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# An Algorithm Solving a Discrete Multicriteria Choice Problem Using Reference Cones

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## 1. Introduction

The Discrete Multicriteria Problems are a subclass of the multicriteria decision making problems. They are characterized by a given finite set of objects (alternatives) and a finite set of the criteria evaluating these objects. The decision maker (DM) most often has to choose the best alternative, taking into account all the criteria simultaneously (choice problem). The difficulty in this case is connected with the fact that the estimates of the alternatives with respect to the different criteria are contradicting.

For the multiple criteria choice problems "optimization motivated" interactive algorithms have been developed [1, 2, 3]. There is good interaction between the DM and the solution process in these algorithms that permits the DM learn more about the decisions advantages. But these methods require from the DM to set his preferences only in the form of aspiration levels for all the criteria. The DM estimates the alternatives from a given set at each iteration. In some cases these alternatives may be spread on the whole set of the non-dominated alternatives.

The algorithm suggested is also interactive, but it provides an easier way of DM preferences setting at each iteration in the form of desired directions for criteria alteration. On the other hand it is more convenient for the DM to evaluate and choose one alternative from a set of close alternatives in the criteria space.

The most precise definition about "solving" a multicriteria decision problems, given in [4] pointed out that finding a solution is the assistance to the DM in the better understanding of the data and the problem solution in order to find the most desirable action. According to this the best "compromise solution" selected depends to a large extend on the DM personality, the way of problem statement and circumstances in which the decision is to be made. Consequently, an important characteristic of the algorithms, designed to solve the discrete multicriteria choice problem is the way of visualisation of the information, which the DM has to evaluate, as well as the representation of some additional features of the criteria and the alternatives that could facilitate his choice.

## 2. General description of the algorithm

The discrete multicriteria choice problem is defined as a set $I$ of $n$ (>1) of deterministic alternatives and a set $J$ of $k$ (>2) criteria be given which define a $n \times k$ decision matrix $A$. The element $a_{ij}$ of the matrix $A$ denotes the evaluation of the alternatives $i \in I$ with respect to the criterion $j \in J$. The evaluation of the alternative $i \in I$ with respect to all the criteria in the set $J$ is given by the vector $(a_{i1}, a_{i2}, \ldots, a_{ik})$. The assessment of all the alternatives in the set $I$ for the criterion $j \in J$ is given by the column vector $(a_{1j}, a_{2j}, \ldots, a_{nj})$.

The solving of this problem is the search for a non-dominated alternative, satisfying the DM to a larger extent with respect to all the criteria.

The alternative $i \in I$ is called non-dominated if there is no other alternative $p \in I$ for which $a_{pj} \geq a_{ij}$ for all $j \in J$ and $a_{pj} > a_{ij}$ for at least one $j \in J$.

Since it is comparatively simple to separate the dominated alternatives, further on we shall assume that the matrix $A$ contains non-dominated alternatives only.

Desired directions of change for the criteria at each iteration are the directions, along which the DM wishes to change the criteria values of the current preferred alternative in order to get a better one.

The algorithm suggested is an interactive algorithm. At each iteration the DM is presented a ranked set $M=\{m_1, m_2, \ldots, m_p\}$ of alternatives, the first alternative being the current preferred alternative. The DM has to estimate the alternatives of this set and to choose one of them either as the best preferred or as a current preferred alternative. In the first case the discrete multiple criteria choice problem is solved. In the second case on the basis of the preferred alternative selected the DM sets the desired directions for alteration of the criteria values in order to search for a new better alternative with respect to all the criteria.

In the criteria space $R^k$ the alternatives can be represented as vectors (points) of this space. When the DM sets the desired directions of change of the criteria values the defining of the set $M$ by alternatives neighbouring to the current preferred alternative (with an index $h$) can be defined in the space $R^k$ on the basis of the alternatives allocation with respect to a convex cone with a vertex in the current preferred alternative. This cone is called a reference cone. The generators of the reference cone, denoted by $V(h)$ are defined on the basis of the directions desired by the DM for change of the criteria values they have in the current preferred alternative. The reference cone $V(h)$ has $k$ generators $v_1, \ldots, v_p, \ldots, v_k$ and may be defined as follows:

$$V(h) = \{v \in R_k \mid v = ah + \sum_{p \in J} \beta_p v^p,\ \beta_p \geq 0\},$$

where $J$ is the set of the criteria indices, and the components $v_j^p$ of the generator $v^p$ are defined according to the relations:

$$v_j^p = \begin{cases} 0 \text{ if } j \neq p, \\ 1 \text{ if } j = p,\ j \in L_h, \\ -1 \text{ if } j = p,\ j \in E_h. \end{cases}$$

Here $L_h$ is the set of indices $j \in J$ of the criteria, the values of which the DM wishes to improve, $E_h$ is the set of indices $j \in J$ of the criteria, the worse values of which the DM accepts,

$$L_h \cup E_h = J.$$

For every alternative $i \in I \mid i \neq h$, a corresponding vector $\mu_i \mid i \in I$ is put, whose components are defined as follows:

$$\mu^i_j = \begin{cases} 1 & \text{if } a_{ij} \geq a_{hj},\ j \in J, \\ -1 & \text{otherwise.} \end{cases}$$

Let the distance $d(V(h), i)$ between the reference cone $V(h)$ and every alternative $i \in I \mid i \neq h$ be defined as

$$d(V(h), i) = \sum_{j \in J} \left| v^i_j - \mu^i_j \right| / 2.$$

From mathematical point of view, $d(V(h), i)$ shows the number of directions by which the alternatives $i \in I \mid i \neq h$ differ from every alternative that belongs to the cone $V(h)$. It is obvious that these alternatives have a distance equal to zero.

From a view point of the multiple criteria choice problem the total number of the directions along which the alternative with an index $i \in I \mid i \neq h$ differs from each alternative belonging to the reference cone $V(h)$ is not so important as the number of directions, where these two alternatives differ, having in mind the criteria the values of which the DM wants to improve. This number is given by the distance $d'(V(h), i)$, defined as follows:

$$d'(V(h), i) = \sum_{j \in L_h} \left| v^i_j - \mu^i_j \right| / 2,\ i \in I,\ i \neq h.$$

On the basis of the distance function $d'(V(h), i)$ an algorithm can be proposed solving the multiple criteria choice problem. With the help of this algorithm the DM has the possibility, setting desired directions of alterations of the criteria values, to estimate iteratively a small subset of alternatives. The alternatives of these subsets are to some extent close to the current preferred alternative with an index h. Thus in the learning process and after that the DM can take into mind such factors that can be hardly formalized. In order to assist the DM in the estimation of the alternatives from the set $M$ it is useful to give additional parameters for each alternative from this set, such as the maximal deterioration of the criteria, which the DM agrees to be weakened, the maximal and minimal feasible values of the criteria and others.

The main steps of the algorithm proposed are as follows:

**Step 1.** Reject all the dominated alternatives and define the decision matrix $A$. Ask the DM to choose an initial preferred alternative and assign h its index. Any alternative can be selected as an initial preferred alternative in Step 1. One acceptable initial preferred alternative can be found optimizing one criterion.

**Step 2.** Ask the DM to define the desired directions for change of the values of the criteria $j \in J$ and the number of alternatives $p$, included in the subset $M$ to be visualized.

**Step 3.** The sets $L_h$ and $E_h$ are formed. $L_h$ includes the indices of the criteria which the DM wishes to improve and $E_h$ includes the indices of the criteria that the DM agrees to be weakened.

Define the set $I' \subset I$ of the indices $i \in I$ of the alternative for which there exists at least one index $j \in L_h$, for which $a_{ij} \geq a_{hj}$.

For each alternative with an index $i \in I'$ determine the values of the distance function $d'(V(h), i)$ and the maximal deterioration $t(i, h)$ of the criteria from the set $E_h$ for this alternative with respect to the current preferred alternative

$$d'(V(h), i) = \sum_{j \in L_h} (1 - \operatorname{sign}(a_{ij} - a_{hj})) / 2,\ \ i \in I',$$

$$t(i, h) = \max_{j \in E_h} (a_{hj} - a_{ij}),\ i \in I'.$$

Rank the alternatives with indices in the set $I'$ in ascending order of the values of $d'(V(h), i) \mid i \in I'$. At equal values of $d'(V(h), i) \mid i \in I'$ for two alternatives, the alternative with a smaller value of $t(i, h)$ occupies a more forefront place. Include all the first $p-1$ alternatives in the set $M$ if $p \le |I'|$ or all the alternatives from the set $I'$ if $p > |I'|$. Take also the current preferred alternative as the first alternative in the set $M$. If the set $M$ contains the current preferred alternative only, pass to Step 4, otherwise – to Step 5.

**Step 4.** Since there does not exist an alternative the value of which coincides for at least one criterion with the desired direction of change, the DM has to decide whether to alter his current preferences or to choose the current preferred alternative as the alternative best preferred. In the first case go to Step 2, while in the second – Go to Step 6.

**Step 5.** Show the set $M$ to the DM. If the DM finds one of the alternatives as the most preferred alternative – Stop. Otherwise ask the DM to choose the preferred alternative and assign h its index. Go to Step 2.

**Step 6.** Stop.

**Remark 1.** Minimizing the distance function $d'(V(h), i)$ on the basis of matrix $A$ in Step 3, the alternatives, comprising the set $M$ are determined. In case any of the criteria are for minimization, matrix $A'$ is used instead of matrix $A$. The last one obtained multiplying the elements of the columns of the matrix $A$, corresponding to the criteria minimized, by $-1$.

**Remark 2.** In DM learning process alternatives close to the current preferred alternative are included in the set $M$ in Step 3. In case the DM feels more confident, he could ask the including of the spread alternatives also in the set $M$. This can be easily done since the alternatives, included in the set $M$, are selected from the set $I' \subset I$ of alternatives arranged in ascending order.

## 4. Concluding remarks

Based on the reference cone approach we have proposed an interactive algorithm for solving discrete multicriteria choice problem. The algorithm enables the DM to realize a convenient and easy understandable way of setting preferences as desired directions of the criteria alteration with respect to a given reference alternative. This algorithm has a very good "learning" influence on the DM, since it helps the taking into account of the criteria significance, their correlation and the possibility for compensation among them.

The algorithm suggested is tested on the basis of problems, available in the references or problems, connected with the privatization evaluations of different industrial enterprises in Bulgaria.

## References

1. Korhonen, P., J. Wallenius, S. Zionts. Solving discrete multiple criteria problems using convex cones. – Management Science, **30**, 1984, 1336–1345.

2. Korhonen, P. A visual reference direction approach to solving discrete multiple criteria problems. – European Journal of Operational Research, **34**, 1988, 152–159.

3. Lotfi, V., T. J. Stewart, S. Zionts. An aspiration level interactive model for multiple criteria decision making. – Computer & Operation Research, **19**, 1992, 671–681.

4. Roy, B. Decision aid and decision making. – European Journal of Operational Research, **45**, 1990, 324–331, 1990.

# Алгоритм для решения дискретной задачи многокритериального выбора, основанный на подходе отправного конуса

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(Р е з ю м е)

В статье представлен интерактивный алгоритм для решения задачи многокритериального выбора, основанный на подходе отправного конуса. Лицо, принимающее решения (ЛПР), задает свои предпочтения k улучшению или допустимому ухудшению стойностей критериев в форме желанных направлений изменения критериев по отношению к выбранной отправной альтернативе. На каждой итерации алгоритм находит подмножество "близких" альтернатив, в котором ЛПР может сделать свой выбор или корректировать свои предпочтения для следующей итерации.