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An Interactive Method for Solving a Class of Multiple Criteria Choice Problems

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1. Introduction

The problems for multicriteria choice can be divided (Vincke [1992]) into two separate classes according to their formal statement: on one hand problems, in which a finite number of explicitly set constraint functions determine implicitly an infinite number of feasible alternatives and on the other hand problems, in which a finite number of alternatives are explicitly set in a table form. The first class of problems are called problems of multiple objective mathematical programming (MOMP). The second class of problems, belonging to multicriteria decision analysis problems, are called problems for multicriteria choice with discrete alternatives (MCCP) as well.

The so-called interactive methods have been most widely used in the solution of MOMP problems. In each of these methods the phases of decision and computation are iteratively executed. In the computation phases, when a certain type of a scalarizing problem is solved, one or several nondominated alternatives are generated, which satisfy to the greatest extent the local preferences of the decision maker (DM) in the decision phase. The DM realizes selection and choice of the best local alternative (the preferred alternative). In case this alternative satisfies his global preferences also, it becomes the best global alternative (the most preferred alternative). Otherwise the DM enters additional information, corresponding to his new local preferences, which is used in the next computation phases searching for new better alternatives.

The outranking methods (see Roubens [11], Roy [12], Braus and Mareschal [1] and the utility theory methods (see Fishburn (1970), Keeney and Raiffa [7], Farquhar [2] and Saaty [14]) are traditional methods for solving MCCP problems and for a wide class of MCCP (especially problems with a great number of criteria and a comparatively small number of alternatives) they have no competitors, at least for the moment. For problems with a great number of alternatives and a small number of criteria, in which the DM can hardly perceive these alternatives as a whole, which makes these problems comparatively close to MOMP problems, methods of a third group are designed, namely interactive methods. They are most often inspired by MOMP methods (see Koksalan, Karvan and Zionts [8], Marcotte and Soland [10], Korhonen [1988], Lotfi, Steward and Zionts [9]; Sun and Steuer [1996], Jaszkiewich and Slowinski [5].

Some difficulties in the first two groups of methods can be overcome in the interactive algorithms on the account of the greater tension and engagement of the DM..

A learning oriented interactive method is proposed in this paper, which decreases considerably the DM's tension and at the same time gives him/her the possibility to control the search process. The defining and ranking of the current sample of alternatives in this method is realized by two different procedures. The determination of the current sample of comparable alternatives is done with the help of a scalarizing problem. A set of alternatives, close to the special reference alternative is found with this problem on the basis of DM's local preferences with the purpose to improve the current preferred alternative. This set of close alternatives (together with the current alternative) is the current sample of alternatives. The alternatives ranking is done with the help of a local outranking procedure (being PROMETEE II type) on the basis of the local inter- and intra-criteria information of the DM. The best alternative is the local preferred alternative, which the DM chooses to improve or to accept as the most preferred alternative.

The discrete multicriteria choice problem is defined as follows: Given a set I of n (>1) deterministic alternatives and a set J of k ($\geq$2) quantitative criteria which define an $n\times k$ decision matrix A. The element a_{ij} of the matrix A denotes the evaluation of the alternatives $i\in I$ with respect to the criterion $j\in J$. The evaluation of the alternative $i\in I$ with respect to all the criteria in the set J is given by the vector (a_{i1}, a_{i2}, ..., a_{ik}). The assessment of all the alternatives in the set I for the criterion $j\in J$ is given by the column vector (a_{1j}, a_{2j}, ..., a_{ij}). The objective is to search for a non-dominated alternative which satisfies mostly the DM with respect to all the criteria simultaneously.

The alternative $I\in I$ is called non-dominated if there is no other alternative $s\in I$ for which $a_{sj}\geq a_{ij}$ for all $i\in J$ and $a_{sj}>a_{ij}$ for at least one $j\in I$.

Since it is comparatively simple to separate the dominated alternatives, we shall assume in the rest of the paper that matrix A contains only non-dominated alternatives.

A current preferred alternative is a non-dominated alternative chosen by the DM at the current iteration. The most preferred alternative is a preferred alternative that satisfies the DM to the greatest degree.

Desired changes for the criteria at every iteration are the values by which the DM wishes to increase some criteria values of the current preferred alternative with respect to his local preferences in order to obtain a better alternative.

A reference alternative is an alternative (it may not exist in reality) obtained from the current preferred alternative and the desired changes for some of its criteria values. The reference alternative and the current alternative differ only in the criteria, which the DM wishes to improve.

A current sample of alternatives is a subset of the nondominated alternatives which includes the preferred alternative and $l-1$ in number alternatives (l being set by the DM), which are nearest to the reference alternative according to some kind of metrics.

A current ranked sample of alternatives is a subset of the nondominated alternatives, obtained from the current sample of alternatives after the alternatives ranking with the help of any procedure on the basis of DM's local preferences.

2. Method description

The interactive method here considered, which solves MCCP with a large number of alternatives preserves the advantage of the interactive methods connected with the availability of DM's possibility to control the process of search for the most preferred alternative, decreasing his tension concerning the necessity to compare directly two or more alternatives in each iteration at the same time. For this purpose instead of one decision phase, in which the DM chooses from a current sample of alternatives the current preferred alternative and gives local information for its improvement, two decision phases are applied. Besides this the computation phase, in which the current sample of alternatives is determined, is replaced by two computation phases respectively. In the first decision phase the DM selects the current preferred alternative and presents his local preferences for the determination of the current sample of alternatives. The local preferences are the desired changes for the criteria. The desired changes for the criteria determine the reference alternative. In the first computation phase solving a scalarizing problem, a current sample of alternatives is defined, that are close to the reference alternative. In case the current preferred alternative is not included in the current sample of alternatives, it is added to this sample. The number of alternatives in the current sample of alternatives is set by the DM in the first decision phase. In order to determine the next current preferred alternative, the DM presents his local inter-criteria information in the second decision phase – i.e. the local weights of the criteria and local intra-criteria information – the indifference and the strict preference thresholds for every criterion (the DM can leave the old weights and thresholds as well). On the basis of this local information a current ranked sample of alternatives is obtained in the second computation phase (with the help of a formal procedure of PROMETEE II type). The first alternative in this ranked sample has to correspond best to DM's local preferences. In case it corresponds to DM's global preferences also, it could become the most preferred alternative.

2.1. Defining the current sample of alternatives

The current sample of alternatives is generated in the first computation phase of each iteration. Let h denotes the index of the current preferred alternative. The following denotations are introduced, connected with the current preferred alternative:

L_h – the set of indices $j \in J$ of the criteria for which the DM wishes to increase their values (desired changes for the criteria) in comparison with their values in the current preferred alternative;

E_h – the rest of the criteria ($E_h = J \setminus L_h$);

Δ_{hj} – desired change of the criterion with an index $j \in L_h$;

i_j^h – the current reference alternative, its components being defined as

$$i_j^h = \begin{cases} \alpha_{hj} + \Delta_{hj} \text{ if } j \in L_h \\ \alpha_{hj} \text{ if } j \in E_h; \end{cases}$$

Δ_j – the difference between the maximal and the minimal value for the criterion with an index j;

$$\Delta_j = \max_{i \in I} a_{ij} - \min_{i \in I} a_{ij};$$

M_1 – the current sample of alternatives; $M_1 = (i_1, i_2, ..., i_p)$. The set M_1 comprises the alternatives that are closest to the reference alternative. Their number p is determined by the DM. One of them is the current preferred alternative. It can be at the beginning, in the middle or at the end, depending on its vicinity to the reference alternative.

The reference alternative is defined in the first decision phase. It is found on the basis of the current preferred alternative and the desired changes for the criteria. The set M_1 or the current sample of alternatives is determined in the first computation phase on the basis of the reference alternative.

The set M_1 can be defined solving the following scalarizing problem:

$$\text{(A):} \qquad \min_{i\in I} S(i, h) = \min_{i\in I} \max_{j\in J} \{ (i^h_j - a_{ij})/\Delta_j \}.$$

When solving problem A, an alternative with an index i_1 from the set M_1 is found, that is nearest to the reference alternative. Simultaneously with this for each of the rest of alternatives the value of $S(i, h)$ is determined, which enables the defining of the indices of the remaining elements in the set M_1. The alternatives with the smallest values of $S(i, h)$ are included in the set M_1. In case the current preferred alternative is not included in the set M_1, it is added in place of the alternative with an index i_p.

2.2. Defining the ranked current sample of alternatives

The current sample of alternatives, included in the set M_1 comprises relatively close alternatives, hence it can be said that they are comparable. Instead of direct comparing of the alternatives in the set M_1 by the DM and selection of a current preferred alternative, it is possible and recommendable to rank the alternatives from this set according to their significance with the help of a formal procedure on the basis of the local information for DM's preferences. The first alternative of this ranked set, satisfying to the greatest extent the DM's local preferences, is probably chosen by him in the next decision phase as the current preferred alternative or the most preferred alternative. Naturally the DM could select another alternative from this set as well, if he considers it really better for him.

In connection with the fact that the alternatives from the current sample of alternatives M_1 are relatively close, the most appropriate formal procedure for their ranking with respect to their importance is the PROMETEE II outranking procedure (Braus and Mareschal [1]). The ranking in this procedure is done on the basis of two types of DM's local preference information. The first type of preference information is the so called intra-criteria information. For each criterion with an index $j\in J$ two types of thresholds are determined by the DM – an indifference threshold q_j and a preference threshold p_j. The indifference threshold q_j for the criterion $j\in J$ is the difference in the criterion values, which has no considerable influence on the DM. The preference threshold p_j for the criterion j is the difference in the criterion values for two alternatives, which expresses explicitly DM's local preferences towards one of them accepted as better by him.

The second type of DM's local preference information is the so called inter-criteria information. In PROMETEE II outranking procedure, information concerning the significance of the separate criteria for the DM is used only as such type of information. This information for the criteria significance is expressed by the weighing coefficients (weights) of the criteria. This outranking procedure, unlike other outranking procedures, as ELECTRE family procedures, does not require a tabu threshold setting. The criteria weights, especially when their number is small, can be directly introduced by the DM. Additional use of any simple procedure is also possible, which will determine more precisely these weights on the basis of the pairwise criteria comparison (Hwang and Kwangsun [4]).

A comparatively simple procedure called procedure B, which ranks the current sample of alternatives M_1 is described below on the basis of the local inter- and intra-information with the help of PROMETEE II.

Let $P_j(i_1, i_2)$ denotes the preference function for the criterion with an index j, describing the intensity of preference for alternative i_1 with respect to alternative i_2 as a function of the difference d_j between the values of this criterion for these two alternatives, where

$$d_j = a_{i_1 j} - a_{i_2 j}$$

The function $P_j(i_1, i_2)$ is defined as follows:

(1) $$P_j(i_1, i_2) = \begin{cases} 0 \text{ if } d_j \le q_j \\ (d_j - q_j) / (p_j - q_j) \text{ if } q_j \le d_j \le p_j \\ 0 \text{ if } d_j > p_j . \end{cases}$$

On the basis of the preference function $P_j(i_1, i_2)$ the preference index $\pi(i_1, i_2)$ among the alternatives i_1, i_2 can be represented as

(2) $$\pi(i_1, i_2) = \sum_{j=1}^{k} w_j P_j(i_1, i_2),$$

where w_j, $j \in J$, are the criteria weights. The preference index $\pi(i_1, i_2)$ can be computed for each two alternatives from the set M_1. For each alternative $i_1 \in M_1$ it is possible to compute the so-called positive and negative outranking flows, denoted as $\Phi^+(i_1)$ and $\Phi^-(i_1)$.

(3) $$\Phi^+(i_1) = \sum_{i \in M_1} \pi(i_1, i) ,$$

$$\Phi^-(i_1) = \sum_{i \in M_1} \pi(i, i_1) .$$

The positive outranking flow $\Phi^+(i_1)$ expresses how the alternative with an index $i \in M_1$ is outranking all the other alternatives, belonging to the set M_1. The negative outranking flow $\Phi^-(i_1)$ expresses how the alternative with an index $i \in M_1$ is outranked by all the other alternatives belonging to the set M_1.

The net outranking flow $\Phi(i_1)$ is used for the complete ranking of the alternatives from the current sample of alternatives:

(4) $$\Phi^+(i_1) = \Phi^+(i_1) - \Phi^-(i_1),$$

where for each two alternatives $i_1, i_2 \in M_1$, i_1 is preferred to i_2, in case $\Phi(i_1) > \Phi(i_2)$, and i_1 is indifferent to i_2 if $\Phi(i_1) = \Phi(i_2)$.

On the basis of these two relations of preference and indifference the alternatives from the set M_1 are ranked and a ranked current sample of alternatives is obtained. This set is denoted by M_2. The set M_2 is represented to the DM for evaluation, suggesting him the first alternative from the set (with the greatest value of $\Phi(i_1)$) as the current preferred alternative. He has to make this choice in the first decision phase of the next iteration.

3. The algorithm scheme

An algorithm can be proposed to solve MCCP on the basis of the scalarizing problem A and the formalized procedure B. On the ground of the scalarizing problem A the DM has the possibility to get a small subset of alternatives, which are to some extent close to the reference alternative, defined by the current preferred alternative and the local desired change of the criteria values. In a learning mode the DM determines a set of samples of alternatives, which could cover to a great degree the whole set of alternatives. The current sample of alternatives is ranked with the help of the formalized outranking procedure B on the basis of the local intra- and inter-criteria information given by the DM. The DM can choose the first alternative from the ranked current sample of alternatives as the most preferred alternative or as the current preferred alternative. The last one serves as a basis for the search of the next current sample of alternatives.

The main steps of the algorithm are:

Step 1. Reject all the dominated alternatives and define the decision matrix A. Set iter = 1 and ask the DM to choose an initial current preferred alternative, denoted as h.

Step 2. If the DM wants to store the current preferred alternative h – check if it has been saved before and in case it has not – add h to LIST – a set of stored preferred alternatives.

Step 3. Ask the DM to define desired changes of the criteria values with respect to the current preferred alternative. Define a reference alternative. Ask the DM to specify the parameter l – the number of alternatives in the current sample of alternatives.

Step 4. Solving the scalarizing problem A, determine the current sample of alternatives M_1.

Step 5. Ask the DM to determine the local indifference and preference thresholds – q_j and p_j, $j \in J$, and the weights w_j, $j \in J$. In case the DM does not want to change these parameters, the old ones remain.

Step 6. Ranking of the current sample of alternatives M_1 using the simple outranking procedure B. The current ranked sample of alternatives M_2 is obtained..

Step 7. Show the current ranked sample of alternatives M_2 to the DM for estimation. If the DM chooses the best-preferred alternative – go to Step 8. In case the DM wants to continue the search procedure, set $iter = iter + 1$, select the current preferred alternative and assign it to h, then go to Step 2.

Step 8. The DM can compare the last preferred alternative with the alternatives obtained and stored in LIST. For this purpose a final sample of alternatives is formed from the alternatives stored in LIST and with the help of the simple outranking procedure B, this sample is ranked.

Step 9. Ask the DM to choose, as the best preferred alternative one of the two alternatives: the last preferred alternative (Step 7) or the first alternative from the last ranked sample of alternatives, then Stop.

Remark 1. Any alternative can be selected as an initial preferred alternative. One acceptable initial preferred alternative can be found optimizing one criterion.

Remark 2. The rejecting of a dominated alternative is done once in the initial phase of the algorithm (Step 1).

4. Conclusion

An interactive learning oriented method for solving a class of multiple criteria choice problems with a large number of alternatives and a small number of quantitative criteria is proposed in the paper. The method enables the DM to screen consecutively and systematically the set of non-dominated alternatives. The method proposed has several advantages, some of them being:

it is user-understandable for the DM, which gives him/her confidence about the suggested decision;

the comparison and the ranking of samples of comparatively close alternatives is done by the DM, which relieves the DM and enables the easier and more realistic setting of his/her local preferences;

it provides a possibility for evaluation of distributed alternatives, stored in the process of solution;

a possibility exists for a relatively easy learning by the DM of the problem solved.

The method has been included in a software system evaluating the efficiency, the financial stability and economic capacity of industrial enterprises from different branches (for some of the branches more than 60). The experts of several Bulgarian investment funds have used this system in the purchase of various state enterprises during the process of wide privatization in the country.

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Интерактивный метод решения класса задач многокритериального выбора

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(Р е з ю м е)

Предлагается интерактивный метод решения класса задач много-критериального анализа с большим числом дискретных альтернатив и малым числом количественных критериев. Лицо, принимающее решение (ЛПР), задает свои предпочтения в

форме желанной перемены стойности критериев.

При помощи скаляризирующей задачи находится малое подмножество сравнительно близких альтернатив. Оно рядится при помощи процедуры на основе заданной ЛВР локальной информации внутри- и междукритериального типа. Это подмножество показывается ЛПР, которое выбирает самую предпочитанную альтернативу или вводит новые предпочтения для улучшения выбранной альтерантивы.

Предложенный метод позволяет ЛПР оценить последовательно и систематически множество недоминированных и сравнительно близких альтернатив. Метод включен в программной системе принятия решений. Он применяется в реальных задачах многокритериального выбора.