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Restoration of a Modulating Function of Quasi-Periodic Processes*

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Introduction

A lot of processes have high degree of periodicity in their performance. This periodicity can be disturbed under the influence of some parallel running, not well known or unfamiliar processes. As a measure of this influence the degree of periodicity distruction can be used, regarded as continuous function of time or its respective representation in the frequency domain. The tremor of the basic tune in a voice signal is a typical example for a similar process.

This tremor is expressed in the continuous alteration of the main tune frequency, the time dependence of this change having explicitly expressed individuality and increased dependence on the psychophysical (emotional) state of the vocal signal source. These relations can be used as an objective estimate in speaker's identification and in the evaluation of speaker's emotional status as well. Some typical processes with slowly changing periodicity are the heart rhythm, the respiration rhythm, the generator frequency, the revolutions or vibrations of different mechanisms.

Process model

The classical approach in the study of quasi-periodical processes in the frequency area, though giving an adequate representation, leads to complicated interpretations of the results. The time-frequency representation – a Fourier transformation or any type of wave transformation can lead to difficult for interpretation data. The present paper considers only the process periodicity, not taking into account its form. This periodicity contains an weight component and a variable which can be described analytically as a determinant of a time stochastic function. In this way, the process can be regarded with respect to the time as a point process, consisting of a series of Dirak pulses:

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(1) $$g(t)=\sum_{j=-\infty}^{+\infty}\delta(t-t_j).$$

Here t_j is the beginning of each new cycle. We assume that every period can be represented in the type

(2) $$t_j - t_{j-1} = T + \alpha\int_{-\infty}^{+\infty} x(t)\,dt,$$

$$|x(t)| < 1,\ \alpha < 1,\ E[x(t)] = 0,\ W_x < \frac{1}{T},$$

where T is the average process period and W_x is the efficient frequency band of $x(t)$. The last one can be defined as the second moment (inertial radius) of the Fourier image:

$$X(f)=F[x(t)],$$

(3) $$W_x = \left[\frac{\int_{-\infty}^{+\infty} f^2 |x(f)|^2\,df}{\int_{-\infty}^{+\infty} |x(f)|^2\,df}\right]^{\frac{1}{2}}.$$

Thus the constraints introduced in the model describe a process without a large deviation from the purely periodical one, with a constant average frequency and slow alterations of the moment frequency. The moment frequency here is accepted as $f(t_j)=1/(t_f - t_{f-1})$.

The purpose of the investigation is to find the function $x(t)$, using the values of t_f, and if it is a random value, its spectral characteristics. In case all the relations are used and it is assumed that $t_0=0$, the process is represented in the form

(4) $$g(t) = \sum_{t=-\infty}^{+\infty}\delta[t - jT - \alpha\int_{\infty}^{t} x(t)\,dt].$$

Replacing $z(t)=t-\alpha\int^t x(t)\,dt$, $z'(t)=dz(t)/dt=1-\alpha x(t)$ is obtained. It is assumed that $z(t)$ is a monotonously increasing function, which means that the period of the process cannot accept negative values. This follows directly from constraints (2) as well.

Having thus introduced a new variable $z(t)$, the beginning of each cycle starts at t_j, for which $z(t)=jT$ and $t_j=Z^{-1}(jT)$.

The variable

$$V_j[z(t)]=U_j[z(t)-jT]$$

is introduced where U is the Heaviside function,

$$U(t)=\begin{cases}1 \text{ if } t\geq 0\\ 0 \text{ if } t<0.\end{cases}$$

Having in mind that $dU(t)/dt=\delta(t)$, the following relation is obtained:

$$\frac{dV_j}{dt}=\frac{dU_j}{dt}\frac{dz}{dt}=\delta[z(t)-jT]\,Z^n,$$

It follows from the definition of V_j, that

$$\frac{dV_j}{dt} = \delta(t - t_j)$$

and in this way the process becomes of the form:

$$g(t) = \sum_t \delta(t - t_j) = \sum_t \delta[z(t) - jT]\, z'(t),$$

$$g(t) = [1 - \alpha\, x(t)\; \sum_t \delta[z(t) - jT]\, .$$

With the introduced variable, the function $\sum_t \delta[z(t) - jT]$ becomes completely periodical with respect to $z(t)$ and it can be developed as a Fourier series.

$$g(t) = [1 - \alpha\, x(t)] \sum_{n=-\infty}^{+\infty} b_n e^{j(2\pi/T)nz(t)}.$$

(5)
$$b_n = \frac{1}{T} \int_{-0}^{T=0} \delta(z)\; e^{-j(2\pi/T)nz} dz\, ,\; dz = \frac{1}{T}\, .$$

$$g(t) = \frac{1}{T} [1 - \alpha\, x(t)] \sum_{n=-\infty}^{+\infty} e^{j(2\pi/T)nz(t)}.$$

Having in mind the symmetry of $g(t)$ with respect to $z(t)$, the expression becomes:

(6)
$$g(t) = \frac{1}{T} [1 - \alpha\, x(t)] [1 + 2 \sum_{n=1}^{\infty} \cos\, (2\pi/T) n [\, t - \alpha \int_0^t x(\tau)\, d\tau]\, .$$

It can be seen in this representation of the process that it is a superposition of several more elementary processes:

a – a constant component,

b – a modulating oscillation $\alpha x(t)$,

c – a network of frequency-modulated oscillations with an average frequency n/T, moment $(1/T)\, n\, [1 - \alpha\, x(t)]$; $n = 1 \div \infty, \beta$ and amplitude-modulated with the modulating oscillation. The frequency modulation is the reason for the blurring of the basic and side spectral lines, but for $\alpha << 1$, the efficient width of this blurring is not considerable and is much narrower than the main frequency and it can be neglected.

The oscillation $g(t)$, represented in the frequency domain, (without the constant term), gets the form

(7)
$$G(f) = \frac{\alpha}{T} \sum_{n=-\infty}^{+\infty} x(f - \frac{n}{T}) + \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\, [f - \frac{n}{T}).$$

The frequency spectrum of $X(f)$ is repeated through intervals of $1/T$.

Restoration of the oscillation $g(t)$

The frequency bands do not cover for $W_x << 1/T$. When $g(t)$ is passed through a low-frequency filter with bound frequencies $\pm W_x$, the output signal gets the form:

$$G'(f) = (\alpha / T) X(f),\;\; X(f) = F^{-1}[G(t)]$$

and can be directly obtained by an inverse Fourier filtration.

When determining $X(f)$, there is always a limited realization, which causes frequency leakage and hence, inaccuracy in the restoration. An weighting function is used in order to decrease these errors.

Let the observation interval be T_0, $N(T_0/T)$ – the number of the cycles observed, $h(t)$ – the weighing function (with spectrum $H(f)$). At window processing, the oscillation considered will get the form:

$$\hat{G}(f) = \hat{G}(f)H(f) = (1/T)\sum_{n=-\infty}^{\infty} H(f-(n/T)) - (\alpha/T))]\sum_{n=-\infty}^{\infty} H(f)X(f-(n/f))$$

While keeping the constraint $T_0 \gg T$, after entering a low-frequency filter with a limited frequency of W_x, the output signal will get the form:

$$\hat{G}(f) = (1/T)\,H(f) - (\alpha/T)\,H(f)X(f).$$

The following expression can be accepted as an estimate of the spectrum of $x(t)$:

(8) $$\hat{X}(f) = \alpha H(f)X(f) = -T\hat{G}(f) + H(f).$$

The introducing of a window leads to the alteration of the spectral characteristics of $X(f)$. The spectral features of the window are added on one side and on the other – the estimate itself is a result of the convolution between the real spectral characteristics and that of the window. The deformations in the spectrum are less when the spectral energy of the window is close to null, which under equal other conditions means that the realization of the process T_0 is longer in comparison with the width of the autocorrelation function $1/W_x$ of the process $X(f)$. If the difference between them is big enough, it can be accepted, that the influence of the window is not so strong and its spectral characteristic $H(f)$ can be accepted with a zero phase in the computations below given:

(9)
$$\hat{G}(f) = \sum_{i=1}^{N} h(t_i)\, e^{-jwt_i} = \sum_{i=1}^{N} h(t_i)\cos wt_i - j\sum_{i=1}^{N} h(t_i)\sin wt_i$$

$$\hat{X}(f) = (-T\sum_{i=1}^{N} h(t_i)\cos wt_i + H(f)) + jT\sum_{i=1}^{N} h(t_i)\sin wt_i.$$

At frequency enable Δf, defined by the parameters of the window, the power spectrum of the process for frequency $m\Delta f$ is defined by the expression:

(10)
$$S_x(m\Delta f) = (T_0^2/N^2)\,[\sum_{i=1}^{N} h(t_i)\cos 2\pi m\Delta f\, t_i - (N/T_0)H(m\Delta f)]^2 +$$

$$(T_0^2/N^2)[\sum_{i=1}^{N} h(t_i)\sin 2\pi m\Delta f\, t_i]^2,\; m = 1 \div (W_x/\Delta f).$$

The average frequency of the process is determined by the expression $T = T_0/N$. As expected, the output parameters, which define the process $X(f)$, are the moments of starting a new cycle t_i, the duration of the sample T_0 and the number of complete cycles N.

The process $x(t)$ can be determined by the expression:

(11) $$\hat{X}(t) = F^{-1}[\hat{X}(f)] = \sum_{m=1}^{N}\hat{X}(m\Delta f)\, e^{j2\pi m\Delta f}.$$

In order to define $\hat{X}(t)$ or the spectrum $S(f)$, it is necessary to define exactly the moments t_i, at which a new cycle is starting.

Conclusion

The study accomplished has shown that rather precise representation of the cyclic alterations in quasi-periodical signals as a result of the action of time-dependent oscillation, is possible. This oscillation is restored both in the time and frequency domain. It is assumed in the investigation that the beginning of each cycle is quite precisely defined. In fact this determination can be a separate procedure based on stochastic methods.

The study will continue with experimental determination of the vocal signals tremor and its dependence on the psychophysical (emotional) status of the speakers.

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Восстановление модулирующей функции квазипериодических процессов

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(Резюме)

Рассматривается исследование квазипериодических процессов, как например тремор основного тона сигнала диктора, сердечный ритм, частота генератора и др. Результаты в области каждого из названных примеров могут найти интересные практические применения. Так например изменение частоты голоса характеризирует психофизические особености сигнала (его эмоциональность) и используется при идентификации дикторов.