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Applications of Reference Point Method for the Analysis of Linear Fractional Programming Problems*

Boyan Metev

Institute of Information Technologies, 1113 Sofia

1. Introduction

The (scalar) linear fractional programming problem can be presented in the following way:

(1) $$\max \left[f(x) = \frac{p(x)}{q(x)} \right]$$

s.t.

$$x \in S \subset R^n,$$

where $p(x)$ and $q(x)$ are linear functions and the set S is defined as follows:

$$S = \{x \mid Ax = b,\ x \geq 0\}.$$

Here A is a real $m \times n$ matrix. We will suppose that S is a bounded polyhedron and $q(x)>0,\ \forall x \in S$. It is well known [1] that the goal function in (1) has a global maximum on S and has not any other local maxima. This maximum is obtained at an extreme point of S. These properties are in the base of the Gilmore and Gomory's algorithm [1] for solving problem (1). This algorithm is a modification of the simplex method for linear programming (LP) problems. Charnes and Cooper have proposed another algorithm [1, 9], based on variables substitution and replacing LP problem.

The multiple objective linear fractional programming problem can be presented in the following way:

$$\max \frac{p_1(x)}{q_1(x)}$$

$$\max \frac{p_2(x)}{q_2(x)}$$

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$$\max \frac{p_s(x)}{q_s(x)}$$

s.t.

$$x \in S.$$

Here S, $p_i(x)$ and $q_i(x)$ are defined as in problem (1). The parameter s determines the number of partial criteria in (2). We will assume that $q_i(x)>0$, $\forall i$, $\forall x \in S$.

Some information about applications of linear fractional programming problems can be found in [1, 4, 8, 9]. In [9] Steuer proposes a well described example of problem (2) however the book does not contain a geral method for an analysis of this problem. In [8] Nykowski and Zolkiewski have proposed for problem (2) a replacing MOLP problem and a corresponding compromise programming procedure for its solving (using ADBASE software). In [4] Dutta, Rao and Tiwari have shown a way to improve the results from [8] for the special case when all $q_i(x)$ coincide.

Problem (1) and problem (2) are treated here in two ways. On one hand linear programming problems are proposed with the following idea. If for problem (1) we know a feasible point (that is not the optimal one) we can find another feasible point, where the goal function has a better value. If for problem (2) we know a feasible point that is not weak efficient, we can find another feasible point that is better with respect to all partial criteria. On the other hand problem (1) and problem (2) can be analyzed with the use of some replacing multiple objective linear programming (MOLP) problem, described in [8]. In this paper Nykowski and Zolkiewski have proposed a compromise programming procedure for analysis of replacing MOLP problems. Instead of such procedure the reference point method [10, 11] is used here for obtaining weak efficient points for the replacing problem and in this way for solving problem (1) and for analyzing problem (2).

2. The scalar linear fractional programming problem – a method with given feasible point

Let us consider the following LP problem

(3)
$$\max t$$

s.t.

$$p(x) = L^0 q(x) + t,$$
$$x \in S$$

Here $p(x)$, $q(x)$ and S are defined as in problem (1); in addition we have

(3a)
$$L^0(x) = \frac{p(x^0)}{q(x^0)},$$

where $x^0 \in S$ is not the point that gives the maximum in problem (3).

Theorem 1. Let $x^0 \in S$ and the condition (3a) hold. Suppose that the point (x^1, t^1) is a solution of problem (3). Then we have

$$\frac{p(x^1)}{q(x^1)} \geq \frac{p(x^0)}{q(x^0)}.$$

The proof can be done supposing the opposite and obtaining a contradiction.

It must be pointed out that the solution of problem (3) does not give always the maximum defined by problem (1). For the point determined by this solution we can write and solve a similar problem and as a result we will get a new better point or we will discover that the searched maximum is obtained. This can be illustrated by numerical examples.

3. The scalar linear fractional programming problem – a method with an auxiliary MOLP problem

Under the assumption $p(x)>0$, $q(x)>0$ $(\forall x \in S)$ we will consider the following problem

(4.a)
$$\max p(x),$$
$$\max (A - q(x))$$

s.t.

$$x \in S, \ \{p(x)>0; \ q(x)>0 \ (\forall x \in S); \ A \geq 0\}.$$

Under the assumption $p(x)>0$, $q(x)>0$ $(\forall x \in S)$ we will consider the problem

(4.b)
$$\max p(x),$$
$$\max q(x)$$

s.t.

$$x \in S, \ \{p(x)>0; \ q(x)>0 \ (\forall x \in S)\}.$$

Here the functions $p(x)$, $q(x)$ and the set S are defined as in problem (1). In addition $A \geq 0$.

Theorem 2. If the conditions $p(x)>0$; $q(x)>0$ $(\forall x \in S)$ hold, then the point x^0 that gives the maximum in problem (1) is a weak efficient point for problem (4a). If the conditions $p(x)<0$; $q(x)>0$ $(\forall x \in S)$ hold, then the point x^0 that gives the maximum in problem (1) is a weak efficient point for problem (4b).

The proof can be done supposing the opposite and obtaining a contradiction.

Theorem 2 shows that instead of solving problem (1) we can consider problem (4a) or problem (4b) N y k o w s k i and Z o l k i e w s k i [8] have presented this result in a more general form – for multiple objective linear fractional programming problems and with assertions that concern the sets of efficient points. For the analysis of the replacing MOLP problem they have proposed a variant of compromise programming procedure and the usage of a special software – ADBASE. Here for such purposes the reference point method [2, 5, 10, 11] will be used.

Having in mind problem (4a) we will consider the following LP problem:

(5.a)
$$\min (D_1 - D_2)$$

s.t.

$$D_1 - D_2 \geq b_1 (r_1 - p(x)),$$
$$D_1 - D_2 \geq b_2 (r_2 - A + q(x)),$$
$$x \in S.$$

Having in mind problem (4b) we will consider another LP problem :

(5.b)
$$\min (D_1 - D_2)$$

s.t.

$$D_1 - D_2 \geq b_1 (r_1 - p(x)),$$
$$D_1 - D_2 \geq b_2 (r_2 - q(x)),$$
$$x \in S.$$

Here the set S is defined as in problem (1). We have also $b_i > 0$, $i = 1, 2$. The parameters r_1 and r_2 are the reference point components. The following conditions must hold

(6a) $r_1 > \max p(x)$, where $x \in S$ – for problems (5a) and (5b),

(6b) $r_2 > \max (A - q(x))$, where $x \in S$ – for problem (5a),

(6c) $r_2 > \max q(x)$, where $x \in S$ – for problem (5b).

Theorem 3. When the conditions (6a) and (6b) hold the solution of problem (5a) determines a point x' that is a weak efficient point for problem (4a). When the conditions (6a) and (6c) hold the solution of problem (5b) determines a point x' that is a weak efficient point for problem (4b).

The proof can be done supposing the opposite and obtaining a contradiction.

Suppose that $p(x)>0$, $q(x)>0$ in problem (1). Theorem 3 and the results from [5] allow us starting from a given feasible point and choosing a suitable reference point to obtain a weak efficient point for problem (4a) and to increase the numerator or to decrease the denominator of the goal function in problem (1). So we can improve the value of this goal function. This idea can be illustrated by numerical example and can be used for solving the general case of scalar linear fractional programming problem.

Theorem 4. Consider problem (4a) and problem (5a). If r_1 and r_2 determine an attainable point in the criterion space that is not a weak Pareto point, then the solution of problem (5a) determines a point f^m in the criterion space that dominates the point $r=(r_1, r_2)$.

Theorem 5. The same assertion as in Theorem 4 is true for problems (4b) and (5b).

The proofs can be done supposing the opposite and obtaining a contradiction.

Roughly speaking the last two theorems show that the weak efficient points of problem (4a) or (4b) are attainable and thus the point giving the maximum in problem (1) is attainable too. The auxiliary MOLP problem has nearly the same dimensionality, there is no need of variable substitution. A standard LP software is sufficient however the auxiliary MOLP problem must be solved several times. This approach could be further developed using the information for solution sensitivity with respect to the reference point changes. For such purposes standard software is sufficient too [6].

4. The multiple objective linear fractional programming problem – using a given feasible point and an auxiliary maximin problem

Having in mind problem (2) let us consider the following LP problem

(7)
$$\max\ (D_1 - D_2)$$

s.t.

$$D_1 - D_2 \leq t_i \quad (i = 1, \ldots, s),$$
$$p_i(x) = L_i^0\, p_i(x) + t_i B_i \quad (i = 1, \ldots, s),$$
$$x \in S,\ D_1 > 0,\ D_2 > 0.$$

Here $p_i(x)$, $q_i(x)$ and S are the same as in problem (2). In addition we have

(8)
$$L_i^0(x) = \frac{p_i(x^0)}{q_i(x^0)} \quad \forall i,$$

where x^0 is a feasible point and is not a weak efficient point. Problem (7) is a generalization of problem (3).

Theorem 6. Consider problem (2) and the corresponding problem (7). Suppose that the conditions (8) hold. Then the solution of problem (7) determines a point (x^1, t^1) such that the following is true

$$f_i(x^1) \geq f_i(x^0),$$

where $f_i(x) = p_i(x)/q_i(x)$, $(\forall i)$.

The proof can be done using direct estimations of $(D_1 - D_2)$ for the domain where the announced inequalities hold, and for domains, where one of these inequalities at least is not true.

It is possible to create a sequence of problems similar to problem (7) in the following way: the solution of given problem from this sequence determines the values L_i^0 for the next problem. This sequence of problems will give a sequence of points in the criterion space. Each point of this sequence will not be worse than the previous one, the sequence will stop at weak Pareto point thus determining a weak efficient point for problem (2).

5. The multiple objective linear fractional programming problem – a usage of an auxiliary (modeling) MOLP problem

Having in mind problem (2) we will consider two cases.. Under the assumption $p_i(x)/q_i(x) > 0$ (for all i and all $x \in S$) we will consider the following MOLP problem

(9a)

$$\begin{array}{ll} \max p_1(x) & \max(-q_1(x)), \\ \max p_2(x) & \max(-q_2(x)), \\ \dots & \dots \\ \max p_s(x) & \max(-q_s(x)) \end{array}$$

s.t.

$$x \in S.$$

Under the assumption $p_i(x)/q_i(x) < 0$ (for all i and all $x \in S$) we will consider another MOLP problem

(9b)

$$\begin{array}{ll} \max p_1(x) & \max q_1(x), \\ \max p_2(x) & \max q_2(x), \\ \dots & \dots \\ \max p_s(x) & \max q_s(x) \end{array}$$

s.t.

$$x \in S.$$

Problems (9a) and (9b) are proposed in [8] where the following is proved.

Theorem 7. [8] If $p_i(x)/q_i(x) > 0$ (for all i and all $x \in S$) then the set of efficient points for problem (2) is a subset of the set of efficient points for problem (9a). If $p_i(x)/q_i(x) < 0$ (for all i and all $x \in S$) then the set of efficient points for problem (2) is a subset of the set of efficient points for problem (9b).

It is possible that for some i we have $p_i(x)/q_i(x) > 0$ (all $x \in S$) and for all the rest j we have $p_j(x)/q_j(x) > 0$ (all $x \in S$). For this case the auxiliary MOLP problem is as follows [8]: for each i the criteria $\max p_i(x)$ and $\max(-q_i(x))$ are taken into account, for each j the criteria $\max p_j(x)$ and $\max q_j(x)$ are taken into account. Thus all the criteria in the auxiliary MOLP problem are determined. In addition $x \in S$.

Then we have a similar theorem: The set of efficient points of problem (2) is a subset of the set of efficient points for the so formulated auxiliary MOLP problem.

It must be pointed that the mentioned theorems concern the sets of efficient points. However it is well known [8,9] that the set of weak efficient points of a multiple objective linear fractional programming problem is easier for handling than the set of efficient points, moreover we do not lose generality. In [8] the auxiliary MOLP problem is treated by compromise programming with the usage of ADBASE software. For the same purpose we will use here the reference point method [10, 11] and especially the approach from [2, 5]. It is important to note that the technique for handling the set of weak efficient points broadly described in [2] will be put in the base of the analysis proposed here.

Suppose that in problem (2) we have

(Cond) $p_i(x)/q_i(x) > 0$, $i = 1, \ldots, h$, and for all $x \in S$.

$p_i(x)/q_i(x) < 0$, $i = h+1, \ldots, s$ and for all $x \in S$.

Then the auxiliary MOLP problem is

(10)
$$\begin{array}{ll} \max p_1(x) & \max (-q_1(x)), \\ \max p_2(x) & \max (-q_2(x)), \\ \ldots & \ldots \\ \ldots & \max (-q_h(x)), \\ \ldots & \max (-q_{h+1}(x)), \\ \max p_s(x) & \max (-q_s(x)) \end{array}$$

s.t.

$$x \in S.$$

From [2, 5] we know that the weak efficient points of problem (10) can be determined by the solutions of the following LP problem

$$\min \ (D_1 - D_2)$$

s.t.

(11)
$$\begin{array}{l} D_1 - D_2 \geq b_1^{nu}(r_1^{nu} - p_1(x)), \\ D_1 - D_2 \geq b_2^{nu}(r_2^{nu} - p_2(x)), \\ \ldots \\ D_1 - D_2 \geq b_s^{nu}(r_s^{nu} - p_s(x)), \\ D_1 - D_2 \geq b_1^{de}(r_1^{de} + q_1(x)), \\ D_1 - D_2 \geq b_2^{de}(r_2^{de} + q_2(x)), \\ \ldots \\ D_1 - D_2 \geq b_h^{de}(r_h^{de} + q_h(x)), \\ D_1 - D_2 \geq b_{h+1}^{de}(r_{h+1}^{de} - q_{h+1}(x)), \\ \ldots \\ D_1 - D_2 \geq b_s^{de}(r_s^{de} + q_s(x)), \end{array}$$

s.t.

$$x \in S.$$

Here the superscript *nu* denotes parameters that concern the numerators in the criterion functions in problem (2), the superscript *de* denotes parameters concerning the denominators in the same problem. All parameters denoted by r are components of the reference point for problem (10). It is known [2] that the reference point in (11) must dominate the ideal point for problem (10). This will assure that the solution of problem (11) determines a weak efficient point for problem (10).

The following theorem is needed because it concerns the set of weak efficient points.

Theorem 8. Suppose that in problem (2) $q_i(x) > 0$, for all i and all $x \in S$. Suppose also that the condition (Cond) holds. Then if the point $x^0 \in S$ is a weak efficient point for problem (2), it is a weak efficient point for problem (10).

The proof can be done assuming the opposite and obtaining a contradiction.

It is known from [2,5] that the value of chosen criterion at an efficient point for problem (10) (obtained for fixed reference point and solving problem (11)) can be improved by changing the corresponding component of the reference point only and solving the so changed problem (11). This allow to move in the set of weak Pareto points, of problem (10) i.e. in the set of weak efficient points of problem (2) too. The choice of the changes in the reference point can be improved using the information for the solution sensitivity with respect to the reference point [7].

In general, the so described results about the usage of unattainable reference points and attainable points in the criterion space could be applied in creating a computational procedure for finding preferable weak efficient points for problem (2).

6. Some examples

Example 1. Let us consider an example [1] of (scalar) linear fractional programming problem. We will use the vector approach and the reference point method. The problem is

$$\max \frac{-2x_1 + x_2 + 2}{x_1 + 3x_2 + 4}$$

s.t.

$$-2x_1 + x_2 \le 4; \quad 2x_1 + x_2 \le 14;$$
$$x_1 \le 6; \quad x_1, x_2 \ge 0.$$

It is evident that $q_i(x) > 0$ for all feasible x but $p(x)$ can change the sign In this case we will use problems (4.a) and (5.a). We assume $A = 0$. In addition $b_1 = b_2 = 1$, and $r_1 = 10$, $r_2 = 1$. Then the problem (5.a) becomes

$$\min\ (D_1 - D_2)$$

s.t.

$$D_1 - D_2 \ge r_1 + 2x_1 - x_2 - 2,$$
$$D_1 - D_2 \ge r_2 + x_1 + 3x_2 + 4,$$
$$-x_1 + x_2 \le 4; \quad 2x_1 + x_2 \le 14,$$
$$x_2 \le 6; \quad x_1 \ge 0, \quad x_2 \ge 0;$$
$$u_1 - u_2 = -2x_1 + x_2 + 2,$$
$$v_1 - v_2 = x_1 + 3x_2 + 4,$$
$$r_1 = 10,$$
$$r_2 = 1.$$

The solution gives $u_1 = 2.75$, $v_1 = 6.25$, $u_2 = 0$, $v_2 = 0$:

The value of the goal function in problem (1) is $2.75:6.25 = 0.44$.

If r_1 increases we must get an increased value of the numerator. Let $r_1 = 12$, $r_2 = 1$. Now the solution gives $u_1 = 3.25$, $v_1 = 7.75$, $u_2 = 0$, $v_2 = 0$.

The value of the goal function is $3.25:7.75 = 0.4193548$.

Here (as in the general case) the goal function has a unique global maximum and has not any other local maxima. If this change of the numerator does not increase the goal function we must try to decrease the denominator. We choose $r_1 = 10$, $r_2 = 4$. From the solution we obtain $u_1 = 2$, $v_1 = 4$, $u_1 = 2$, $v_1 = 4$. The value of the goal function is $2 : 4 = 0.5$. This is the searched maximum. Further increasing of r_2 does not change the solution. Further increasing of r_1 increases the value of the numerator, but the goal function value decreases.

Example 2. We will use here an example of C h o o, considered in [9]. This book contains a very good description of the weak efficient set, but there is not a full algorithm for analysis of the problem. The very problem is

(12)
$$\max\ f_1 = \frac{x_1}{x_2},$$
$$\max\ f_2 = x_3,$$
$$\max\ f_3 = \frac{-(x_1 + x_3)}{1 + x_2}$$

s.t.

$$1 \le x_1, x_2, x_3 \le 4.$$

The weak efficient set E^w is [9]

$$E^w = U_1 \cup U_2 \cup U_3 \cup U_4 \cup U_5,$$

where

$$U_1 = \{x \in S \mid x = (a, b, c), a = bc\},$$
$$U_2 = \{x \in S \mid x = (4, b, c), bc > 4),$$

U_3 is the set of all convex combinations of the points
(1, 4, 4), (1, 4, 1), (4, 4, 1), (4, 4, 4),
U_4 is the set of all convex combinations of the points
(4, 1, 4), (1, 1, 4), (1, 4, 4), (4, 4, 4).
U_5 is the set of all convex combinations of the points (4, 1, 1), (4, 1, 4), (1, 1, 1).

A part of the experimental results is given in Table 1. The first column contains the row number. The next three columns contain the components of some initial feasible point (in the argument space) and the next three – the corresponding criteria values. The parameters of problem (7) for this feasible point are computed and $B_i \equiv 1$. The so formulated problem (7) is solved and its solution determines a new feasible point. The components of the new feasible point are printed in the next triad of columns. The corresponding criteria values are printed in the last triad of columns.

It can be seen that the solution of problem (7) gives each time a weak efficient point. It must be noted that this point is obtained each time with the first solution of problem (7) Following formulations (10) and (11) an auxiliary MOLP and a corresponding LP problems (with reference point) are considered for problem (12). The initial reference point is given by

$$r_1 = 5,\quad r_2 = 1,\quad r_3 = 5,\ r_4 = 1,\quad r_5 = 6.$$

Table 1

No	Initial feasible point						Feasible point obtained with the problem (7) solution					
	x_1	x_2	x_3	f_1	f_2	f_3	x_1	x_2	x_3	f_1	f_2	f_3
1	3	4	1	0.75	1	−0.8	3	4	1	0.75	1	−0.8
2	2	4	3	0.5	3	−1.0	2	4	3	0.5	3	−1.0
3	1	3	4	0.333	4	−1.25	1	3	4	0.333	4	−1.25
4	2	2	4	1	4	−2.0	2	2	4	1	4	−2.0
5	3	1	2	3	2	−2.5	3	1	2	3	2	−2.5
6	3	1	3	3	3	−3.0	3	1	3	3	3	−3.0
7	2.5	1	3	2.5	3	−2.75	4	1.58	3.048	2.53	3.048	−2.73
8	2	1	3	2	3	−2.5	4	1.92	3.15	2.08	3.15	−2.45
9	2	2	1	1	1	−1	4	4	1	1	1	−1
10	3	3	1	1	1	−1	4	4	1	1	1	−1
11	3.5	1.5	1	2.33	1	−1.8	2.42	1	1.09	2.42	1.09	−1.76
12	2.5	3	1	0.8333	1	−0.875	3.347	4	1.014	0.84	1.014	−0.872
13	1	2	2	0.5	2	−1	2.33	4	2.33	0.58	2.33	−0.93
14	1	2	3	0.5	3	−1.333	2.555	4	3.555	0.64	4	−1.222
15	2	2	2	1	2	−1.333	4	3.804	2.195	1.05	2.195	−1.29
16	2	1	3	2	3	−2.5	4	1.92	3.15	2.08	3.15	−2.45

Table 2 shows the results obtained for various reference points.

Table 2

No	Reference point components					Obtained feasible point			Corresponding values of the criteria		
	r_1	r_2	r_3	r_4	r_5	x_1	x_2	x_3	f_1	f_2	f_3
1	5	1	5	1	6	2	3	1	0.6667	1	−0.75
2	6	1	5	1	6	2	3	1	0.6667	1	−0.75
3	8	1	5	1	6	3	2	1	1.5	1	−1.333
4	10	1	5	1	6	4	1	1	4	1	−2.5
5	5	2	5	1	6	1	2.5	1	0.4	1	−0.571428
6	5	3	5	1	6	1	2	1	0.5	1	−0.6667
7	5	5	5	1	6	1	1	1	1	1	−1.0
8	5	1	6	1	6	1	3	2	0.333	2	−0.75
9	5	1	7	1	6	1	2.5	2.5	0.4	2.5	−1.0
10	5	1	8	1	6	1	2	3	0.5	3	−1.3333
11	5	1	10	1	6	1	1	4	1	4	−2.5
12	5	1	5	2	6	1	3	1	0.333	1	−0.5
13	5	1	5	3	6	1	2	1	0.5	1	−0.666
14	5	1	5	4	6	1	1	1	1	1	−1.0
15	5	1	5	1	7	2.5	3.5	1	0.714286	1	−0.7777
16	5	1	5	1	8	3	4	1	0.75	1	−0.8
17	5	1	5	1	9	4	4	1	1	1	−1.0
18	5	5	5	1	7	1	1.5	1	0.6666	1	−0.8
19	5	5	5	1	8	1	2	1	0.5	1	−0.6666
20	5	5	5	1	10	1	3	1	0.333	1	−0.5
21	5	5	5	1	12	1	4	1	0.25	1	−0.4

All points with components x_1, x_2, x_3, shown in this table are weak efficient for the auxiliary MOLP problem however only bold faced of them are weak efficient for problem (12). It can be seen that monotone changes in a reference point component can lead to a weak efficient point for problem (12). The underlined numbers in the last three columns are the maxima of the corresponding criteria

7. Some comments

The proposed ways for solving the scalar and the vector linear fractional programming problems seem to be satisfactory. They use standard LP software only. They do not need a variable substitution. A correct formulation of the auxiliary LP or MOLP problem and a proper use of attainable or reference points is needed. The dimentionality of the auxiliary LP or MOLP problems does not increase significantly. In general these results show that multiple objective linear programming tools can successfully be used for analysis of other mathematical programming problems.

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Применение метода эталонной точки для анализа дробно-линейных задач программирования

Боян Метев

Институт информационных технологий, 1113 София

(Р е з ю м е)

В работе рассматривается скалярная и векторная задачи дробно-линейного программирования. Для этих задач предлагаются два способа анализа. При первом способе, решая вспомогательную задачу линейного программирования, осуществляется переход от заданной допустимой точки (она находится в аргументном пространстве и не является слабоэффективной) к новой допустимой точке, которая лучше исходной. При втором способе используется вспомогательная задача многокритериального линейного программирования (МКЛП). С помощью эталонной точки осуществляется движение в множестве слабоэффективных точек этой МКЛП задачи. Это множество содержит точки, которые экстремизируют целевую функцию (скалярной задачи), или множество слабоэффективных точек (векторной задачи). Во всех случая в исчислениях используются стандартные задачи линейного программирования.