether the obtained joint coordinates are within the range of variation;
- determine the possible range of variation for the three angles γ,β and α;
- compute the dexterity index for this point.

This algorithm can be applied for every point of the workspace. Using the above-mentioned algorithm several graphs for the dexterity of the considered SCARA type robot have been obtained. In this case we mean *Z*-dexterity index. In Fig.2 is shown a 3-D map of the dexterity over the workspace of the robot. Fig.3 shows several areas with different dexterity index. The area with dexterity index 1 corresponds to the dextrous workspace. In Fig. 4 is shown a 3-D graph of dexterity index over the workspace of the robot. The design parameters of the robot are as follows: L_0=500 mm, L_1=400 mm, L_2=250 mm, L_4=150 mm. The ranges of variation of the joint coordinates are: θ_1=-110 deg÷90 deg, θ_2=0 ÷155 deg, θ_3=0 ÷ 360 deg, L_3=0÷150 mm.

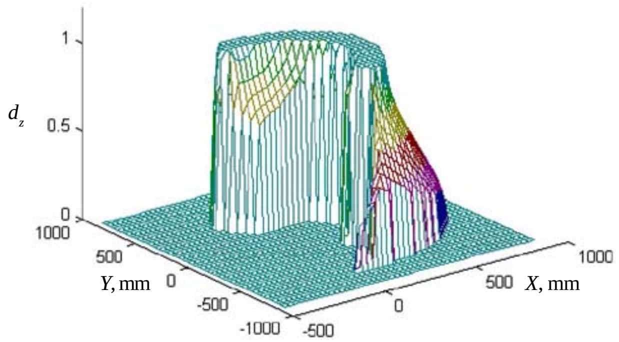

Fig. 2. Dexterity index over the workspace

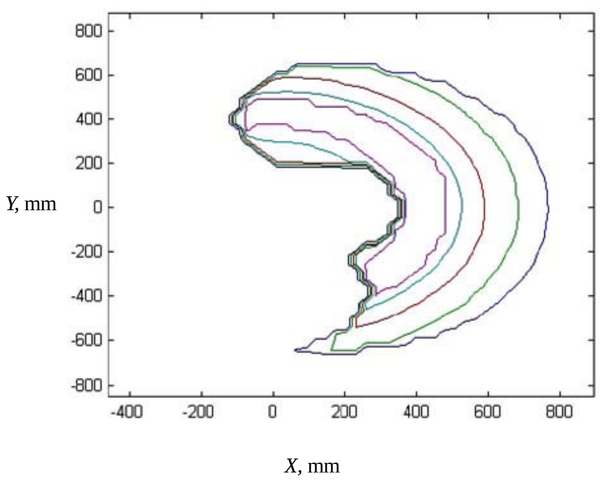

Fig. 3. Areas with different dexterity index

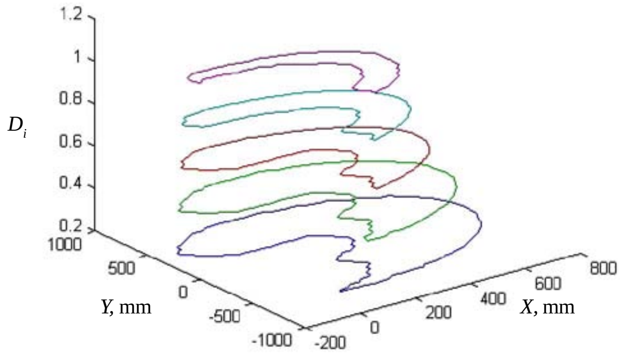

Fig. 4. 3-D graph of areas with different dexterity index

In fact, Figures 2, 3 and 4 give a clear idea of the dextrous and reachable workspaces of the manipulator.

A mean dexterity index can be introduced and can be defined as:

$$D_m = \frac{\sum_1^n D}{n}, \tag{6}$$

where n is the number of the points for which the dexterity index is computed. The obtained mean dexterity index for the considered manipulator using 682 points is $D_m = 0.6590$.

3 Manipulability

The concept of manipulability of a manipulator was introduced by Yoshikawa [9]. The manipulability is defined as the square root of the determinant of the product of the manipulator Jacobian by its transpose, i.e.,

$$\mu = \sqrt{\det(\mathbf{JJ}^T)}. \tag{7}$$

The manipulability μ is equal to the absolute value of the determinant of the Jacobian in case of square Jacobian. Using the singular value decomposition the manipulability can be written as follows:

$$\mu = \sigma_1\sigma_2\ldots\sigma_r , \tag{8}$$

where σ_i are singular values of the Jacobian (see next section).

The value of the determinant depends on the used units, i.e., the manululability index will have different values for the different used units. Because of that, the determinant cannot give a practical measure of the degree of ill-conditioning [2] of the Jacobian. Therefore it is convenient to apply the normalised mobility index, which can be written as follows:

$$\mu_n = \frac{\mu_1\mu_2 \cdot\cdot\cdot \mu_r}{\max\{\mu_1\mu_2 \cdot\cdot\cdot \mu_r\}}, \tag{9}$$

where $\max\{\mu_1\mu_2 \cdot\cdot\cdot \mu_r\}$ is the largest manipulability index within the workspace of the manipulator which is obtained by using equation (8).

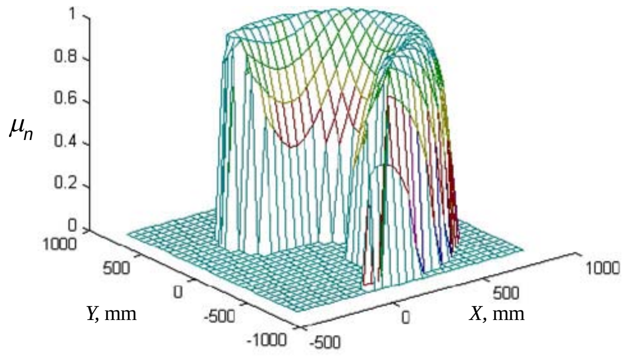

Fig. 5. The normalised manipulability index

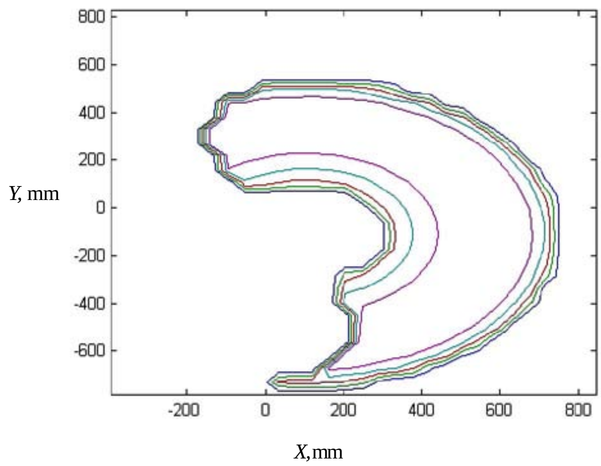

Fig. 6. Areas with different manipulability index

The normalised manipulability index is bounded between 0 and unity. In Fig.5, Fig.6 and Fig.7 is shown the normalised manipulability index over the workspace with constant orientation for the considered SCARA type robot. The manipulability graphs for the other orientations are similar.

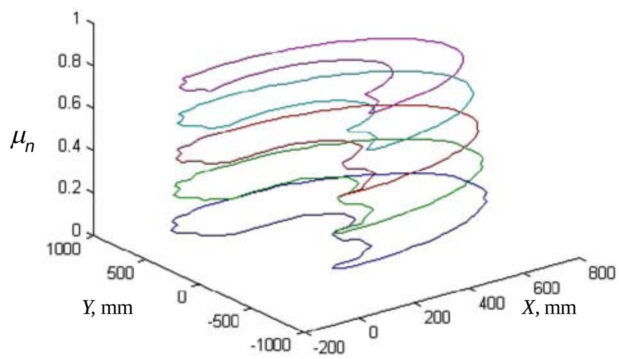

Fig. 7. 3-D graph of areas with different normalised manipulability index

4 Condition number

When the determinant of the Jacobian is equal to zero, it means that the manipulator approaches singularities. However, the actual value of the determinant cannot be used as a practical measure of the degree of ill-conditioning [2]. For this purpose it is convenient to use the condition number of the Jacobian. It is well known from the singular value decomposition theorem that an arbitrary matrix can be represented as follows [2]:

(10) $$A = USV^T,$$

where U is a mxm orthogonal matrix; V is a nxn orthogonal matrix; S is a mxn diagonal matrix which has the following elements: $\sigma_{ij}=0$ for $i \neq j$ and $\sigma_{ii}=\sigma_i \geq 0$.

The elements σ_i are known as singular values of the matrix A. The condition number is a measure for the linear independence of the columns of the matrix. The condition number of a matrix A of full rank can be defined as:

(11) $$r_{cond_d} = \frac{\sigma_{max}}{\sigma_{min}},$$

where σ_{max} and σ_{min} are the largest and the smallest singular values of the matrix A, respectively.

If the matrix A has not full rank then $\sigma_{min}=0$ and the condition number cond(A) is infinite.

Referring to the robots the condition number of a matrix can be used as a performance index applied to the Jacobian of the manipulator. In this case, it is more convenient to use the reciprocal condition number, $r_{cond_d}(\mathbf{J})$, which is bounded between 0 and 1. Obviously the conditioning index depends on the joints coordinates of the manipulator and therefore it is posture-dependant. The reciprocal conditioning index for a manipulator can be defined as follows:

(12) $$r_{cond} = \frac{1}{\mathrm{cond}(\mathbf{J})}.$$

The variation of the reciprocal conditioning index over the workspace with constant orientation of the considered manipulator is shown in Fig.8. The values of the reciprocal conditioning index near to zero indicate singularities.

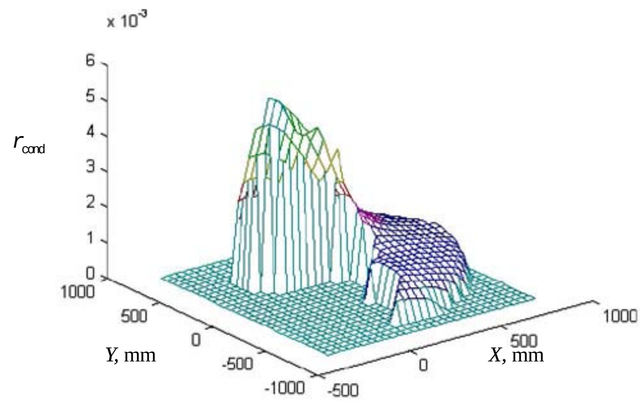

Fig. 8. The reciprocal condition number

5 Minimum singular value.

As another measure can be used the minimum singular value. In most cases the use of the minimum singular value is efficient for indication whether the determinant is near to zero [2]. The minimum singular value changes more radically near singularities than the other singular values. Fig. 9 shows the changes of the minimum singular values (msv) of the Jacobian over the workspace with constant orientation. The orientation is the same as for the obtained reciprocal condition numbers.

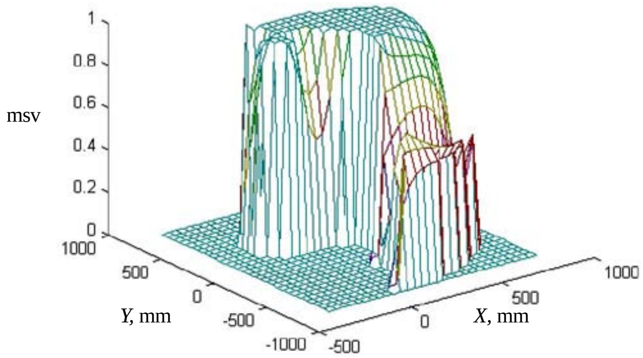

Fig. 9. The changes of the minimum singular values (msv)

6 Conclusion

In this paper, performance indexes are discussed as measures of kinematic capabilities of manipulators. Dexterity and manipulability of manipulators are considered. All obtained results for the performance indexes are graphically visualised. The presented graphs allow a comparison of the different performance indexes, i.e., what kind of similarities exist. Some of the known indexes are considered and some new indexes are introduced. The presented graphical examples for the performance of a manipulator can be easily interpreted and also they can help in application and design of manipulators.

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Качественные индексы манипуляционных роботов

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(Р е з ю м е)

Рассматриваются некоторые коэффициенты, которые используются для определения качественных характеристик манипуляционных систем – коэффициент ориентируемости, число обусловенности и минимальная снигулярная стойность. Для этих коэффициентов показаны тримерные и двумерные графики в оперативной области робота. Полученные результаты предсталяют значительный интерес при применении роботов в разнообразных технологических операциях.