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An Interactive Method for Solving Problems of Multicriteria Choice with Discrete Alternatives

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1. Introduction

The idea discussed is to select one alternative from a set of discrete alternatives that satisfies to the greatest extent the DM's preferences about the values of a given criteria set in the considered class of problems. This class of problems, belonging to multicriteria decision analysis (MCDA) problems (V i n c k e [16]) are called problems for multicriteria choice with discrete alternatives (MCCP).

Depending on the ways of deriving and processing the information about DM's preferences, as well as on the assumption that there really exists (or does not exist) limited comparability among the alternatives, the methods solving MCCP can be divided into three groups:

– methods, in which the global preferences of the DM are aggregated through the synthesis of one generalized criterion (the multiattribute utility theory methods [F i s h b u r n [3], K e e n e y and R a i f f a [7], F a r q u h a r [2] and the analytical hierarchy process methods (S a a t y [14]);

– methods, in which the DM's global preferences are aggregated through the synthesis of one or several generalized relations of preferences (the outranking methods (R o u b e n s [11], R o y [12], B r a n s and M a r e s c h a l [1]);

– methods, in which the preferences of the DM are iteratively aggregated by direct or indirect comparison of two or more alternatives (K o k s a l a n, K a r v a n and Z i o n t s [8], M a r c o t t e and S o l a n d [10], K o r h o n e n [6], L o t f i, S t w a r d and Z i o n t s [9]; S u n and S t e u e r [15], J a s z k i e -w i c h and S l o w i n s k i [5]).

For problems with a great number of alternatives and a small number of criteria, in which the DM can hardly perceive these alternatives as a whole, which makes these problems comparatively close to multiple objective mathematical programming problems, the interactive methods have been widely used.

In each of these methods the phases of decision and computation are iteratively executed. In the computation phases the nondominated set of alternatives is reduced to a sample of neighbouring to a current or a reference alternative. The problem in the development of this phase is to use such mathematical tools that do not require a very precise qualitative information from the DM about his preferences and the alterations in them and are characterized with highspeed. In the decision phases the DM estimates whether any of the alternatives in the sample presented satisfies him and introduces some preferential information which is intended to improve the alternatives generated in the next computation phase. It is expected on his side to make multicriteria comparison of the presented alternatives from the current sample.

In order to decrease the difficulties of the DM, connected with the direct evaluation of the current sample of alternatives, these alternatives have to be comparable for the DM. This tension can be further decreased, if the DM is assisted by any formalized procedure, able to rank the alternatives of the current sample of alternatives on the basis of the local preferences. In J a s z k i e w i c h, S l o m i n s k i [5] the DM is supported by an approach, in which a local outranking procedure is included (ELECTRE 4 type) for the defining and ranking of the current sample of alternatives.

In the learning oriented interactive method suggested the determination of the current sample of comparable alternatives is done with the help of reference cones. The alternatives ranking is done by a local outranking procedure (being PROMETEE II type).

1. Method description

The problem for multicriteria choice with discrete alternatives is defined as follows: A set I of n (>1) deterministic alternatives and a set J of k ($\geq$2) quantitative criteria be given which define an $n\times k$ decision matrix A. The element a_{ij} of the matrix A denotes the evaluation of the alternatives $i\in I$ with respect to the criterion $j\in J$. The evaluation of the alternative $i\in I$ with respect to all the criteria in the set J is given by the vector (a_{i1}, , a_{i2}, ..., a_{ik}). The assessment of all the alternatives in the set I for the criterion $j\in J$ is given by the column vector (a_{1j}, a_{2j} , ..., a_{ij}). The objective is to search for a non-dominated alternative which satisfies mostly the DM with respect to all the criteria simultaneously.

The alternative $i\in I$ is called non-dominated if there is no other alternative $s\in I$ for which $a_{sj} \geq a_{ij}$ for all $j\in J$ and $a_{sj} > a_{ij}$ for at least one $j\in I$.

A number of algorithms are known for separation of the dominated alternatives (S u n and S t e u e r [15]). Their complexity is measured by $O(kn^2)$. We shall assume in the rest of the paper that matrix A contains only non-dominated alternatives.

A current preferred alternative is a non-dominated alternative chosen by the DM at the current iteration. The most preferred alternative is a preferred alternative that satisfies the DM to the greatest degree.

Desired directions of change for the criteria at each iteration are the directions, along which the DM wishes to change the criteria values of the current preferred alternative in order to get a better one.

A current sample of alternatives is a subset of the nondominated alternatives which includes the preferred alternative and (l–1) number of alternatives (l being set by the DM), which are nearest to the reference alternative according to some kind of metrics.

A current ranked sample of alternatives is a subset of the nondominated alternatives, obtained from the current sample of alternatives after the alternatives ranking with the help of any procedure on the basis of DM's local preferences.

In the interactive method here considered, instead of one decision phase, in which the DM chooses from a current sample of alternatives the current preferred alternative and gives local information for its improvement, two decision phases are applied. Besides this the computation phase, in which the current sample of alternatives is determined, is replaced by two computation phases respectively. In the first decision phase the DM selects the current preferred alternative and presents his preferences for the determination of the current sample of alternatives. These preferences are the desired directions of change of the criteria values. They determine the reference cone. In the first computation phase a current sample of alternatives is defined by the alternatives that belong to this cone and are close to the current preferred alternative. The number of alternatives in the current sample of alternatives is set by the DM in the first decision phase (parameter l). In order to determine the next current preferred alternative, the DM presents in the second decision phase his local inter-criteria information – i.e. the local weights of the criteria and local intra-criteria information - the indifference and the strict preference thresholds for every criterion. On the basis of this local information a current ranked sample of alternatives is obtained in the second computation phase (with the help of a formal procedure of PROMETEE II type). The first alternative in this ranked sample has to correspond best to DM's local preferences. In case it corresponds to DM's global preferences also, it could become the most preferred alternative.

2.1. Defining the current sample of alternatives

The current sample of alternatives is generated in the first computation phase of each iteration. Let h denotes the index of the current preferred alternative. The following denotations are introduced, connected with the current preferred alternative:

L_h – the set of indices $j \in J$ of the criteria for which the DM wishes to increase their values (desired changes for the criteria) in comparison with their values in the current preferred alternative;

E_h – the rest of the criteria ($E_h = J \setminus L_h$);

In the criteria space $\boldsymbol{R}^k$ the alternatives can be represented as vectors (points) of this space. When the DM sets the desired directions of change of the criteria values the set M by alternatives neighbouring to the current preferred alternative can be defined in space $\boldsymbol{R}^k$ on the basis of the alternatives allocation with respect to a convex cone with a vertex in the current preferred alternative. This cone is called a reference cone. The generators of the reference cone, denoted by $V(h)$ are defined on the basis of the directions desired by the DM for change of the criteria values. The reference cone $V(h)$ has k generators $v^1, \dots, v^p, \dots, v^k$ and may be defined as follows:

$$V(h) = \{v \in \boldsymbol{R}^k \mid v^p = a_h + \sum_{p\in J} \beta_p v^p,\ \beta_p \geq 0\},$$

where the components v_j^p of the generator v^p are defined according to the relations:

$$v_j^p = \begin{cases} 0 \text{ if } j \neq p, \\ 1 \text{ if } j = p,\ j \in L_h, \\ -1 \text{ if } j = p,\ j \in E_h. \end{cases}$$

For every alternative $i \in I \mid i \neq h$ a corresponding vector $\mu_j^i \mid i \in I$ is put, whose components are defined as follows:

$$\mu_j^i = \begin{cases} 1 \text{ if } a_{ij} \geq a_{hj}, \\ j \in J, \\ -1 \text{ otherwise.} \end{cases}$$

Let the distance $d(V(h), i)$ between the reference cone $V(h)$ and every alternative $i \in I \mid i \neq h$ be defined as

$$d(V(h), i) = \sum_{j\in J} \left| v_j^i - \mu_j^i \right| / 2.$$

From mathematical point of view, $d(V(h), i)$ shows the number of directions by which the alternatives $i\in I \mid i\neq h$ differ from every alternative belonging to the cone $V(h)$. It is obvious that these alternatives have a distance equal to zero.

From a view point of the multiple criteria choice problem the total number of directions along which the alternative with an index $i\in I \mid i\neq h$ differs from each alternative belonging to the reference cone $V(h)$ is not so important as the number of directions, where these two alternatives differ, having in mind the criteria, the values of which the DM wants to improve. This number is given by the distance $d'(V(h), i)$, defined as follows:

$$d'(V(h), i) = \sum_{j\in L_h} \left| v_j^i - \mu_j^i \right| / 2 .$$

On the basis of the distance function $d'(V(h), i)$ a set M_1 is formed. It contains the current preferred alternative h, and $l-1$ alternatives belonging to the reference cone ($d'(V(h), i) = 0$). In case their number is smaller, alternatives with the least distance ($d'(V(h), i)$ are added. The sample M_1 thus obtained is presented to the DM for estimation in the first decision phase.

2.2. Defining the ranked current sample of alternatives

The current sample of alternatives, comprises relatively close alternatives, hence it can be said that they are comparable. Instead of direct comparing of the alternatives in the set M_1 by the DM and selection of a current preferred alternative, it is possible and recommendable to rank the alternatives from this set according to their significance with the help of a formal procedure on the basis of the local information for DM's preferences. The first alternative of this ranked set, satisfying to the greatest extent the DM's local preferences, is probably chosen by him in the next decision phase as the current preferred alternative or the most preferred alternative. Naturally the DM could select another alternative from this set as well, if he considers it really better for him.

In connection with the fact that the alternatives from the current sample of alternatives M_1 are relatively close, the most appropriate formal procedure for their ranking with respect to their importance is the PROMETEE II outranking procedure (B r a n s and M a r e s c h a l [1]). The ranking in this procedure is done on the basis of two types of DM's local preference information. The first type of preference information is the so called intra-criteria information. For each criterion with an index $j\in J$, two types of thresholds are determined – an indifference threshold q_j and a strict preference threshold p_j. The indifference threshold q_j for the criterion $j \in J$ is the difference in the criterion values, which has no considerable influence on the DM. The strict preference threshold p_j for the criterion j is the difference in the criterion values for two alternatives, which expresses explicitly DM's local preferences towards one of them accepted as better by him. The second type of DM's local preference information is the so called inter-criteria information. In PROMETEE II outranking procedure, information concerning the relative importance or weights of the criteria is used only as such type of information. These quantitative weights have to be set directly by the DM. This outranking procedure, unlike other outranking procedures, as ELECTRE family procedures, does not require a tabu threshold setting.

A comparatively simple procedure called procedure B, defining the criteria weights and ranking the current sample of alternatives M_1 is described below on the basis of the local inter- and intra-criteria information.

In procedure B the information about the criteria weights can be entered and obtained in three ways (according to the Conflict Analysis Method (Van Huglenbreck (1995)): a) the DM is able to give quantitative weights so that their sum is equal to 1; b) the DM is able to define only the ranking order of criteria relative importance. The expected weights, if a uniform distribution is assumed, are given by the formula:

$$w_j = \sum_{s=r}^{k} (1 / r)$$

where r is the priority level of ranking of the criteria j (with r=1 for the most important and r =k for the least important criterion); c) In this case he/she is asked to compare the criteria two by two and the weights are derived from the eigenvector of the pairwise comparison matrix. Two scales are possible: a three point scale ("<", "=", ">") or a nine-point scale as in the AHP method of Saaty.

The main steps, included in the procedure, are three:

Step 1. Enrichment of the preference structure by introducing the preference function $P_j(i_1, i_2)$. The preference function $P_j(i_1, i_2)$ represents for the criterion j the degree of preference of the alternative i_1 with regard to the alternative i_2 as a function of the difference d_j between the values of this criterion for these two alternatives, where:

$$d_j = a_{i_1 j} - a_{i_2 j} \, .$$

The function $P_j(i_1, i_2)$ is defined as follows:

$$P_j(i_1, i_2) = \begin{cases} 0 \text{ if } d_j < q_j \, , \\ (d_j - q_j) \, / \, (p_j - q_j) \text{ if } q_j < d_j < p_j \, , \\ -1 \text{ if } d_j > q_j \, . \end{cases}$$

Step 2. Enrichment of the dominance relations. A valued outranking relation is built taking into account all the criteria. For each pair of alternatives, belonging to the current sample of alternatives M_1, the overall degree of preference of an alternative over the other one is obtained computing the multicriteria preference indices and outranking flows.

The multicriteria preference index $\pi(i_1, i_2)$ among the alternatives i_1 and i_2 is defined as follows:

$$\pi(i_1, i_2) = \sum_{j=1}^{k} (1 / r)$$

where $w_j \mid j \in J$ are normalized criteria weights. The multicriteria preference index, (i_1, i_2) measures how the alternative i_1 is preferred to alternative i_2 taking into account all the criteria. The multicriteria preference indices are computed for each two alternatives from the set M_1.

Outranking flows $\Phi^+(i_1)$ and $\Phi^-(i_1)$ and $\Phi(i_1)$ associated with the alternative i_1 are computed as follows:

$$\Phi^+(i_1) = \sum_{i \in M_1} \pi(i_1, i),$$

$$\Phi^-(i_1) = \sum_{i \in M_1} \pi(i, i_1),$$

$$\Phi(i_1) = \Phi^+(i_1) - \Phi^-(i_1).$$

The positive outranking flow $\Phi^+(i_1)$ expresses how the alternative i_1 is outranking all the other alternatives, belonging to the set M_1 (the power of i_1). The negative outranking flow $\Phi^-(i_1)$ expresses how the alternative i_1 is outranked by all the other alternatives belonging to the set M_1 (the weakness of i_1). The outranking net flow $\Phi(i_1)$ expresses the real power of the alternative i_1.

Step 3. Exploitation for decision aid.

Each two alternatives from the set M_1 can be compared with the help of the outranking net flow $\Phi(i_1)$, using two binary relations only – a strict preference relation P and an indifference relation I. For each two alternatives $i_1, i_2 \in M_1$ one of the following conditions is satisfied:

$$i_1 \text{ P } i_2 \text{ if } \Phi(i_1) > \Phi(i_2),$$
$$i_1 \text{ I } i_2 \text{ if } \Phi(i_1) = \Phi(i_2).$$

On the basis of these two relations, the alternatives from the set M_1 are completely ranked. This set is denoted by M_2. The set M_2 is represented for evaluation to the DM, suggesting him the first alternative from the set as the current preferred alternative. He can make this choice in the first decision phase of the next iteration.

3. The algorithm scheme

The main steps of the algorithm are:

Step 1. Reject all the dominated alternatives and define the decision matrix A. Set $iter = 1$ and ask the DM to choose an initial current preferred alternative, and assign h its index.

Step 2. If the DM wants to store the current preferred alternative h – check if it has been saved before and in case it has not – add h to LIST – a set of stored preferred alternatives.

Step 3. Ask the DM to define the desired directions for change of the values of the criteria $j \in J$ and to specify the parameter l – the number of alternatives in the current sample of alternatives.

Step 4. The sets L_h and E_h are formed. Define the set $I' \subset I$ of the indices $i \in I$ of the alternative for which there exists at least one index $j \in L_h$, for which $a_{ij} \geq a_{hj}$. For each alternative with an index $i \in I'$ determine the values of the distance function $d'(V(h),i)$ and the maximal deterioration $t(i, h)$ of the criteria from the set E_h for this alternative with respect to the current preferred alternative

$$d'(V(h),i) = \sum_{j \in L_h} (1 - \operatorname{sign}(a_{ij} - a_{hj})) / 2, \quad i \in I',$$

$$t(i,h) = \max_{j \in E_h}(a_{hj} - a_{ij}), \quad i \in I'.$$

Rank the alternatives with indices in the set I' in ascending order of the values of $d'(V(h),i) \mid i \in I'$. At equal values of $d'(V(h),i) \mid i \in I'$ for two alternatives, the alternative with a smaller value of $t(i,h)$ occupies a more forefront place. Include all the first $l-1$ alternatives in the set M_1 if $l \leq |I'|$ or all the alternatives from the set I' if $p > |I'|$. Take also the current preferred alternative as the first alternative in the set M_1. If the set M_1 contains the current preferred alternative only, pass to Step 5, otherwise – to Step 6.

Step 5. Since there does not exist an alternative the value of which coincides for at least one criterion with the desired direction of change, the DM has to decide whether to alter his current preferences or to choose the current preferred alternative as the alternative best preferred. In the first case go to Step 3, while in the second one – Stop.

Step 6. Defining with the help of the procedure B the local weights $w_j \mid j \in J$ setting directly the quantitative weights by the DM or setting the rank order of the criteria relative importance, or comparing the criteria two by two (pairwise comparison). In case the DM does not want to change these parameters, the old ones remain.

Step 7. Ask the DM to determine the local indifference and preference thresholds – q_j and $p_j \mid j \in J$. In case the DM does not want to change these parameters, the old ones remain.

Step 8. Ranking of the current sample of alternatives M_1 using the procedure B. The current ranked sample of alternatives M_2 is obtained.

Step 9. Show the current ranked sample of alternatives M_2 to the DM for estimation. If the DM chooses the best-preferred alternative – go to Step 9. In case the DM wants to continue the search, set *iter=iter+1,* assign the current preferred alternative to *h* and go to Step 2.

Step 10. If the DM does not hesitate that it is really the most preferred alternative, Stop, otherwise he can compare it with the alternatives obtained and stored in LIST. For this purpose a final sample of alternatives is formed from the alternatives stored in LIST and the last alternative found. With the help of the simple procedure B, this sample is ranked.

Step 11. Show the last ranked sample of alternatives to the DM for estimation and choose the most preferred alternative. Stop.

Remark 1. Any alternative can be selected as an initial preferred alternative. One acceptable initial preferred alternative can be found optimizing one criterion.

Remark 2. The rejecting of a dominated alternative is done once in the initial phase of the algorithm (Step 1). A number of algorithms are known (S u n and S t e u e r [15]). Their complexity is measured by $O(kn^2)$.

4. Conclusion

An interactive learning oriented method for solving problems of multiple criteria choice with a large number of discrete alternatives and a small number of quantitative criteria is proposed in the paper. This method decreases considerably the DM's tension and at the same time gives him/her the possibility to control the search process. The method proposed has several advantages, some of them being:

– The algorithm enables the DM to realize a convenient and easy understandable way of setting preferences at each iteration in the form of desired directions for criteria alteration with respect to a given reference alternative.

– This algorithm has a very good "learning" influence on the DM, since it helps the taking into account of the criteria significance, their correlation and the possibility for compensation among them.

– Provides a possibility for evaluation of distributed alternatives, stored in the process of solution.

The method has been included in a software system evaluating the efficiency of various state enterprises during the process of wide privatization in Bulgaria.

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Интерактивный метод решения задач мультикритериального выбора с дискретными альтернативами

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(Резюме)

В работе предложен интерактивный метод решения класса задач мультикритериального выбора с большим числом дискретных альтернатив и и малым числом количественых критерий. Лицо, принимающее решение (ЛПР), задает свои предпочтения в форме желаемых направлений перемены стойностей критериев по отношении выбранной отправной альтернативы.

При помощи подхода отправного конуса находится небольшое подмножество сравнительно близких альтернатив. Это подмножество ранкуется аутранкиращей процедурой, на основе заданной ЛПР локальной преференциальной информации внутренно- или междукритериального типа. Так полученное множество представляется ЛПР, которое выбирает наиболее предпочитаемую альтернативу или вводит свои новые предпочтения для улучшения выбранной альтернативы.

Предложенный метод позволяет ЛПР оценять последовательно и систематично множество недоминиранных и сравнительно близких альтернатив.