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Some Mathematical Aspects of Control of Robots*

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Introduction

The problem of servo-control of manipulating robots is treated by many authors and the results of the efforts could be summarized in three main groups of control methods [1]: linear servo-control methods, model-reference adaptive control (MRAC) methods and learning control methods. The classification is not the absolute one, but it gives some advantages. The group of the MRAC methods divides itself in two subgroups: computed torque methods, involved in [2 and 3], and feed-forward servo-control methods [4-6]. From mathematical point of view, the application of computed torque or feed-forward servo-control method leads to different types of differential equations. Nevertheless in both cases the main problem is to prove the stability of the closed-loop control system.

Maybe for the first time, the idea of rejecting the stability condition was launched in [7], where it is shown that the requirement the closed-loop system to be asymptotically stable does not lead to good performance behaviour of the controlled manipulator, i.e. it does not give good tracking of the desired trajectory. Furthermore the problem of how to apply the results of stability theory to the control of robots manipulators is not considered in [7] and this is made in the presented paper.

Notations and problem statement

The motion of a robot manipulator is described by the second order differential equation [3]:

$$A(\theta)\ddot{\theta} + b(\theta, \dot{\theta}) + g(\theta) = \tau(t, \theta, \dot{\theta}), \tag{1}$$

where $\theta \in R^n$ is the vector of the joint co-ordinates; $A(.) \in R^{n \times n}$ – the generalized inertia matrix; $b(.,.) \in R^n$ – the Coriolis' and centrifugal forces vector; $g(.) \in R^n$ – the gravitation

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forces vector; $\tau(.,.,.) \in R^n$ – the control actions vector, and n – the number of the degrees of freedom of the robot manipulator.

The dots over the letters denote the respective time derivatives. The equation (1) is the mathematical model describing the motion of a robot manipulator. The problem to be solved, from a mathematical point of view, is: Let a desired trajectory:

$$\theta^* = \theta^*(t), \quad \dot{\theta}^* = \dot{\theta}^*(t) \tag{2}$$

be given. A control function has to be found so that the equation (1), resolved with respect to the performance error is asymptotically stable.

This problem has been solved by many authors implying several restrictions and assumptions. In [8, 9] PD and PID servo-control laws are applied to equation (1) in order the point-to-point task motion to be solved, and in (10, 11) a strong proof of the stability of the closed loop system is given. The computed torque method is applied for the first time in [1, 2] and later the control system stability is proven in [12–14]. The common method is the Liapunov's second one. In [4, 5, 15–19] one could find the same method application to the stability of (1) with respect to the performance error, in the case when the feed forward control method is chosen. The difference between the authors approaches consists in the manner of the Liapunov's function choice. All of them use the property that the b() term in (1) is a quadratic form with respect to the joint velocities, with a skew-symmetric matrix. The last property is employed also to assure the stability of (1) in case predictive-adaptive [20] and decentralized adaptive control [21] laws are applied. In such a case the mathematics problem seems to be solved.

From a practical point of view, the problem that should be solved, is: let a desired trajectory:

$$\theta^* = \theta^*(t), \quad \dot{\theta}^* = \dot{\theta}^*(t) \tag{2'}$$

be given. A control function has to be found so that the performance error, i.e., the difference between the actual values of the joint co-ordinates and velocities and the desired ones, are small enough.

The difference between the problems is partially discussed in [7] and it is shown there that the stability condition is not a sufficient condition for a good behaviour of the closed-loop control system with respect to the performance error. The main reasoning in [7] is that the time interval of a robot motion is always a final one while the stability condition guarantees the properties of the solution in infinity.

The problem discussed in this paper is related to another restriction, must be implied ti (1), so that the mathematical model is made closer to the real motion of a robot manipulator. In this case it will be shown that the performance error is bounded without use of the direct method of Liapunov.

Results

The discussion further on is founded on the fact that the joint co-ordinates, velocities and accelerations of the robot manipulator are restricted. Therefore, there exist three bounded domains $\Theta, \dot{\Theta}$ and $\ddot{\Theta}$ of R^n so that:

$$\theta \in \Theta, \ \dot{\theta} \in \dot{\Theta}, \ \ddot{\theta} \in \ddot{\Theta} \qquad \Theta, \ \dot{\Theta}, \ \ddot{\Theta} \subset R^n. \tag{3}$$

The control function τ in (1) is built as a sum of two parts [1]:

$$\tau(t, \theta, \dot{\theta}) = \tau^*(t, \theta, \dot{\theta}) + \tau^f(t, \theta, \dot{\theta}). \tag{4}$$

Here

(5) $\tau^* = \tilde{A}(\theta)\ddot{\theta}^*(t) + K_d[\dot{\theta}^*(t) - \dot{\theta}(t)] + K_p[\dot{\theta}^*(t) - \dot{\theta}(t)]$, $\tilde{A}(\theta) = \mathrm{diag}\, A(\theta)$,

and

(6) $$\tau^f = [A(\theta) - \tilde{A}(\theta)]\,\ddot{\theta} + b(\theta, \dot{\theta}) + g(\theta).$$

The matrices K_d and K_p are constant and diagonal ones and all of their diagonal elements are positive.

In the case when the equations (5) and (6) are exact the closed-loop equation (1) is decoupled and in [1] it is shown how to choose K_d and K_p so that the equation

(7) $$\tilde{A}(\theta(t))\,\ddot{\varepsilon} + K_d\dot{\varepsilon} + K_p\varepsilon = 0$$

is asymptotically stable with respect to the performance error .

In every real case the equations (5) and (6) are not exact and express an estimation of the actual values of the control actions desired. In [1] one can also find an accurate investigation of the influence of the uncertainties on the solution behaviour of (7). If the inaccuracy of the estimation of the $A(\theta)$ matrix is neglected (the influence of this inaccuracy is very small [1]), then the equation (7) becomes nonhomogeneous:

(8) $$\tilde{A}(\theta(t))\,\ddot{\varepsilon} + K_d\dot{\varepsilon} + K_p\varepsilon = e$$

where $e(t, \theta, \dot{\theta})$, function is:

i) a bounded function with respect to the time (physical reasons);

ii) a bounded function with respect to the joint co-ordinates (sin and cos);

iii) a linear function with respect to the joint velocities.

Therefore it follows from (3) that the solution of (8) is bounded if the solution of (7) is asymptotically stable.

It is easy to be seen that the propositions i, ii and iii are correct. The first one of them is due to the restricted feed forward component of the control function. The second proposition does not need any explanations. The third one needs maybe, some more comments. The $b(.,.)$ term of (1), as it was above mentioned, is a quadratic form with respect to the joint velocities and it contains sin-s and cos-s with respect to the joint co-ordinates. Using the well known formula $x^2 - y^2 = (x+y)(x-y)$, it is clear that statement iii holds at least in the sense of enormity.

Therefore the following statement holds: Let the control function is built as it is shown in (4), (5) and (6). Let the gain matrices in (5) satisfy the respective conditions obtained in [1]. Let also holds (3). Then the solution of (8) is bounded.

Discussion

Conditions (3) are not only natural ones. In fact equation (1) does not describe the motion of any real mechanical system. So one can speak about a mathematical model of the motion of a robot manipulator only in the case when equation (1) is furnished with conditions (3).

On the other hand, it seems to appear a closed-loop reasoning. If conditions (3) do not exist, the stability of (8) is not obvious, because it is not clear if the right-hand side of (8) is a bounded function. If restrictions (3) are implied it is not feasible if the stability condition is a necessary one, because (3) directly guarantees that the performance error is bounded. In fact it is not true, because if the gain matrices in (5) are not properly chosen, the performance error will be enormous, even if it is restricted. The problem about the performance error evaluation is discussed partially in [7] and more details can be found in [1]. In this sense the results here must be considered as complementary ones to the results published in [1] and [7].

Conclusion

The problem of completeness of the mathematical model describing the motion of a robot manipulator is discussed. Some natural restrictions was employed so that the mathematical description is made closer to the reality. It is shown that the performance error (the deviation from a desired trajectory) is bounded, even if the time interval of the motion is large.

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Некоторые математические аспекты управления роботами

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(Р е з ю м е)

Математическое моделирование физических процессов – это общепринятый научный метод описания природы. Основной вопрос, которому надо ответить в этом случае, это вопрос о нахождении обхвата математической модели. Такой ограниченный обхват математической модели встречается очень часто. Например, закон Гука можно применить только к относительно малым деформациям, но он не может объяснить эффектов пластических деформаций. В настоящей работе сделана попытка найти некоторые ограничения применимости математической модели движения робота-манипулятора. Здесь рассмотрены существующие физические ограничения и их отражение на результатах теории управления роботами.