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Multiterminal Network Flows with Inverse Linear Constraints in Engineering Networks*

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1. Introduction

The necessity of using and studying models that are similar to reality leads to the development of new network flows, generalizing the already known network structures. The multiterminal network flow is a natural development of the classical flow with one source and one sink and fixed upper limits of the flow function on each arc [1, 2]. There exists a number of engineering and economic problems, the solution of which implies the use of more universal multiterminal flows for the interpretation and modeling of real systems [1, 4]. The fluid supply networks and in particular - the water- and gas supply networks, the telecommunication networks, the transport networks are similar systems. Network flows with inverse linear constraints are introduced and investigated in [3] when defining the lower bounds of the capacity on subsets of the network arcs (ILC-flows) for more exact and adequate modeling of real physical networks and for increase in their reliability and functionality. The multiterminal network flows with inverse linear constraints are a logical generalization of this class of flows with respect to the real practical problems. The main difference between the classical flow and the ILC-flow determines the principal difference in the corresponding multiterminal flows and the necessity for independent study of the multirerminal network flows with inverse linear constraints.

2. Conditions for the existence of a multiterminal ILC-flow

In [3] an ILC-flow on a network with one source s and one sink t only is considered. Let the network has a set of sources $S=\{s\}$ and sinks $T=\{t\}$ and intermediate nodes, i.e., $G_M = [G(N,U), S, T]$. The sets S and T are called poles and the ILC-flow between them – a multipole ILC-flow (MP ILC-flow).

A MP ILC-flow from an arbitrary source $s \in S$ towards an arbitrary sink $t \in T$ will be called every arbitrary function $f: U \to R^+$, for which the following conditions are

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satisfied: for each $x \in N$

(1) $$f(x,N) - f(N,x) = \begin{cases} v(x) \text{ if } x \in S \\ 0 \text{ if } x \notin S,\ x \notin T \\ -v(x) \text{ if } x \in T, \end{cases}$$

(2) $$\sum_{(x,y)\in D_i} b_i(x,y)\ f(x,y) \ge c_i,\ i \in I,$$

(3) $$f(x,y) \ge 0;\ (x,y) \in U,$$

where:

$$f(x,N) = \sum_{y\in A(x)} f(x,y),\ f(x,N) = \sum_{y\in B(x)} f(y,x),$$

$$A(x) = \{y \mid (x,y) \in U\}, \qquad B(x) = \{y \mid (y,x) \in U\};$$

v – dimensions or power of the flow;

I – the set of indices of the constraints (2);

c_i, $i \in I$ – real positive numbers;

D_i, $i \in I$ – subsets of U such that for each $j, k \in I$,

$$D_i \cap D_k = \varnothing,\ \bigcup_{i\in I} D_i = U.$$

It follows from the equation for flow reservation that $v(S) = v(T)$.

The MP ILC-flow can be reduced to a respective two-pole ILC-flow similar to the classical MP ILC-flow. For this purpose the network G_M is expanded to $G_M(N', U')$, with the addition of two nodes s' and t' and the arcs (s', S) and (T, t'), for each $i \in I'$

(4) $$D_i^* = \begin{cases} D_i \text{ if } i \in I \\ (s', S) \cup (T, t') \text{ if } i = i'; \end{cases}$$

(5) $$c_i^* = \begin{cases} c_i \text{ if } i \in I \\ R^* \text{ if } i = i',\ R^* < c' |S||T|; \end{cases}$$

(6) $$i' = I \cup \{i'\}.$$

It follows from the last three relations that every MP-ILC-flow can be continued to a two-pole ILC-flow, so that the theoretic results for the last flow are spread over the case discussed – with a set of sources S and a set of sinks T. It is necessary to use cutting sets only, which block all the paths from S towards T.

Let a non-negative number $a(x)$, called a proposition, is assigned to each $x \in S$, and a non-negative number $b(x)$, called a search – to every node $x \in T$. The following problem for constraints feasibility is solved:

(7) $$f(x,N) - f(N,x) \ge a(x),\ x \in S,$$

(8) $$f(x,N) - f(N,x) = 0,\ x \notin S,\ x \notin T,$$

(9) $$f(x,N) - f(N,x) \le b(x),\ x \in T,$$

(10) $$\sum_{(x,y)\in D_i} b_i(x,y)\ f(x,y) \ge c_i,\ i \in I,$$

(11) $$f(x,y) \ge 0;\ (x,y) \in U.$$

The relations below stated give the necessary and sufficient conditions for a MP ILC-flow feasibility under constraints (7)–(11).

Lemma 1. The necessary condition for the existence of a MP ILC-flow f subject to constraints (7)–(11) is the validity of each one of the following relations:

(12) $$b(T\cap\bar{X}(r)) - a(S\cap\bar{X}(r)) \geq c(r),\ r\in G_0,$$

(13) $$b(T\cap\bar{Y}(r)) - a(S\cap\bar{Y}(r)) \geq c(r),\ r\in G_0,$$

where G_0 is the set of indices of all the cutting sets $U(r)\in U$ between S and T.

Proof. If there exists a MP ILC-flow satisfying constrains (7)–(11), after the summation of these relations with respect to $x\in X(r)$, it is obtained:

$$b(T\cap\bar{X}(r)) - a(S\cap\bar{X}(r)) \geq f(X(r), \bar{X}(r)) - f(\bar{X}(r), X(r)).$$

From this relation and the relations $v = f(X, \bar{X}) - f(\bar{X}, X)$ and $v \geq c(r)$ extracted in [3], (12) follows. The necessity of (13) is analytically proved. ∎

The transformation of a MP ILC-flow to a two-pole ILC-flow is done passing from $G(N,U)$ towards $G(N',U')$ in the following way:

(14) $$D_i^* = \begin{cases} (s',x) \text{ if } x\in S,\ i\in I^*\backslash I, \\ (x,t') \text{ if } x\in T,\ i\in I^*\backslash I, \\ D_i - \text{otherwise}; \end{cases}$$

(15) $$c_i^* = \begin{cases} a(x) \text{ if } D_i=(s',x),\ i\in I^*\backslash I, \\ b(x) \text{ if } D_i=(x,t'),\ i\in I^*\backslash I, \\ c_i - \text{otherwise}; \end{cases}$$

at that each of the arcs $(x, y)\in(s', S)\cup(T, t')$ is assigned the respective indices $i\in I^*\backslash I$; $D_i^* = (x,y)$ and the coefficients $b_j(x, y)\in R'$.

Let G_0 is the set of indices of all the cutting sets between s' and t' in $G(N', U')$ and for each of the cutting sets with indices $r\in G_0$ and $r'\in G_0$, the following is true:

(16) $$U(r) = U^*(r)\backslash\{(s',S)\cup(T,t')\},$$

(17) $$U(r)\neq\varnothing.$$

Theorem 2. The sufficient condition for the existence of a MP ILC-flow under constraints (7)–(11) is the simultaneous satisfying of any of the relations (12) or (13) and the following equality:

(18) $$c(r)\leq b(T) \text{ for each } r'\in G_0.$$

Proof. Let the relations from Lemma 1 be true. Then from (14) and (15) it follows that for the section $U(r'')=(T, t)$

(19) $$c(r'') = b(T).$$

From (18), (19) and the sufficient condition for maximality of the cutting set extracted in [3] it follows that $U(r'')$ is a maximal cutting set, for which

$$v = c(r'') = b(T)$$

and all the arcs of the cutting set-section (T, t') are saturated.

From Lemma 1, the relations (14)—(17) and the assumption for (12) it follows that there exists a flow f^* from s towards t in $G(N^*,U^*)$, which saturates the arcs (T, t'). The conditions (8), (10) and (11) are fulfilled in the contraction of the flow f^* on U. Since $a(x)\leq f^*(s', x)$, $x\in S$, $b(x) = f^*(x, t')$, $x\in T$, and $f^*(x, N) - f^*(N, x) = f(x, N) - f(N, x)$ for S and T respectively, conditions (7) and (9) are also satisfied. ∎

3. Flow interpretation of a class of fluid supply systems

The mathematical models of many engineering systems are naturally and adequately formulated in terms of flow programming. This is explained with the fact that on one hand there exists physical interpretation of the network characteristics and on the other – under certain conditions there is analogy between variables and relations, characterizing engineering networks of different character. For example, the flow function can be interpreted as a real physical flow – electrical current in electric systems, volumetric flow or just flow – in fluid systems, rate – in mechanical systems, heat flow – in heating systems.

The fluid supply systems are one of the most important subsystems in the infrastructure of almost all the modern societies. The name fluid system underlines the generality of the approach for analysis, synthesis and control of these systems independently on the properties of the fluid environment – water, gas, oil, oil products, etc. Their common character is defined basically by the unity of the physical regularities of hydro- and gas dynamics, mechanics and thermodynamics, connecting the parameters of the continuous media in a network. These systems determine to a great extent the welfare, the economic prosperity and security of entire states. In the last few years of transition, the problems connected with them are particularly actual for our country. It is recently expected that their significance will increase, mostly due to the fact that the distribution networks – in their main part, have to be renewed – as water supply or to be constructed – as town gas supply systems of low pressure.

The water supply systems are an important class of fluid supply systems. They usually include many sources – lakes, rivers, basins, wells, reservoirs; stations of main and auxiliary generators of pressure; branches, valves and other fluid control and regulating devices, intake basins; a distributing network of fluid conducts; a set of consumers – living places, industrial plants, districts in big cities. A generalized configuration of a water supply system is shown in Fig. 1.

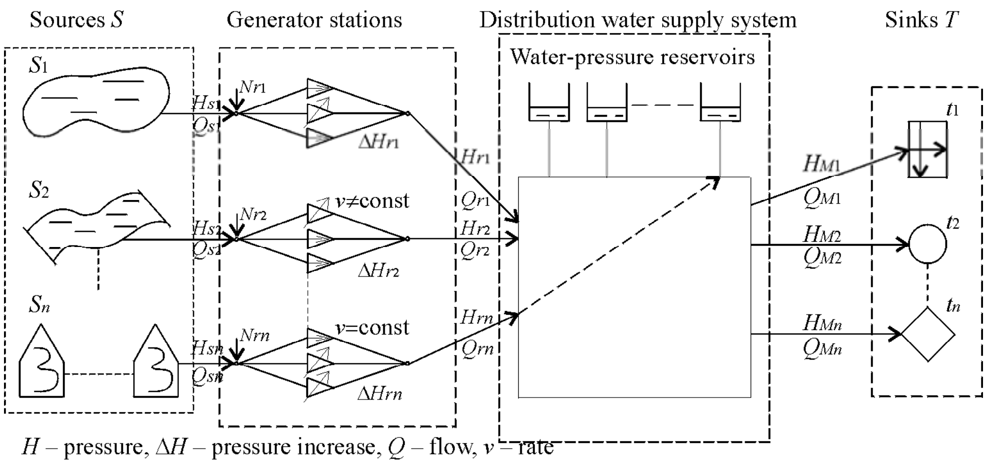

Fig. 1

The main purpose of the water supply systems is the complete satisfying of the rapidly growing consumption of water under some constraints for reliable and economical functioning. From a theoretical viewpoint they are complex nonlinear systems, built by interacting subsystems with dominating network structure. The analysis, the optimal design and control of the water supply systems depends strongly on the development of comparatively not complex – achieved by appropriate facilitations and assumptions, and hence – approximate methods and tools for adequate interpretation and modeling. The facilitations are based mainly on decomposition, the application of a row of empirical relations and aggregation. Due to the illustrativity, the logical basis and the high efficiency, the network flows are appropriate objects for operational research in water supply systems modeling and are an efficient solution of some classes of engineering problems for water transfer and distribution. The including of additional linear constraints is sometimes implied in engineering practice, for example – in the uniting or merging the outputs of separate network subdivisions of the system satisfying the common demands; constraints on the capacities in different modes of water transfer and distribution; constraints on common resources; constraints on the budget; mixing of water flows with different characteristics in the so called multiproduct network flows, etc. In this case the development and application of more general flows, in which the efficiency of classical methods and algorithms is maximally used, is appropriate.

In Fig. 2 an equivalent network flow is shown of the generalized configuration of the water supply system from Fig. 1.

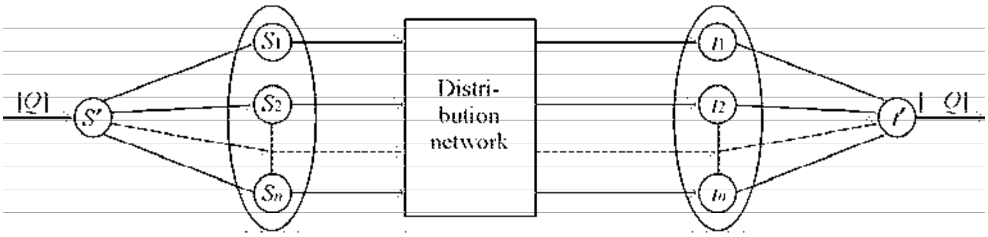

Fig. 2

The passing from multiterminal water supply systems towards two-pole ones is realized by appropriate aggregation of the set of sources S and sinks T to one source s' and one sink t. The following functional constraints are introduced, ensuring the necessary total amount of water mass in one district i (subdivisions, subnetwork, path) from the network:

$$\sum_{(x,\,y)\in D_i} l_i(x,y)\, Q(x,\,y) \geq M_i,\ i\in I,$$

where I is the set of districts, in which the network is separated; M_i, $i\in I$, are real positive numbers, accounting the minimal feasible water mass in a given district of the network; D_i, $i\in I$, are sets of arcs, corresponding to water pipes, generators of pressure or water distributing devices on separate districts of the network, subsets of the complete set of arcs U such that for each $j, k\in I$,

$$D_i \cap D_k = \varnothing, \bigcup_{i\in I} D_i = U;\ \varnothing \text{ – an empty set;}$$

$$Q(x,\,y),\ (x,\,y)\in D_i \text{ – a flow of the arc } (x,\,y);$$

$$l_i(x,y)=\begin{cases}\in R' \text{ if } i\in I \text{ and } (x,y)\in D_i\\ =0 \text{ otherwise;}\end{cases}$$

R' – a set of the fluid conducts lengths, the sign in front of each length being defined by the direction of the water flow.

The coefficient l_i can be interpreted, besides length, as loss in the pressure, water resistance, production expenses, cost, etc. For one source and a sink, and the conditions above given, the water flow is adequately and naturally modeled with the help of a two-pole ILC-flow. In reality the water supply network includes a set of sources and a set of sinks, which makes the two-pole ILC-flows inapplicable. In this real, but more general case it is particularly appropriate and only possible to use multiterminal ILC-flows in flow programming terms. At that all the theoretic, methodological and algorithmic results for a two-pole flow can be applied for the case considered – with a set of sources and a set of sinks. The source s' and the sink t' added are modelled with the help of external flows [Q]. The first problem for analysis, optimal design and control of the water supply networks is reduced to the determination of a minimal multipole ILC-flow and the flows in the respective water conducts, pressure generators or water distributing devices. The second basic problem is connected with the finding of the minimal value of this minimal flow, as well as with the respective water distribution. The distribution found must satisfy the continuity equation at the same time.

4. Conclusion

The following two inferences can be made.

– The multiterminal ILC-flows (MP ILC-flows) investigated are a natural generalization of the two-pole ILC-flows. The multiterminal ILC-flow is reduced to a two-pole ILC-flow, which, due to the characteristics of the last flow, is not a trivial problem. In spite of the classical multiterminal network structure, the multipole ILC-flow exists under certain conditions. The necessary and sufficient conditions for the multiterminal ILC-flow existence are stated and extracted.

– The real water supply systems in the general case include a lot of sources – lakes, rivers, basins, boreholes, wells, reservoirs and a set of consumers – cities, industrial plants, districts of large towns. In engineering practice additional constraints are implied with the purpose to increase their reliability, functionality and efficiency. In this case the use of multipole ILC-flows for interpretation and modeling of water supply systems is appropriate and only possible as well in flow programming terms.

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Многополюсные потоки с обратными линейными ограничениями в инженерных сетях

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(Р е з ю м е)

Предложен и исследован класс сетевых потоков, названных многополюсными потоками с обратными линейными ограничениями или МП ОЛО-потоками. При этом потоке сеть имеет несколько источников и стоков и значении дуговых потоковых функций неограниченных сверху и ограниченных снизу множеством линейных неравенств с действительными ненулевыми коэффициентами. Показано, что этот поток можно свести к двуполюсному ОЛО-потоку. При этом, все теоретические результаты ОЛО-потоков распространяются на исследованном потоке. Использованы лишь рассекающие множества дуг, блокирующие все цепи от множества источников в множество стоков. Доказаны необходимые и достаточные условия существования МП ОЛО-потока.

Исследован реальный объект приложения МП ОЛО-потока – флуидные снабдительные системы. Реализована подходящая и адекватная интерпретация одного класа этих систем– система водоснабжения. Показано, что включение дополнительных линейных ограничений с целью улучшения надежности и функционирования системы водоснабжения делает использование предложеных сетевых потоков не только целесообразным, но и единственно возможным в терминах потокового программирования.