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MODELS AND SOFTWARE ARCHITECTURES OF DECISION SUPPORT SYSTEMS

ABSTRACT

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Title: **Models and Software Architectures of Decision Support Systems**

INTRODUCTION

The usage of methods and models of multi-criteria decision-making is important because they provide a systematic approach for evaluating alternatives. Such are the Weighted Sum Model and the Weighted Product Model, and they are the focus of this dissertation. They stand out with their simplicity, transparency and wide applicability in various processes, such as business, engineering, social, management and others. Despite their wide applicability, the classical models can't fully reflect the actual conditions in which decision-makers (DMs) operate. In practice, situations often arise in which the DMs have different levels of competence, while the desired solution should satisfy requirements related to key indicators derived from available objective data. These features determine the need for developing modifications that not only increase the precision and objectivity of the decisions made, but also facilitate the evaluation process and the structuring of the decision-making task.

To ensure the practical applicability and accessibility of such modifications, it is appropriate to implement them in web-based applications. Because of this, it is necessary to design an appropriate software architecture as a base for the development of a specific application and to meet the requirements for flexibility and scalability. By applying well-established principles and best practices in the design of the architectures, the risk of creating chaotic and hard-to-maintain systems can be minimised.

Considering the close relationship between decision-support models and software architectures, it becomes clear that a comprehensive understanding is required of both the decision-making processes and the software tools that implement these models.

The dissertation is structured in the following 4 chapters.

In Chapter 1 is presented an analysis of decision-making methods and software architectures for implementing web applications. Various concepts and classifications of multi-criteria analysis methods are introduced. Special attention is given to the weighted sum method and the weighted product method. A comparison between the two methods is provided, and some of their modifications are discussed. Particular emphasis is placed on the classifications of decision support systems (DSS). The second part of Chapter 1 focuses on software architectures for implementing web applications. An analysis of software architecture concepts, architectural styles, and patterns is presented. Based on this analysis, conclusions are drawn, which serve as the basis for formulating the aim and objectives of the present dissertation.

In Chapter 2 are described the proposed modifications of the weighted sum model and the weighted product model, motivated by the need for a more precise and relatively objective formation of group or, in particular, individual decisions. The first modification considers the differences in the competence of decision-makers with respect to various criteria by using competence-based weighting coefficients. The second modification introduces the usage of objective data, combining it with the preferences of the decision-makers. In this way, the subjectivity is reduced, and the process of evaluating alternatives is facilitated. This modification applies only to the weighted sum model. Essentially, it does not change the mathematical model, but adds a formalization of the evaluation generation within the method's implementation algorithm. The third modification introduces a new type of weighting coefficients for the alternatives, based on key criteria that represent critical requirements of the specific task. These coefficients allow the suppression or amplification of aggregated results, by adding non-compensatory characteristics to the weighted sum and weighted product models. At the same time, the number of subjective evaluations is reduced, and emphasis is placed on the importance of the key requirements of the task. This modification applies to both methods and it affects both the algorithm for their application and the mathematical models. Each of the proposed modifications is described following an identical structure.

In Chapter 3 is given a description the proposed and designed two different software architectures of the DSS. The design of two architectures for the same system is motivated by the need to be evaluated and selected an appropriate set of technologies for development and operating environment, considering the technological requirements, the expected workload and the usage context. Based on the modifications described in Chapter 2 are formulated combined and generalized mathematical models. These models define the core of the decision-making business logic. A generalized algorithm that facilitates the mathematical models is proposed, and its crucial steps are described in pseudocode. The functionality of the DSS is designed using adapted UML diagrams. The roles, the functional requirements and the use cases of the system are defined. The necessary forms and the interaction sequence between the user and the system are also described.

In Chapter 4 are presented the experiments conducted with the proposed modifications of the usage algorithm and the mathematical models of the weighted sum model and the weighted product model. The experiments with the three modifications, the generalized algorithm, and the developed DSS prototype are described. The experiments are conducted on three different decision-making tasks. For each task are formulated different scenarios, implemented using the same objective input data and task parameters.

The conclusion summarizes the obtained results and outlines some directions for future research.

CHAPTER 1. ANALYSIS OF DECISION-MAKING METHODS AND SOFTWARE ARCHITECTURES FOR IMPLEMENTING WEB APPLICATIONS

Multiple Criteria Decision Making (MCDM) methods offer mathematically grounded approaches for problems involving multiple criteria and alternatives. Studies present the wide application of these methods across various practical fields [Taherdoost and Madanchian, 2023]. Statistical research conducted across different industries shows that MCDM methods are primarily used for strategic and tactical decision-making, while they are applied less frequently at the operational level [Khan et al., 2018].

The development of web-based Decision Support Systems opens new opportunities for integrating established MCDM methods, providing an intuitive interface, centralized data management, and accessibility from any location [Magabaleh et al., 2024]. Such a system would allow decision-makers to focus on the essence of the problem, while the other tasks, related to automating routine operations, such as input data processing, normalization, and aggregation to be delegated to the software system.

1.1. Multi-Criteria Decision-Making methods

In recent years, there has been a growing trend towards integrating MCDM with data analytics, artificial intelligence, and machine learning to improve decision-making processes [Amini et al., 2024]. This reflects the shift towards more data-driven approaches in various fields.

1.1.1. Concepts and classifications of Multi-Attribute Decision-Making methods

Unlike MODM, the goal of MADM is to make a decision over a discrete set of alternatives characterized by multiple, often conflicting attributes/criteria. In the dissertation, the research subject is focused on the Weighted Sum Model (WSM) and the Weighted Product Model (WPM).

1.1.2. Weighted Sum Model and Weighted Product Model

One of the earliest mathematical formulations of the WSM precursor can be found as a sum of products used to determine expected effectiveness when selecting a direction for development in the fields of economics and statistics. This formulation is described in the work of [Churchman and Ackoff, 1954]. The roots of the WPM can be traced back to Bridgman in 1922 in the context of dimensional analysis and are later adapted by Miller and Starr in 1969 for decision-making problems [Triantaphyllou and Mann, 1989].

1.1.3. Decision-making algorithm using the Weighted Sum Model and the Weighted Product Model

The WSM and WPM methods share the same algorithm for the overall decision-making process [Bozorg-Haddad et al., 2021]. In the reviewed literature, the algorithm is often presented with variations in structure and level of detail, and the scope of exposition varies depending on the source [Baker et al., 2001].

1.1.4. Comparison of the Weighted Sum Model and the Weighted Product Model

A comparison between WSM and WPM, showing their characteristics, advantages and disadvantages, is presented in Table 1.4.

Таблица 1.4. A comparison between WSM and WPM.

Characteristics	WSM	WPM
Aggregation	**Sum** of products of alternative evaluations and criteria weights. Linear (additive).	**Product** of the scores raised to the power of the criteria weights. Nonlinear (multiplicative).
Compensation	Full compensation. A high score on one criterion can compensate a low score on another.	Limited compensation. Low values on one criterion are difficult to be compensated.
Behavior towards high scores	Amplifies.	Amplifies the result slightly.
Behavior towards low scores	Reduces the score.	Reduces the score more significantly (strong penalty effect, especially for scores close to 0).
Normalization of scores	Necessary (to make evaluations dimensionless and comparable).	Less frequently needed. Not mandatory [Tofallis, 2014].
Data requirements	They can be positive or negative.	Only positive values, as 0 eliminates the alternative.
Sensitivity to measurement interval	High. A criterion with a high value may dominate [Podvezko, 2011].	Lower, but very small values significantly reduce the result.
Interpretation	A sum of weighted scores is easier to be intuitively interpreted.	A product of weighted scores is more difficult to be perceived intuitively.
Applications	Projects evaluation; Supplier selection; Financial analysis. The most commonly used method in practice, suitable for a wide range of tasks [Taherdoost, 2023].	Engineering, Risk Management. Less commonly used, but useful when it is important to avoid unit inconsistencies and when low scores should be penalized more severely [Nasr et al., 2022].
Advantages	- Easy to understand and implement; - Widely used.	- Avoids overcompensation; - Suitable for tasks where each criterion is critically important; - Suitable for heterogeneous units of measurement.
Disadvantages	- Allows overcompensation - May distort results at extreme values	- Sensitive to very low values (score tends to 0) - More difficult for understanding and interpretation.

1.1.5. Modifications of the Weighted Sum Model and the Weighted Product Model

As some of the oldest and fundamental methods in MCDM, the WSM and WPM methods have undergone numerous modifications and adaptations to other approaches. Big share of these changes is aimed at: (1) extending or altering the set of attributes to which the methods are applied; (2) combining them with methods for handling uncertainty and ambiguity; and (3) developing hybrid models. Over time, the weighted sum and weighted product methods have been enhanced through various modifications. Their evolution demonstrates that they are not static or outdated methods, but they are dynamically adapted to new practical needs [Greco et al., 2025]. As a result, they are becoming more flexible, widely applicable, and suitable for integration into DSS.

1.1.6. Decision support systems

The evolution of software-based DSS is presented as a genealogical tree, with the main types categorized by [Arnott and Pervan, 2014]. The reviewed works demonstrate the wide applicability and practical value of implementing MCDM methods in a software tool [Petković et al., 2025; Cinelli et al., 2022].

1.2. Software architectures for web applications implementation

1.2.1. Concepts for software architectures, architectural styles and patterns

Over the time, various architectural styles have emerged in software engineering, defining the fundamental principles for system design [Garlan и Shaw, 1994]. Specific implementations within these styles are realized through architectural patterns. In practice, different styles and patterns are often combined, allowing their advantages to be leveraged while compensating for their limitations [Richards и Ford, 2020].

1.2.2. Software architectural styles and architectural patterns

Some of the main architectural styles used in the design of web applications, along with commonly used patterns, can be identified based on various sources such as [Garlan and Shaw, 1994]. It is important to note that, in practice, there is no clear distinction between an architectural style and an architectural pattern, neither a clear boundary exists between where the architectural style ends and the where the architecture itself begins [Perry and Wolf, 1992].

1.2.3. Characteristics for comparison of software architectures

In the design of web applications, the selection from the wide variety of interrelated architectural styles and patterns constitutes a complex multi-criteria task on its own. Research in this direction has been conducted by [Galster et al., 2010]. In this regard, it is necessary to consider key characteristics that can serve as criteria for comparing software architectures and for defining non-functional requirements.

1.3. Conclusions

As a result of the conducted review, focused on the Weighted Sum Method and the Weighted Product Method, it is ascertained that both methods are clearly defined, well-established, and appear in various classifications. Their wide application and numerous modifications indicate that their study and further development are promising. The diverse approaches to subjective and objective evaluation, normalization, and weight determination provide flexibility and adaptability to different tasks, both for individual and group decision-making. Although the two methods are highly popular, their application to operational, everyday decisions remains limited. This creates a premise for implementing WSM and WPM in a software tool, where modifications aimed at facilitating their use by decision-makers could bring significant practical value. Since these models involve intensive computational operations, often in real time, as well as a need for transparency and collaboration among multiple stakeholders, the choice of an appropriate software architecture becomes a key factor. The architectural solution must ensure not only correct data processing and aggregation but also high performance, reliability, and user convenience.

1.4. Goal and tasks

The goal of the dissertation is to propose models and software architectures for decision support systems. To achieve the goal, the following tasks need to be completed:

1) to perform an analysis of the Weighted Sum Method and the Weighted Product Method, as well as of the main software architectural styles and patterns for developing web applications;
2) to propose a modification of the WSM and WPM algorithm and models, considering the different competence domains of the DMs;
3) to propose a modification of the WSM algorithm formalizing the generation of alternatives scores;
4) to propose a modification of the WSM and WPM algorithm and models formalizing the generation of coefficients that give an advantage in the overall performance of the alternatives;
5) to propose a generalized algorithm and combined mathematical models that incorporate the modifications defined as objectives;
6) to design different software architectures that satisfy conflicting requirements for the decision support system;
7) to develop prototypes of a DSS in accordance with the designed architectures, including a module for results interpretation;
8) to determine the practical applicability of the proposed modifications, implemented in web application software architectures, through numerical testing of real-world tasks.

CHAPTER 2. MODIFICATIONS TO THE WEIGHTED SUM MODEL AND THE WEIGHTED PRODUCT MODEL

2.1. Modification to WSM and WPM for Group Decision-Making considering DMs' different competence domains

When structuring tasks for multi-criteria group decision-making, it is assumed that an exhaustive set of attributes/criteria is available, ensuring that as many as possible aspects of the problem under consideration are taken into account [Keeney and Gregory, 2005]. This condition requires the usage of criteria from various knowledge domains. Consequently, when a group of competent individuals formed, it is expected that they are proven experts in their respective fields of knowledge, although they do not always possess the same level of competence.

2.1.1. Algorithm implementing the modification of the WSM and WPM methods, considering DMs' different competence domains

The proposed algorithm, that implements the modification of WSM and WPM considering the competences of the DM, is illustrated by a schematic diagram shown in Fig. 2.1.

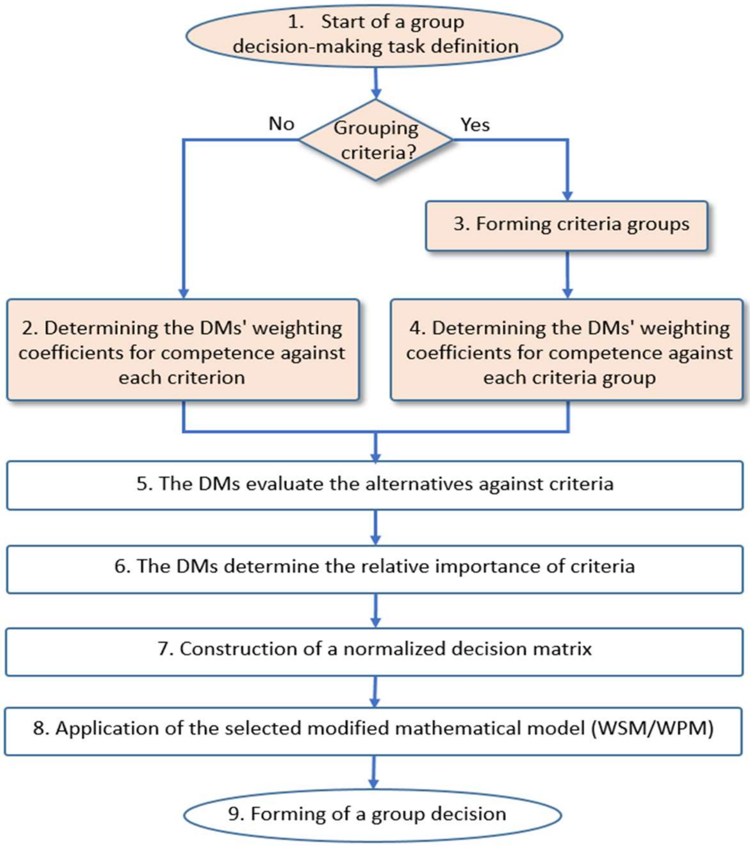


Fig. 2.1. Algorithm implementing the modification of the WSM and WPM methods, considering DMs' different competence domains.

2.1.2. Modified mathematical models of the WSM and WPM methods, considering DMs' different competence domains

In this dissertation, a modification is proposed, by which the global task-specific competence coefficient of a DM [Borissova et al., 2018] is transformed from a scalar value into a vector. Thus, instead of considering the individual contribution of a DM to the scores by criteria with equal importance, the coefficient expressing its competence λ^k, will be considered as a vector $\lambda_j^k = \{\lambda_1^k, \lambda_2^k, \ldots, \lambda_j^k, \ldots\}$. The elements of the new vector contain coefficients corresponding to the level of competence of the DM with in accordance to each criterion.

In this formulation, the vector $\{\lambda^k\}$ is transformed into the matrix (two-dimensional array) $\{\lambda_j^k\}$. Therefore, the mathematical formulation of the proposed modified model of the Weighted Sum Model takes the following form:

$$A_{i\,ModifiedWSM}^{GDM} = \sum_{k=1}^{K} \sum_{j=1}^{J} \lambda_j^k w_j^k a_{i,j}^k \tag{2.1}$$

subject to:

$$\sum_{j=1}^{J} w_j^k = 1, \ \ \forall k = 1, 2, \dots, K \tag{2.2}$$

$$\sum_{k=1}^{K} \lambda_j^k = 1, \ \ \forall j = 1, 2, \dots, J \tag{2.3}$$

$$a_{i,j}^k, w_j^k, \lambda_j^k \ \in [0, 1] \tag{2.4}$$

where: $A_{i\,ModifiedWSM}^{GDM}$ expresses the aggregated score of the i-*th* alternative, calculated by the thus modified WSM method; the index i denotes the current alternative out of a total of I; the index k denotes the *k-th* DM out of a total of K; the index j denotes the *j-th* criterion out of a total of J; λ_j^k is a weighting coefficient that reflects the level of competence of the k-*th* DM according to the j-*th* criterion; w_j^k is a weighting coefficient that reflects the relative importance of the j-*th* criterion according to the k-*th* DM; $a_{i,j}^k$ is the evaluation given to the i-*th* alternative by the k-*th* DM according to the j-*th* criterion.

Following the same logic, the mathematical formulation of the proposed modified model of the WPM takes the form:

$$A_{i\,ModifiedWPM}^{GDM} = \prod_{k=1}^{K} \prod_{j=1}^{J} \left(a_{i,j}^k\right)^{w_j^k \lambda_j^k} \tag{2.5}$$

subject to the same restrictions (2.2), (2.3) and different range for the values of the evaluation scores:

$$a_{i,j}^k = (0,1] \tag{2.6}$$

where: $A_{i\,ModifiedWPM}^{GDM}$ expresses the aggregated score of the i-*th* alternative, calculated by the thus modified WPM method.

By application of **Step 3** and **Step 4** of the algorithm in Fig. 2.1, the mathematical expressions (2.1) – (2.4) are transformed in a way to take into account the competence coefficients of the DM by groups. The modification of the WSM is expressed as follows:

$$A_{i\,ModifiedWSM}^{GDM} = \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{m_l=1}^{M_l} \lambda_l^k w_{m_l}^k a_{i,m_l}^k \tag{2.7}$$

subject to:

$$\sum_{l=1}^{L} \sum_{m_l=1}^{M_l} w_{m_l}^k = 1, \qquad \forall k = 1, 2, \dots, K \tag{2.8}$$

$$\sum_{k=1}^{K} \lambda_l^k = 1, \qquad \forall l = 1, 2, \dots, L \tag{2.9}$$

$$a_{i,m_l}^k, w_{m_l}^k, \lambda_l^k \ \in [0, 1] \tag{2.10}$$

where: the index l denotes the l-*th* group of criteria out of a total of L; m_l denotes the m-*th* criterion of the l-th group of criteria, containing M_l count of criteria; $w_{m_l}^k$ is a coefficient that reflects the relative importance of the m_l-*th* criterion according to the k-*th* DM; λ_l^k is a weight coefficient that reflects the level of competence of the k-*th* DM relative to the l-*th* group of criteria; a_{i,m_l}^k is the evaluation score given to the i-th alternative by the k-th DM in accordance to the m_l-th criterion.

Corresponding with the above, the mathematical model of the WPM method takes the following form:

$$A_{i\ ModifiedWPM}^{GDM} = \prod_{k=1}^{K} \prod_{l=1}^{L} \prod_{m_l=1}^{M_l} (a_{i,m_l}^k)^{\lambda_l^k w_{m_l}^k} \qquad (2.11)$$

subject to the same restrictions (2.8), (2.9) and different range for the values of the evaluation scores:

$$a_{i,m_l}^k = (0,1] \qquad (2.12)$$

2.2. Modification of WSM formalizing the generation of alternatives scores

2.2.1. Algorithm implementing the modification of the WSM method formalizing the generation of alternatives scores

The proposed modification can be used to solve practical problems, by following the steps of the algorithm presented with diagram in Fig. 2.3.

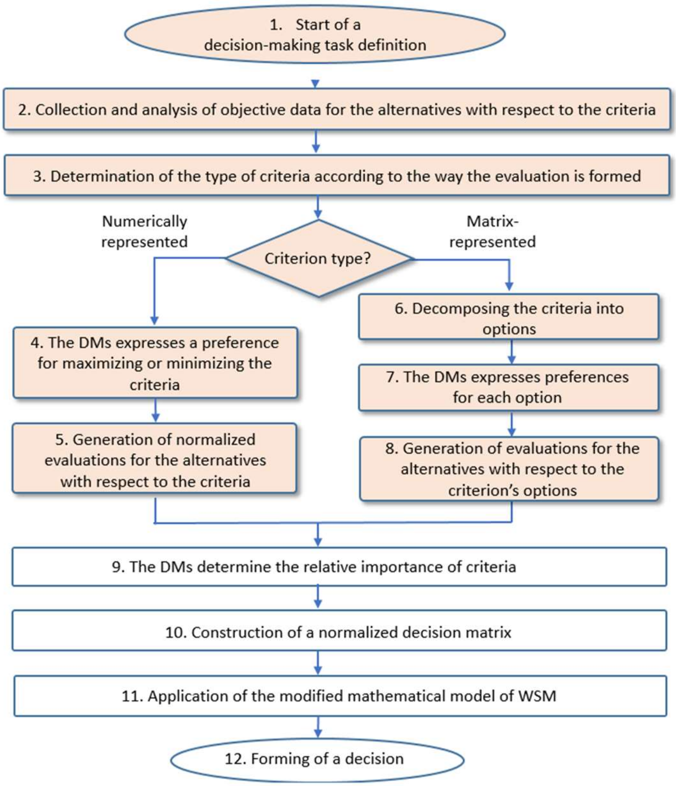


Fig. 2.3. Algorithm implementing the modification of the WSM method formalizing the generation of alternative scores.

2.2.2. Modified mathematical model of the WSM method formalizing the generation of alternative scores

In the present dissertation, a modification of the WSM is proposed, allowing the integration of two approaches for generating evaluations of alternatives in accordance to the criteria. The generated evaluations are based on objective data while simultaneously considering the individual preferences/opinions of the DMs. In the general case of group decision-making, the mathematical model of the modified WSM takes the following form:

$$A_{i\ ModifiedWSM}^{GDM} = \sum_{k=1}^{K} \sum_{j=1}^{J} w_j^k \beta_{i,j}^k \tag{2.13}$$

subject to:

$$\sum_{j=1}^{J} w_j^k = 1,\ \forall k = 1, 2, \dots, K \tag{2.14}$$

$$\beta_{i,j}^k, w_j^k \ \in [0, 1] \tag{2.15}$$

where: $A_{i\ ModifiedWSM}^{GDM}$ expresses the aggregated score of the i*-th* alternative, calculated by the thus modified WSM method; the index i denotes the current alternative out of a total of I; the index k denotes the *k-th* DM out of a total of K; the index j denotes the *j-th* criterion out of a total of J; w_j^k is a weighting coefficient that reflects the relative importance of the j*-th* criterion according to the k*-th* DM; $\beta_{i,j}^k$ s the generated score of the i*-th* alternative, taking into account the preferences of the k*-th* DM towards the j*-th* criterion. Depending on the type of the criterion numerically represented or matrix represented, two types of generated scores are distinguished:

$$\beta_{i,j}^k = \begin{cases} (b_{i,j}^k)^{norm} & b \ \in [0, 1] \\ c_{i,j}^k & c \ \in \ [0, 1] \end{cases} \tag{2.16}$$

where: $(b_{i,j}^k)^{norm}$ is the generated score based on the criteria of numerical type; $c_{i,j}^k$ is the generated score based on the criteria of matrix type. Depending on the preference of the DM, the score $(b_{i,j}^k)^{norm}$ can take one of the two normalized values after applying an appropriate normalization technique:

$$(b_{i,j}^k)^{norm} = \begin{cases} maximize\ b_{i,j}^k \\ \\ minimize\ b_{i,j}^k \end{cases} \tag{2.17}$$

The score against matrix-represented criteria, $c_{i,j}^k$ is calculated as follows:

$$c_{i,j}^k = \frac{\sum_{r_j=1}^{R_j} s_{r_j}^k v_{i,r_j}}{R_j S} \tag{2.18}$$

subject to:

$$v_{i,r_j} \in 0|1 \tag{2.19}$$

$$\sum_{r_j=1}^{R_j} v_{i,r_j} \neq 0, \qquad \forall i = 1, 2, \dots, I \tag{2.20}$$

$$\sum_{i=1}^{I} v_{i,r_j} \neq 0, \qquad \forall r_j = 1, 2, \dots, R_j \tag{2.21}$$

where: the counter r_j denotes the current option out of the total number R_j contained in the *j-th* matrix-represented criterion; $s_{r_j}^k$ is the vector with scores that reflect the importance of the

r_j-*th* option according to the k-*th* DM, relative to a predefined rating scale with a maximum score S; v_{i,r_j} is the value of the i-*th* alternative in accordance to the r_j-*th* option of the j-*th* matrix represented criterion.

2.3. Modification of WSM and WPM formalizing the generation of coefficients giving advantage in the overall performance of the alternatives

2.3.1. Algorithm implementing the modification of the WSM and WPM methods formalizing the generation of coefficients giving advantage in the overall performance of the alternatives

The algorithm that describes the proposed modification is presented in Fig. 2.4.

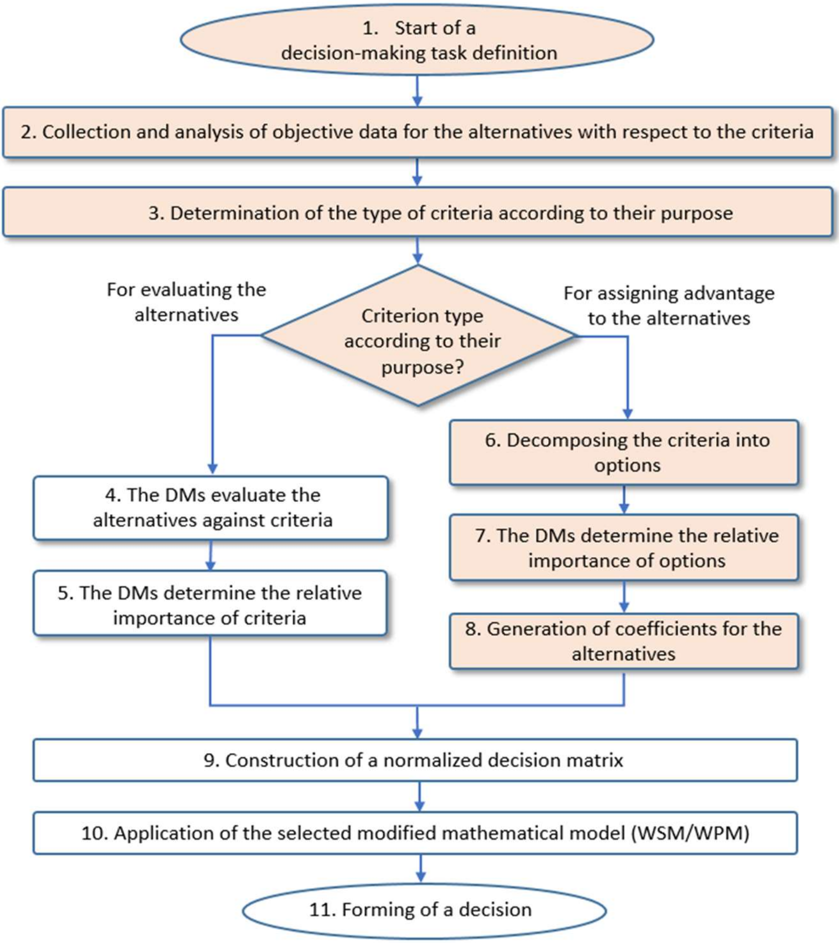


Fig. 2.4. Algorithm implementing the modification of the WSM and WPM methods formalizing the generation of coefficients giving advantage in the overall performance of the alternatives.

2.3.2. Modified mathematical models of the WSM and WPM methods formalizing the generation of coefficients giving advantage in the overall performance of the alternatives

This modification explores the possibility of omitting the assignment of evaluation scores for certain criteria. Instead, it allows the expression of preference by introducing a new coefficient that reflects the relative importance among the options offered by the alternatives. In this way, in combination with the data and the newly introduced coefficient, it becomes possible to more precisely express the overall performance of each alternative.

In the general case of group decision-making, the mathematical model of the modified Weighted Sum Method, in accordance with the above, takes the following form:

$$A_{i\,ModifiedWSM}^{GDM} = \sum_{k=1}^{K} \left(\psi_i^k \sum_{j=1}^{J} w_j^k \, a_{i,j}^k \right) \tag{2.22}$$

$$\psi_i^k = \prod_{n=1}^{N} \sum_{r_n=1}^{R_n} p_{r_n}^k v_{i,r_n} \tag{2.23}$$

subject to:

$$\sum_{r_n=1}^{R_n} p_{r_n}^k = 1, \ \forall n = 1,2,\dots,N; \quad \forall k = 1,2,\dots,K \tag{2.24}$$

$$v_{i,r_n} \in 0|1 \tag{2.25}$$

$$\sum_{r_n=1}^{R_n} v_{i,r_n} \neq 0, \qquad \forall i = 1,2,\dots,I \tag{2.26}$$

$$\sum_{i=1}^{I} v_{i,r_n} \neq 0, \qquad \forall r_n = 1,2,\dots,R_n \tag{2.27}$$

$$\sum_{j=1}^{J} w_j^k = 1, \qquad \forall k = 1,2,\dots,K \tag{2.28}$$

$$a_{i,j}^k, w_j^k \ \in [0,1] \tag{2.29}$$

where: $A_{i\,ModifiedWSM}^{GDM}$ expresses the aggregated score of the i-*th* alternative, calculated by the thus modified WSM method; the index i denotes the current alternative out of a total of I; the index k denotes the *k-th* DM out of a total of K; the index j denotes the *j-th* criterion out of a total of J; the index n denotes the n-*th* criterion out of a total of N criteria, intended for giving priority; ψ_i^k is the generated weight coefficient for the k -*th* DM, which combines the relative importance of the r_n-*th* option of the n-*th* criterion and the v_{i,r_n} value at the i-*th* alternative; the index r_n denotes the r-*th* option of the n-*th* criterion, containing R_n options; $p_{r_n}^k$ is a weight coefficient that reflects the importance of the r_n-*th* option according to the k -*th* DM; v_{i,r_n} is the value of the i-*th* alternative in accordance to the r_n-*th* option of the n-th matrix represented criterion, intended for giving priority to the alternatives; w_j^k is a weighting coefficient that reflects the relative importance of the *j-th* criterion according to the k-*th* DM; $a_{i,j}^k$ is the evaluation given to the i-*th* alternative by the k-*th* DM in accordance to the *j-th* criterion.

The mathematical model of the modified Weighted Product Model takes the following form:

$$A_{i\,ModifiedWPM}^{GDM} = \prod_{k=1}^{K} \psi_i^k \prod_{j=1}^{J} \left(a_{i,j}^k \right)^{w_j^k} \tag{2.30}$$

Subject to the same calculation of the coefficient for the advantage of alternatives (2.23) and restrictions (2.24) – (2.28) with one additional restriction concerning the score.

$$a_{i,j}^{k} = (0,1] \tag{2.31}$$

where: $A_{i\ ModifiedWPM}^{GDM}$ expresses the aggregated score of the i-th alternative, calculated by the thus modified WPM method.

2.4. Conclusions

The proposed modifications of the Weighted Sum Model and the Weighted Product Model introduce three innovations: 1) consideration the personal competence of the DMs; 2) generating alternative scores and 3) applying an additional advantage of alternatives with respect to key criteria. All three can be combined within a single generalized algorithm and mathematical model, allowing for the simultaneous application of all modifications or various combinations of them.

The modifications can be considered as an essential step in the implementation of a decision support system. Operating in a web environment, such a system would significantly contribute to the automation of certain processes and the facilitation of decision-making in the context of group decisions. For the implementation of such a tool, in addition to the availability of models and algorithms, also an appropriate software architecture is required.

CHAPTER 3. DESIGN OF SOFTWARE ARCHITECTURES OF DECISION SUPPORT SYSTEMS

3.1. Mathematical models of combined modifications of the WSM

By combining the proposed modification (2.1) – (2.4), considering the competence of the DMs for each criterion separately through weighting coefficients, together with the modification (2.13) – (2.21), formalizing the generation of alternative scores, the following combined model of the modified Weighted Sum Model is proposed:

$$A_{i\ CombinedWSM}^{GDM} = \sum_{k=1}^{K} \sum_{j=1}^{J} \lambda_j^k w_j^k \beta_{i,j}^k \tag{3.1}$$

subject to the restrictions (2.2), (2.3) and the following additional condition:

$$\beta_{i,j}^{k} = \begin{cases} a_{i,j}^{k} & a \in [0,1] \\ (b_{i,j}^{k})^{norm} & b \in [0,1] \\ c_{i,j}^{k} & c \in [0,1] \end{cases} \tag{3.2}$$

For the generation of $(b_{i,j}^{k})^{norm}$ and $c_{i,j}^{k}$, both the formulations (2.17) and (2.18), and the restrictions (2.19) – (2.21) are valid.

This formulation uses the notation system introduced in Chapter 2, with the addition that $A_{i\ CombinedWSM}^{GDM}$ expresses the aggregated score of the i-th alternative, calculated using the combined model of the modified Weighted Sum Model.

By combining the formulation in (3.1) with the modification that formalizes the coefficient for advantage of the alternatives, the following generalized model of the modified Weighted Sum Method is proposed:

$$A_{i\,GeneralizedWSM}^{GDM} = \sum_{k=1}^{K}\left(\psi_i^k \sum_{j=1}^{J} \lambda_j^k w_j^k \beta_{i,j}^k\right) \qquad (3.3)$$

where: $A_{i\,GeneralizedWSM}^{GDM}$ expresses an aggregated score of the i-*th* alternative, calculated using the generalized model of the modified Weighted Sum Model.

For the generation of ψ_i^k, the condition (2.23) is valid, as well as the restrictions (2.24) – (2.29).

Considering the possibility of grouping the criteria in order to assist administration by assigning a general competence coefficient of the DMs to a group of criteria, the combined model and the generalized model take the following forms:

$$A_{i\,CombinedWSM}^{GDM} = \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{m_l=1}^{M_l} \lambda_l^k w_{m_l}^k \beta_{i,m_l}^k \qquad (3.4)$$

$$A_{i\,GeneralizedWSM}^{GDM} = \sum_{k=1}^{K}\left(\psi_i^k \sum_{l=1}^{L} \sum_{m_l=1}^{M_l} \lambda_l^k w_{m_l}^k \beta_{i,m_l}^k\right) \qquad (3.5)$$

subject to the restrictions (2.8), (2.9) and the following additional condition:

$$\beta_{i,m_l}^k = \begin{cases} a_{i,m_l}^k & a \in [0,1] \\ (b_{i,m_l}^k)^{norm} & b \in [0,1] \\ c_{i,m_l}^k & c \in [0,1] \end{cases} \qquad (3.6)$$

The generalized decision matrix, to which the generalized model incorporating all the proposed modifications of the WSM (3.4) is applied, takes the form shown in Table 3.1:

Table 3.1. Decision matrix for a generalized model of the modified Weighted Sum Model.

Criteria			Weights for DMs' competences			Criteria normalization - maximize/minimize			Weights for criteria importance			Alternatives' scores and coefficients						
Type 1	Type 2	Criterion										A_1			...	A_I		
			DM^1	...	DM^K	DM^1	...	DM^K	DM^1	...	DM^K	DM^1	...	DM^K	...	DM^1	...	DM^K
For evaluating	Subjective	C_1	λ_1^1		λ_1^K	-			w_1^1	...	w_1^K	$a_{1,1}^1$	...	$a_{1,1}^K$	...	$a_{1,1}^1$	...	$a_{1,1}^K$
		...	...		...				...	...	...	...	$a_{i,j}^k$	...	...	...	$a_{i,j}^k$	...
		C_j	...		...				...	...	...	$a_{1,j}^1$	...	$a_{1,j}^K$	...	$a_{I,j}^1$	...	$a_{I,j}^K$
	Numerically represented	C_j	...		...	max/min	...	max/min	...	...	...	$b_{1,j}^1$	...	$b_{1,j}^K$	...	$b_{I,j}^1$	...	$b_{I,j}^K$
		...	...	λ_j^k	...	...	...	...	...	w_j^k	...	...	$b_{i,j}^k$	...	...	...	$b_{i,j}^k$	...
		C_j	...		...	max/min	...	max/min	...	...	...	$b_{1,j}^1$	...	$b_{1,j}^K$	...	$b_{I,j}^1$	...	$b_{I,j}^K$
	Matrix-represented	C_j	...		...	-			...	...	...	$c_{1,j}^1$	...	$c_{1,j}^K$	...	$c_{I,j}^1$	...	$c_{I,j}^K$
		...	...		...				...	...	...	...	$c_{i,j}^k$	...	...	...	$c_{i,j}^k$	...
		C_J	λ_J^1		λ_J^K				w_J^1	...	w_J^K	$c_{1,J}^1$	...	$c_{1,J}^K$	...	$c_{I,J}^1$	...	$c_{I,J}^K$
For advantage		C_n	-			-						ψ_1^1	...	ψ_1^K	ψ_i^k	ψ_I^1	...	ψ_I^K
		...																
		C_N																

Based on the thus defined mathematical models of the combined and generalized Weighted Sum Method, a generalized algorithm is developed, which forms the basis of the business logic of the DSS prototype.

3.2. Generalized algorithm implementing combining modifications

The generalized algorithm aims to integrate all the modifications of the WSM and WPM methods proposed in the dissertation into a single decision support system. The algorithm is presented in Fig. 3.1.

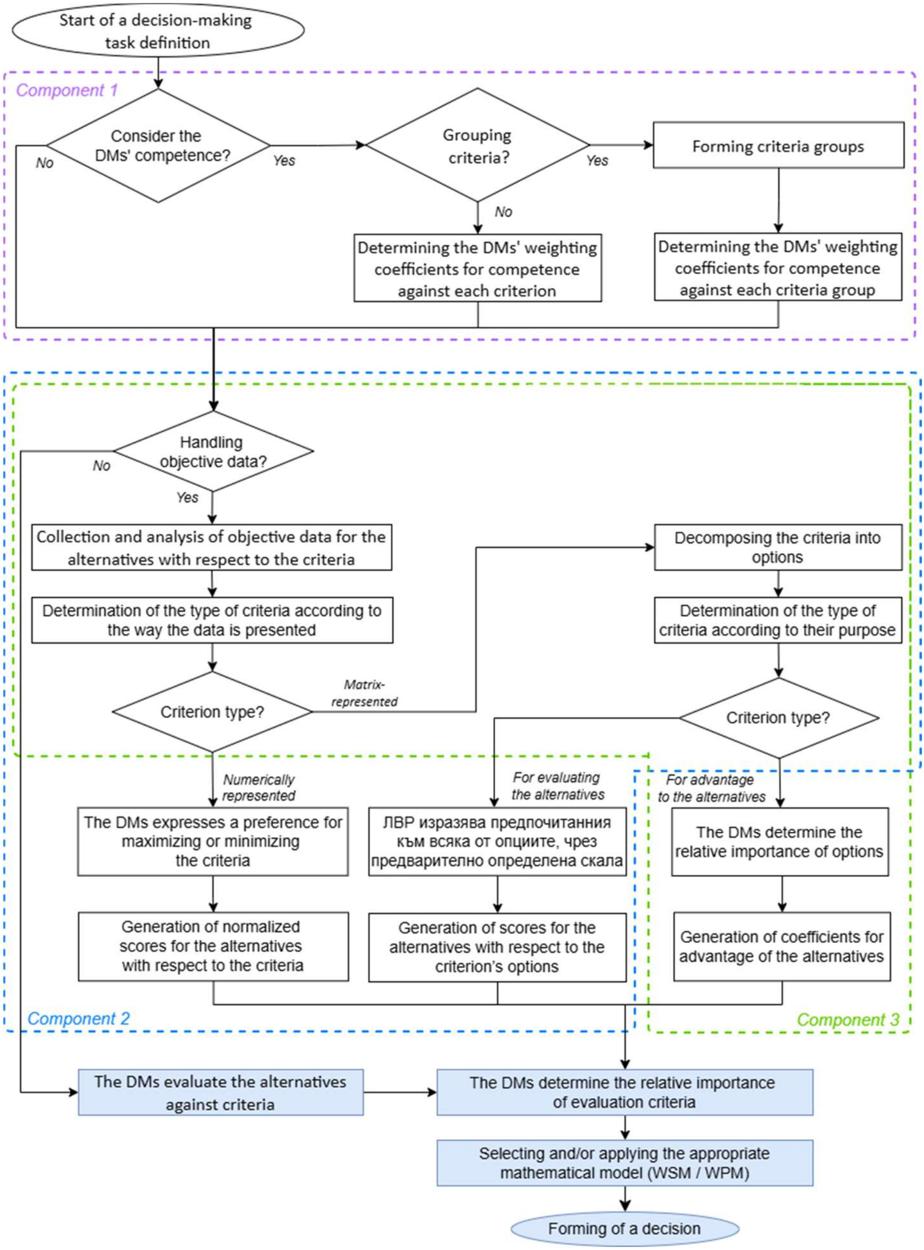


Fig. 3.1. Generalized algorithm implementing the combination of the proposed modifications.

In the context of software engineering, three main components are derived from the generalized algorithm, corresponding to each of the proposed modifications. **Component 1** encapsulates the modification that considers the competence of the DMs. **Component** 2 and **component** 3 reuse some common modules, like the module for decomposing criteria into options and for classifying the criteria. The logic for combining the modified mathematical models, depending on the architectural approach and the technologies used, can be implemented in various programming languages. Therefore, the pseudocode, which defines the logic for filtering and providing a selection of the most appropriate model for the task, is of particular interest, it is illustrated in Fig. 3.8.

```
Begin Mathematical Model Reduction & Selection       // Selection from eligible models
 if λ == true && β == true && Ψ == true then
      if L == true then
             Eligible model is WSM (3.5)
   else
             Eligible model is WSM (3.3)
 if λ == true && β == true && Ψ == false then
      if L == true then
             Eligible model is WSM (3.4)
   else
             Eligible model is WSM (3.1)
 if λ == true && β == false && Ψ == true then
      if L == true then
             For future development
   else
             For future development
 if λ == true && β == false && Ψ == false then
      if L == true then
             Choice between WSM model (2.7) and WPM model (2.11)
   else
             Choice between WSM model (2.1) and WPM model (2.5)
 if λ == false && β == true && Ψ == true then
      For future development
 if λ == false && β == true && Ψ == false then
      Eligible model is WSM (2.13)
 if λ == false && β == false && Ψ == true then
      Choice between WSM model (2.22) and WPM model (2.30)
 if λ == false && β == false && Ψ == false then
      Choice between the classics: WSM model (1.3) and WPM model (1.7)
End
```

Fig. 3.8. Pseudocode describing the filtering component and selecting a model suitable for the task.

The different combinations of the components allow the results to be aggregated using one of 13 different models – 9 with the WSM method and 4 with the WPM, as shown by the references in Fig. 3.8.

3.3. Designing the functionality of a decision support system

As a result of the preliminary requirements analysis, all functions covered by the DSS are defined, along with the rules for their execution. All stakeholders, their roles, and the required user interface are identified. Three roles are distinguished:

- Administrator of decision-making task;
- Decision makers;

- System role, that brings together the activities performed automatically by the system, following the designed logic and proposed algorithms.

The role of the administrator and his interaction with the DSS is illustrated by an UML Use Case Diagram, presented in Fig. 3.9.

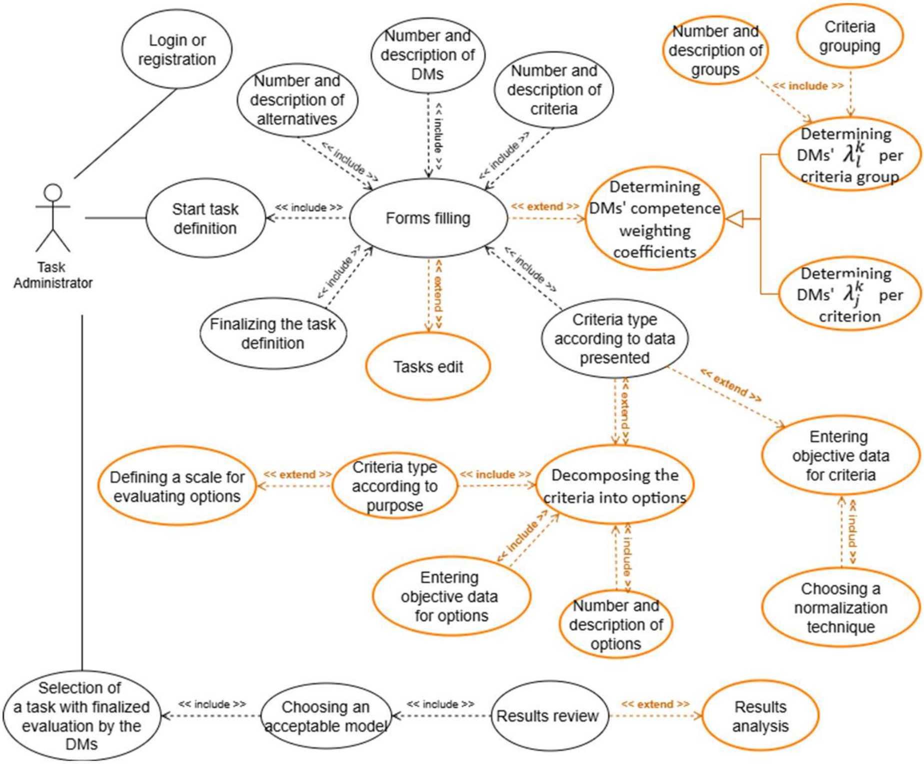


Fig. 3.9. Adapted UML Use Case diagram for the task administrator role.

The role of the DM is described with an analogical diagram, presented in Fig. 3.10.

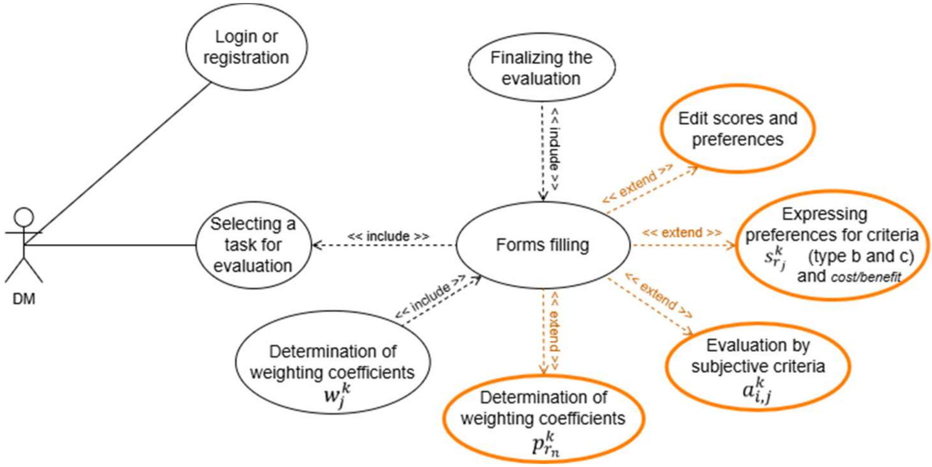


Fig. 3.10. Adapted UML Use Case diagram for the role of the DM.

The automated operations performed by the DSS are presented in Fig. 3.11.

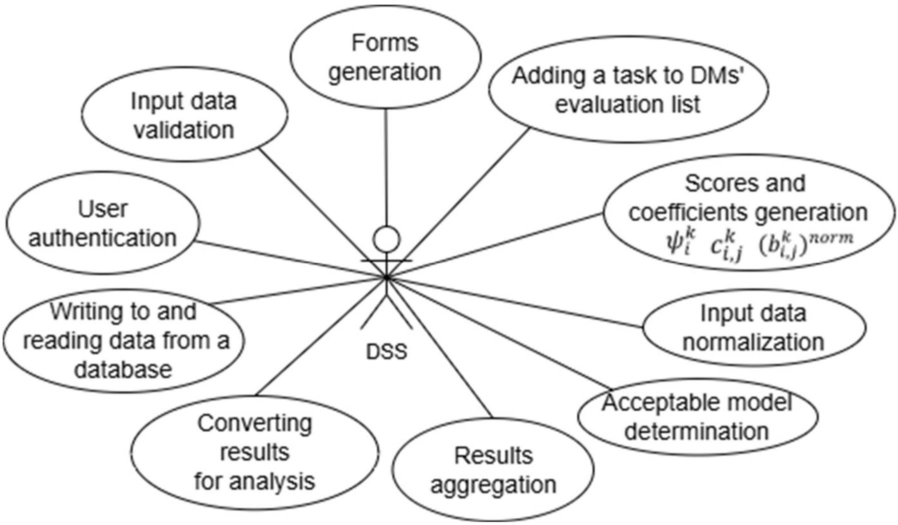


Fig. 3.11. Adapted UML Use Case diagram for the system role.

The dynamically generated forms in the DSS structure the user interaction, while the transitions between them define the system's operational logic. This is illustrated with an adapted state diagram in Fig. 3.12.

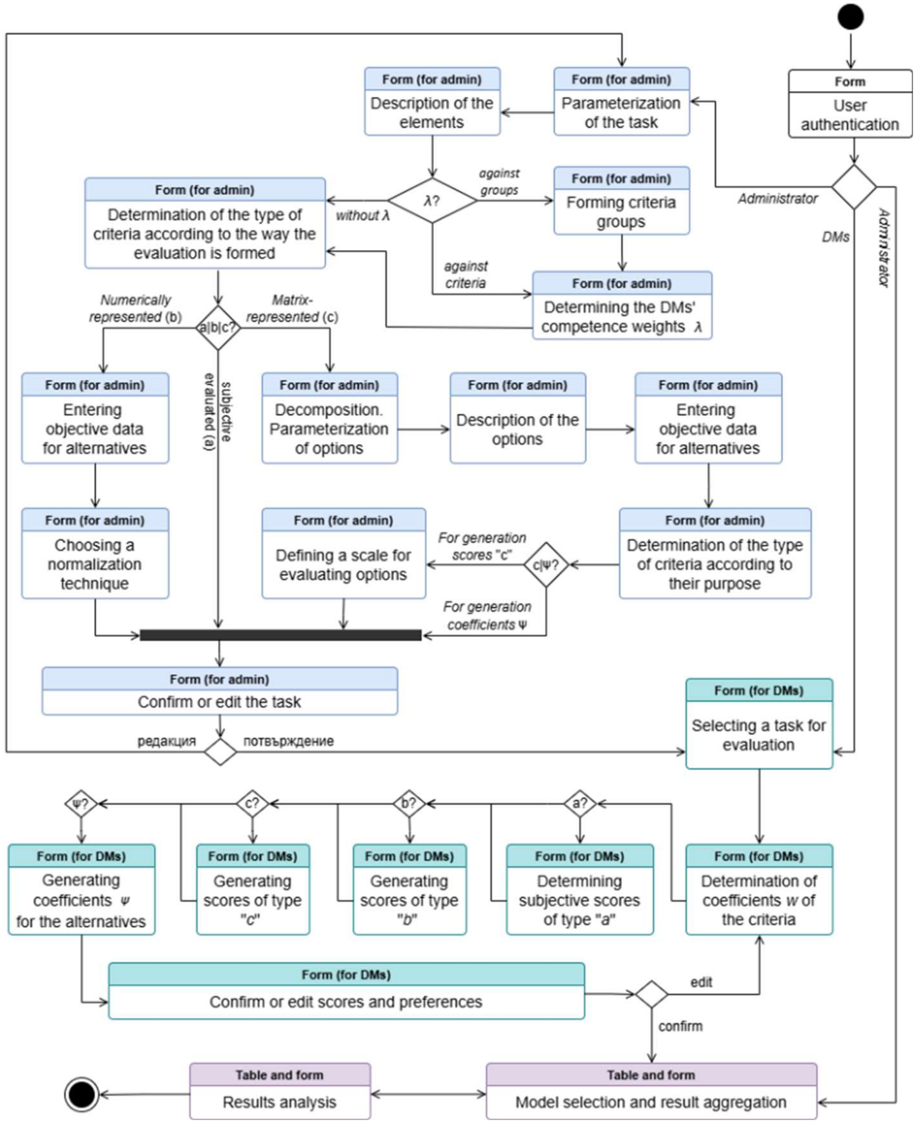


Fig. 3.12. Adapted UML State Machine diagram of the user interface.

Dynamically generated forms are created in real time while the user interacts with the system. This means that the form is not pre-prepared (static), but it is generated according to specified conditions, parameters, and the context of the specific task. The process is **interactive** because the system responds to the user's actions and selections again in real time, adapting the content and structure of the form. The process is also **parallel**, as multiple users can simultaneously create and fill out forms with different parameters from their own accounts without causing conflicts or data loss.

3.4. Web-based DSS with monolithic three-tier architecture

The proposed three-tier architecture follows the classical three-tier architectural pattern, which provides a clear separation of concerns and infrastructure, facilitates maintainability, and enforces a structured approach to system development. From the perspective of code structuring and deployment, it is chosen a monolithic architectural style, and the data flow is managed synchronously. The topology of the three-tier architecture used for the development of the DSS prototype is shown in Fig. 3.13. [Dimitrova et al., 2021].

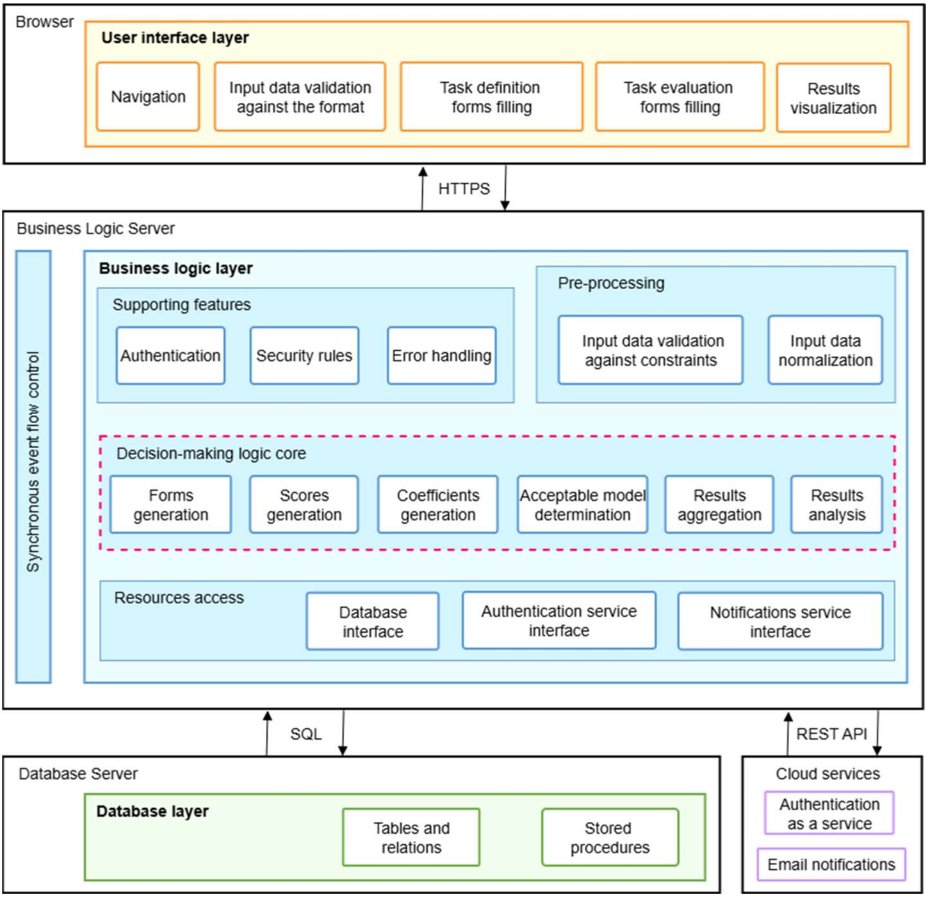


Fig. 3.13. Topology of the designed three-layer architecture of the DSS.

3.5. Web-based DSS with serverless architecture

The proposed serverless architecture adopts the architectural pattern of distributed functions (FaaS), which are triggered by specific user actions or internal system events - an asynchronous event flow management. This design provides high scalability, minimal need for managing hardware and software infrastructure, and very high adaptability to varying system workloads. The topology of the serverless architecture on which the developed DSS prototype is based is shown in Fig. 3.14. [Dimitrova et al., 2024, a].

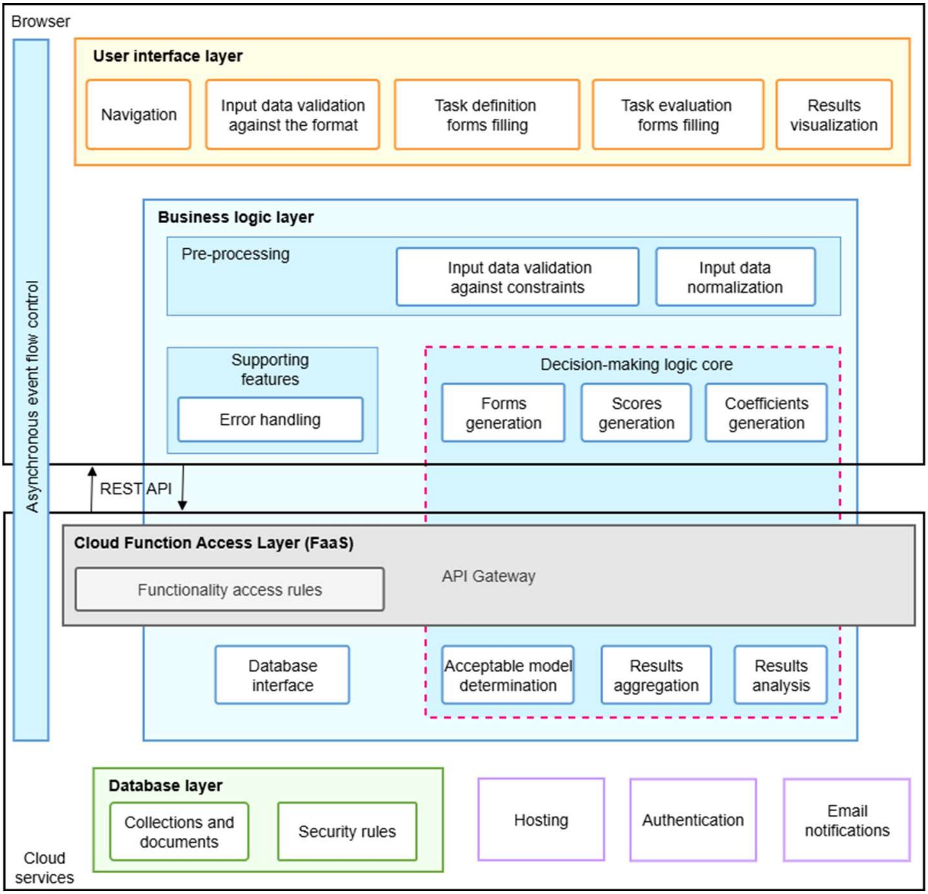


Fig. 3.14. Topology of the designed serverless architecture of the DSS.

3.6. Comparison of the proposed software architectures

A comparison of some key features of the adapted three-tier and serverless architectures of the DSS prototypes is presented in Table 3.1.

Table 3.1. Comparison between the adapted three-tier and serverless architectures.

Characteristics	Three-tier architecture	Serverless architecture
Infrastructure	Monolithic. Concentrated on a single server	Distributed between browser and cloud functions
Deployment	Requires server maintenance and management	No need for direct server management
Scalability	Limited by server resources	Automatic in accordance to the load
Security	Centralized access control	Applicable to the brosew and the cloud services
Performance	Slower, due to the need for more network communication	Faster, due to less need for network communication
Development	Clearer structure, no development constraints	More complex structure, with limitations from cloud functions
Support	More difficult, requires server administration	Easier, delegated to the cloud
Event Handling	Synchronous	Asynchronous

3.7. Conclusions

Designing two different software architectures for the same DSS aims to support the selection of an architectural approach. This choice should be based not only on the functional requirements, but also on non-functional characteristics such as infrastructure capabilities, the technological expertise of the team, and the expected scale of the system.

The monolithic three-tier architecture is a logical choice for institutions that maintain their own server infrastructure and is sufficient for academic purposes such as experiments, training, and demonstrations. In contrast, the serverless distributed architecture provides high scalability under dynamic workloads. It is suitable for organizations that rely on cloud services and need to support a large number of users, such as corporate or public institutions.

CHAPTER 4. NUMERICAL TESTING OF THE MODIFIED METHODS

4.1. Numerical testing of the modification considering DMs' different competence domains

Numerical testing of the proposed modification, which considers the differences in the areas of competence of the DMs, described in Chapter 2, Section 2.1, is performed on both the WSM and WPM methods.

4.1.1. Testing scenarios

In order to examine the specifics of the modification that considers the level of competence of the DMs, two scenarios for the Weighted Sum Model are investigated using the developed prototype. In the first scenario (Scenario 1), a problem is formulated and solved based on the classical WSM model (1.3) – (1.5). The second scenario (Scenario 2) is implemented by formulating and solving a problem based on the proposed modification, in accordance with (2.7) for WSM and (2.11) for WPM. The modification of the Weighted Product Model is tested and compared with the modification of the Weighted Sum Model.

4.1.2. Case study

The specific decision-making task for selecting an e-commerce platform concerns a small company that has initially identified three B2C platforms suitable for its goals, namely: WooCommerce (**A1**), Shopify-Basic (**A2**), and BigCommerce Essentials-Standard (**A3**). Considering the need to facilitate the administration process in group decision-making, it is proposed that the evaluation criteria to be divided into groups. Table 4.1 presents both the groups and the corresponding criteria that form each group.

Table 4.1. Criteria for alternatives evaluation grouped by competence domain

Group 1: Common criteria	
C1	Pricing model
C2	Payment options variety
C3	Number of staff accounts
C4	Support channels
C5	Number of free templates
Group 2: Commercial criteria	
C6	Inventory management
C7	Simplified shopping process
C8	Shipping management
C9	Analytics tools
C10	Discounts and promotions management
Group 3: Technical criteria	
C11	Deployment (DevOps)
C12	Data and Payment Process Security
C13	Responsive Design
C14	Integrations with external applications
C15	Speed and performance
C16	SEO
Group 4: Marketing criteria	
C17	UX/UI design templates personalization
C18	Customer reviews management - feedbacks and rating options
C19	Advertisement channels

A group of experts has been defined whose competencies fully cover the formed criteria groups as follows: **DM-1** - business owner; sales manager (**DM-2**); software engineer (**DM-3**); and marketing manager (**DM-4**). At the same time, different weighting coefficients are assigned in a way that reflects the individual competencies of each DM. The competency weighting coefficients for the DMs, determined according to constraint (2.3), for Scenario 2 are presented in Table 4.2.

Table 4.2. Weighting coefficients for DMs' competence, according to the criteria groups.

Criteria groups	Weiting coeficients for DMs' competences			
	DM-1	DM-2	DM-3	DM-4
Group 1 (Comman criteria)	0.25	0.25	0.25	0.25
Group 2 (Commercial criteria)	0.15	0.50	0.15	0.20
Group 3 (Technical criteria)	0.14	0.14	0.57	0.15
Group 4 (Marketing criteria)	0.25	0.25	0.10	0.40

For each DM, an individual decision matrix is generated, through which the expert determines the weighting coefficients for the importance of the criteria and, accordingly, provides evaluations of the performance of each alternative in accordance to the defined criteria. The input data for the specific group decision-making task is presented in a generalized group decision matrix, as shown in Table 4.3:

Table 4.3. A generalized decision matrix in group decision-making considering DMs' different competence domains.

Criteria						DMs'competences weights				Alternatives' scores												
Group	No	Importanse weights								**A1** (WooComerce)				**A2** (Shopify)				**A3** (BigComerce)				
		DM 1	DM 2	DM 3	DM 4	DM 1	DM 2	DM 3	DM 4	DM 1	DM 2	DM 3	DM 4	DM 1	DM 2	DM 3	DM 4	DM 1	DM 2	DM 3	DM 4
Group 1	C1	0.10	0.03	0.02	0.01					0.43	0.38	0.62	0.57	0.24	0.62	0.52	0.57	0.24	0.62	0.52	0.57
	C2	0.06	0.08	0.02	0.07					0.14	0.33	0.05	0.29	0.14	0.33	0.05	0.33	0.24	0.14	0.33	0.10
	C3	0.05	0.02	0.05	0.01	0.25	0.25	0.25	0.25	0.10	0.10	0.10	0.19	0.29	0.14	0.10	0.19	0.33	0.29	0.29	0.24
	C4	0.07	0.04	0.10	0.04					0.40	0.14	0.50	0.33	0.81	0.50	0.57	0.62	0.81	0.50	0.57	0.62
	C5	0.02	0.05	0.03	0.07					0.24	0.19	0.19	0.24	0.24	0.33	0.19	0.29	0.29	0.05	0.24	0.33
Group 2	C6	0.08	0.10	0.07	0.04					0.39	0.45	0.47	0.44	0.35	0.88	0.51	0.53	0.75	0.35	0.69	0.77
	C7	0.03	0.07	0.04	0.06					0.50	0.51	0.49	0.48	0.62	0.58	0.49	0.53	0.49	0.42	0.51	0.55
	C8	0.05	0.08	0.02	0.03	0.15	0.50	0.15	0.20	0.41	0.33	0.44	0.39	0.52	0.67	0.55	0.61	0.48	0.40	0.49	0.52
	C9	0.10	0.06	0.06	0.06					0.66	0.71	0.59	0.61	0.69	0.89	0.66	0.71	0.60	0.71	0.59	0.61
	C10	0.03	0.07	0.03	0.06					0.35	0.42	0.47	0.38	0.55	0.72	0.71	0.62	0.51	0.62	0.55	0.49
Group 3	C11	0.08	0.03	0.09	0.02					0.56	0.52	0.95	0.40	0.61	0.40	0.30	0.45	0.59	0.39	0.30	0.44
	C12	0.07	0.08	0.06	0.04					0.61	0.69	0.71	0.59	0.48	0.59	0.60	0.49	0.51	0.55	0.59	0.41
	C13	0.04	0.03	0.06	0.06	0.14	0.14	0.57	0.15	0.31	0.40	0.65	0.25	0.42	0.50	0.59	0.62	0.36	0.55	0.57	0.45
	C14	0.03	0.05	0.07	0.04					0.69	0.72	0.85	0.74	0.33	0.42	0.33	0.50	0.45	0.46	0.40	0.42
	C15	0.03	0.04	0.04	0.05					0.35	0.40	0.62	0.33	0.55	0.61	0.50	0.46	0.51	0.60	0.49	0.44
	C16	0.03	0.03	0.05	0.08					0.51	0.46	0.63	0.44	0.53	0.57	0.49	0.60	0.49	0.50	0.38	0.50
Group 4	C17	0.03	0.06	0.09	0.09					0.32	0.37	0.41	0.39	0.47	0.42	0.39	0.39	0.51	0.41	0.42	0.44
	C18	0.06	0.04	0.05	0.07	0.25	0.25	0.10	0.40	0.66	0.63	0.72	0.82	0.67	0.72	0.74	0.81	0.76	0.77	0.67	0.79
	C19	0.04	0.04	0.05	0.10					0.61	0.62	0.71	0.62	0.34	0.45	0.55	0.49	0.77	0.82	0.88	0.85

4.1.3. Input data analysis

Based on the data from Tables 4.3 - 3 alternatives, 4 DMs and 19 criteria distributed across 4 groups, an analysis of the input data is conducted before applying the mathematical models of WPM and the two scenarios of the WSM.

Each DM has indicated as the most important criteria those in the group where it has the highest competence, this is seen from Fig. 4.1.

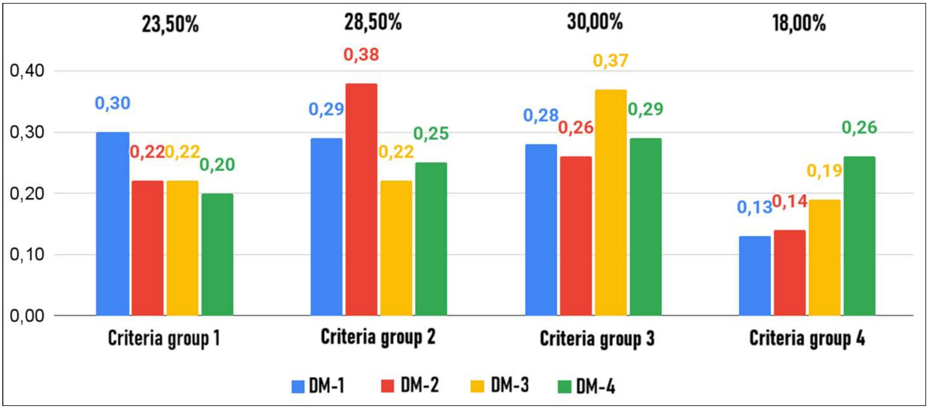


Fig. 4.1. Ratios of criteria importance coefficients determined by the DMs, relative to the criteria groups and the percentage ratio between the criteria groups.

The preferences to the alternatives, expressed only through ratings, excluding the influence of the importance and competence coefficients, are presented in Fig. 4.2.

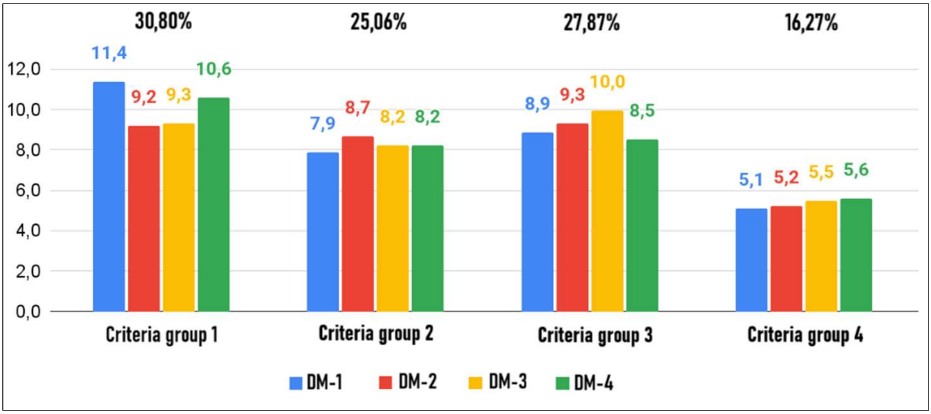
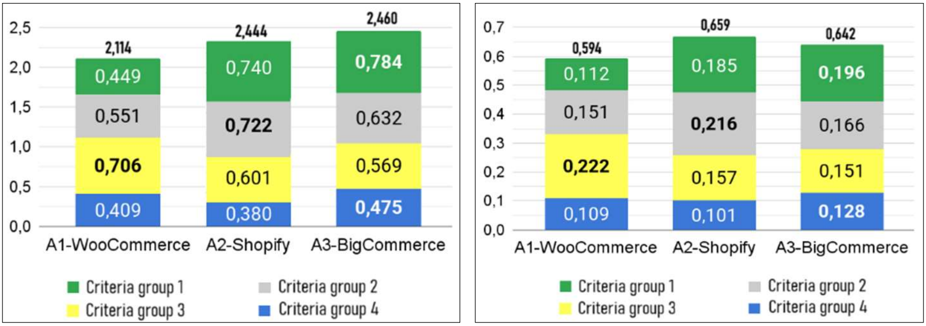


Fig. 4.2. Ratios of the evaluations determined by the DMs for the alternatives, according to the groups of criteria and the percentage ratio between the groups of criteria.

The preliminary analysis of the input data is crucial for the interpretation of the results obtained by applying the mathematical models of the group decision-making methods.

4.1.4. Results of the numerical testing of the modified WSM

In Scenario 1, the utility function of the classical Weighted Sum Model (1.3) – (1.5) is used. In Scenario 2, using the same data and grouping the criteria (to facilitate task administration), the modified Weighted Sum Model (2.7) – (2.10) is applied. The results of group decision rank the alternatives in both scenarios, as illustrated in Fig. 4.5 (a, b).



a) *Scenario* 1 b) *Scenario* 2

Fig. 4.5. Ranking of e-commerce platforms in Scenario 1 and Scenario 2, using the modified WSM.

The key reason for the shift in the ranking of alternatives is observed in Criteria Group 2, where the weight assigned to the opinion of the sales manager (DM-2) is the highest (0.50 out of 1.00 – see Table 4.2). Preliminary analysis of the input data from Table 4.3 shows that all of DM-2's ratings for the alternatives according to the criteria in this group favor A2-Shopify (total 3.740). Additionally, from Fig. 4.1 it can be seen that he is the person who allocated the highest distribution of importance coefficients for the criteria constituting Group 2 (0.38 out of 1.00). Therefore, this combination has led to an increase in the ratio between the ratings of alternatives A2 and A3 for the criteria in the group. This change is sufficient to result in a different ranking of the alternatives.

4.1.5. Results of numerical testing of the modified WPM

The modification of the Weighted Product Model is tested using the input data defined in Table 4.2 and Table 4.3. The preliminary analysis illustrated in Fig. 4.1 - Fig. 4.4 is also valid in this case. The results obtained from the group decision rank the alternatives as illustrated in Fig. 4.10.

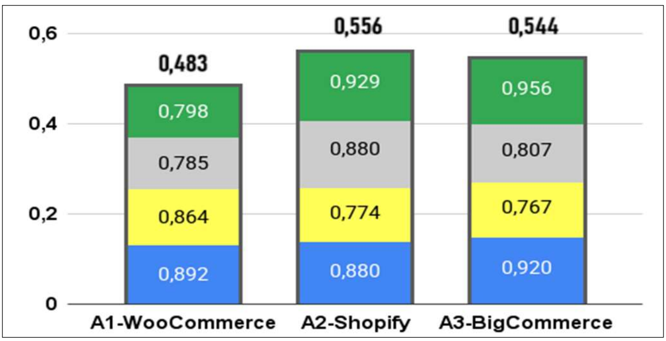


Fig. 4.10. Ranking of e-commerce platforms using the modified WPM.

It can be noted that the ranking obtained by the modified WSM is retained. A comparison of the influence of the individual DMs, global to the task, in the two methods is presented with the diagram in Fig. 4.13.

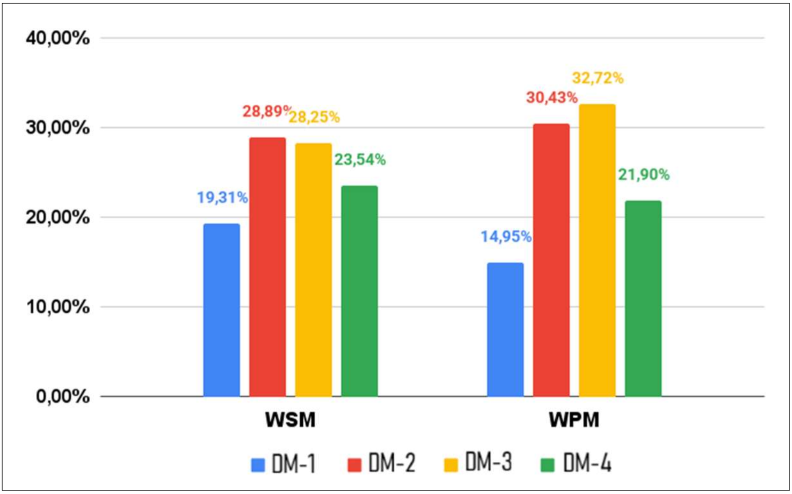


Fig. 4.13. Comparison of the influence of DMs in modified WSM and WPM.

4.2. Numerical testing of the modification formalizing the generation of alternative scores

In order to determine the applicability of the proposed modification of the weighted sum method, formulated using the model (2.13) – (2.21) (Chapter 2, item 2.2), as well as the applicability of the combined model variant (3.4), which includes the same modification, a task is defined for selecting monitors as part of the office equipment of a medium-sized IT company.

4.2.1. Testing scenarios

Three scenarios are considered in the context of group decision-making. All of them are based on the same importance coefficients for the criteria and the same objective data for the alternatives. The first two scenarios (Scenario 1 and Scenario 2) are based on the modification of the WSM described in Chapter 2, Section 2.2 (2.13) – (2.21). In Scenario 1, the generation of scores based on the utility of the criteria (2.17) depends entirely on the preferences of the task administrator. Scenario 2 demonstrates the possibility of individually determining the utility of the criteria. Additionally, Scenario 2 tests the generation of scores with in accordance to the decomposed criteria when the DM's preferences toward the options change (2.18). In Scenario 3, the task is solved by combining the modification for generating scores with the modification that considers the competence of the DMs in different domains of competence, according to the combined model (3.4). The input data in Scenario 3 is similar to those in Scenario 2, with the difference that a different normalization technique is used. The objective of this scenario is to demonstrate the applicability of the generalized algorithm presented in Chapter 3, Section 3.1, and the flexibility of the designed decision support system.

4.2.2. Case study

The task of selecting suitable office equipment is carried out under the following initial conditions: 3 alternatives, 3 DMs, and 3 groups of criteria, containing a total of 12 criteria, 4 of which are decomposed into 13 options. The following 3 types of monitors have been pre-selected as suitable alternatives: Dell U2724DE (**A1**), Samsung U32J590 (**A2**), and MSI MEG 342C (**A3**). The individuals forming the final group decision are as follows: (1) a specialist from the finance department (**DM-1**); (2) a human resources specialist (**DM-2**); (3) a technical staff member – software engineer (**DM-3**). The criteria for selecting the appropriate type of monitor are organized into three thematically related groups, as shown in Table 4.4.

Table 4.4. Grouped criteria for evaluating alternatives, type of criteria and coefficients for their importance determined by the DMs.

Criteria groups	Criteria	Criteria type	Weiting coefcients for criteria impotrance		
			DM-1	DM-2	DM-3
Group 1: Financial criteria	C1: Price	number	0,19	0,07	0,03
	C2: Warranty	number	0,16	0,12	0,01
	C3: Power Consumption	number	0,11	0,07	0,01
Group 2: Health care and ergonomics	C4: Eye Care Technologies	matrix	0,10	0,19	0,15
	C5: Tilt Angle	number	0,04	0,07	0,05
	C6: Portrait Mode	number	0,02	0,09	0,15
Group 3: Technical criteria	C7: Screen size	matrix	0,09	0,08	0,20
	C8: Panel Type	matrix	0,08	0,06	0,10
	C9: Resolution	matrix	0,09	0,06	0,10
	C10: Refresh Rate	number	0,04	0,06	0,05
	C11: Brightness	number	0,04	0,06	0,05
	C12: Color Gamut	number	0,04	0,07	0,10

The options into which the matrix-represented criteria for the specific task are decomposed are shown in Table 4.5.

Table 4.5. Matrix-represented criteria decomposed into options.

C4: Eye Care Technologies	
O1	Blue Light Filter
O2	Anti-Flicker
O3	Ambient Light Sensor
O4	Anti-glare Coating
C7: Screen size	
O5	Screen size 27”
O6	Screen size 31.5”
O7	Screen size 34.2”
C8: Panel Type	
O8	IPS
O9	VA
O10	OLED
C9: Resolution	
O11	UW-QHD
O12	UHD
O13	QHD

The preferences expressed by the DMs regarding whether the criterion is improved if it is maximized or minimized are presented in Table 4.6.

Table 4.6. Numerically represented criteria and their units of measurement, values for the alternatives and preferences of the DMs for their utility in the three scenarios.

Criteria	Unit of measure-ment	Values for alternatives			Scenario 1 - preferences			Scenario 2- preferences			Scenario 3- preferences		
		A1	A2	A3	DM-1	DM-2	DM-3	DM-1	DM-2	DM-3	DM-1	DM-2	DM-3
C1	EUR	505	290	1247	cost	cost	cost	cost	cost	benefit	cost	cost	benefit
C2	Monts	36	24	36	benefit	benefit	benefit	benefit	benefit	benefit	benefit	benefit	benefit
C3	kW. /h	24,4	59	42,3	cost	cost	cost	cost	cost	cost	cost	cost	cost
C5	Degree	26	17	25	benefit	benefit	benefit	cost	benefit	benefit	cost	benefit	benefit
C6	Availability	1	0	0	cost	cost	cost	cost	cost	benefit	cost	cost	benefit
C10	Hz	120	60	175	benefit	benefit	benefit	cost	benefit	benefit	cost	benefit	benefit
C11	cd/m²	350	270	250	cost	cost	cost	cost	cost	benefit	cost	cost	benefit
C12	% sRGB	100	99,1	139	benefit	benefit	benefit	cost	benefit	benefit	cost	benefit	benefit

The data about the options, together with the evaluation points for the importance of the options determined by the DMs, are presented in Table 4.7.

Table 4.7. Matrix-presented criteria, decomposed into options and their values for the alternatives together with the evaluation points determined by the DMs for their importance in the three scenarios.

Criteria		Values of the options			Evaluation points for options importance								
					Scenario 1			Scenario 2			Scenario 3		
No	**Options**	**A1**	**A2**	**A3**	DM-1	DM -2	DM -3	DM -1	DM -2	DM -3	DM -1	DM -2	DM -3
C4	O1	1	0	1	4	5	5	4	5	5	4	5	5
	O2	0	1	1	4	5	5	4	5	5	4	5	5
	O3	1	0	1	2	2	4	2	2	4	2	2	4
	O4	1	1	1	2	3	5	2	3	5	2	3	5
C7	O5	1	0	0	4	3	3	4	3	**1**	4	3	1
	O6	0	1	0	4	3	4	4	3	**5**	4	3	5
	O7	0	0	1	2	3	5	2	3	**3**	2	3	3
C8	O8	1	0	0	4	4	2	4	4	2	4	4	2
	O9	0	1	0	4	3	3	4	3	**1**	4	3	1
	O10	0	0	1	2	2	5	2	2	5	2	2	5
C9	O11	0	0	1	1	2	5	1	2	5	1	2	5
	O12	0	1	0	2	5	5	2	5	**2**	2	5	2
	O13	1	0	0	4	3	2	4	3	2	4	3	2

It should be noted that the options take a Boolean value, reflecting the presence of the corresponding feature in each of the alternatives. The DMs express their preferences regarding the options using a scale from 1 to 5.

In accordance with Scenario 3, for the defined groups of criteria, weight coefficients reflecting the competence level of the DMs are assigned, as shown in Table 4.8.

Table 4.8. Weighting coefficients for DMs' competence, according to the groups of criteria.

Criteria groups	DM -1	DM -2	DM -3
Group 1: Financial criteria	0.62	0.21	0.17
Group 2: Health care and ergonomics	0.26	0.40	0.34
Group 3: Technical criteria	0.30	0.15	0.55

For each of the three described scenarios, individual forms for entering the DMs' preferences are generated based on the input data, on the basis of which the scores for the alternatives are calculated.

4.2.3. Results of numerical testing of the modified WSM

The modification affects the algorithm for implementing the Weighted Sum Model by formalizing an approach for generating scores without changing the mechanism for aggregating attributes in the final result. Therefore, the interpretation of the results is focused on the steps for generating scores within the environment of the developed prototype of the decision support system.

In accordance with **Step 4** of the algorithm for implementing the modification (Chapter 2, item 2.2.1), a form for preferences for numerical criteria is generated. In Scenario 1, the form is filled in only by the administrator, and the entered preferences apply to all DMs. A view of the form is shown in Fig. 4.14.

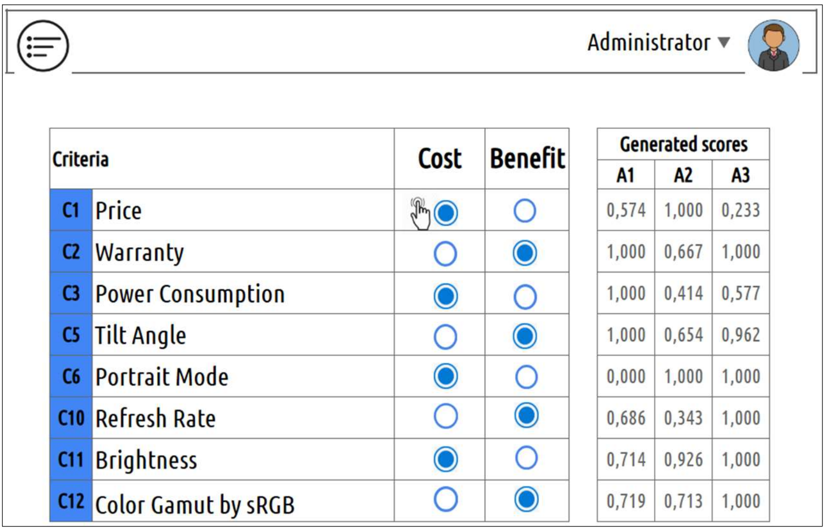


Fig. 4.14. Form for expressing preferences for utility of numerical criteria. View from the prototype of a decision support system.

When a criterion is matrix-represented, according to Step 7 of the algorithm, each decision maker is allowed to express his preferences regarding the alternatives' options. For this purpose, the decision support system prototype generates an individual form for each DM. An example view for DM-1 in Scenario 1 is shown in Fig. 4.15.

DM-1

Criteria and options		Score	Generated scores		
			A1	A2	A3
C4	Eye Care Technologies		0,400	0,300	0,600
O1	Blue Light Filter	4			
O2	Anti-Flicke	4			
O3	Ambient Light Sensor	2			
O4	Anti-glare Coating	2			
C7	Screen size		0,267	0,267	0,133
O5	Размер на екрана 27"	4			
O6	Размер на екрана 31.5"	4			
O7	Размер на екрана 34.2"	2			
C8	Panel Type		0,267	0,267	0,133
O8	IPS	4			
O9	VA	4			
O10	OLED	2			
C9	Resolution		0,267	0,133	0,067
O11	UW-QHD	1			
O12	UHD	2			
O13	QHD	4			

Fig. 4.15. Form for expressing preferences for alternative options, according matrix-represented criteria. View from the prototype of the DSS.

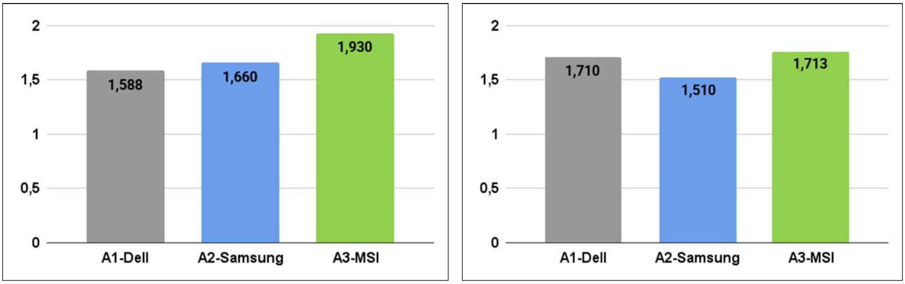
Thus generated scores, together with the criteria importance coefficients, are summarized in the decision matrix in Fig. 4.16.

Administrator

Criteria		Weighted coefficients for criteria importance			Generated scores								
					A1 Dell U2724DE			A2 Samsung U32J590			A3 MSI MEG 342C		
		DM-1: Financial Officer	DM-2: Human Resources	DM-3: Software Engineer	DM-1	DM-2	DM-3	DM-1	DM-2	DM-3	DM-1	DM-2	DM-3
C1	Price	0,19	0,07	0,03	0,574			1,000			0,233		
C2	Warranty	0,16	0,12	0,01	1,000			0,667			1,000		
C3	Power Consumption	0,11	0,07	0,01	1,000			0,414			0,577		
C4	Eye Care Technologies	0,10	0,19	0,15	0,400	0,500	0,700	0,300	0,400	0,500	0,600	0,750	0,950
C5	Tilt Angle	0,04	0,07	0,05	1,000			0,654			0,962		
C6	Portrait Mode	0,02	0,09	0,15	0,000			1,000			1,000		
C7	Screen size	0,09	0,08	0,20	0,267	0,200	0,200	0,267	0,200	0,267	0,133	0,200	0,333
C8	Panel Type	0,08	0,06	0,10	0,267	0,267	0,133	0,267	0,200	0,200	0,133	0,133	0,333
C9	Resolution	0,09	0,06	0,10	0,267	0,200	0,133	0,133	0,333	0,333	0,067	0,133	0,333
C10	Refresh Rate	0,04	0,06	0,05	0,686			0,343			1,000		
C11	Brightness	0,04	0,06	0,05	0,714			0,926			1,000		
C12	Color Gamut by sRGB	0,04	0,07	0,10	0,719			0,713			1,000		

Fig. 4.16. Decision matrix for Scenario 1. View from the DSS prototype.

After applying the model of WSM (2.13) to the decision matrix, the following ranking of alternatives is obtained for Scenario 1 and Scenario 2, as shown in Fig. 4.17



a) *Scenario* 1 b) *Scenario* 2

Fig. 4.17. Ranking of monitors in Scenario 1 and Scenario 2, using modified WSM.

4.2.4. Results of numerical testing of the combined model of the WSM

In Scenario 3, by changing the normalization technique and including the DMs' competence weight coefficients, the aggregated decision matrix takes a different form, as illustrated in Fig. 4.21.

Administrator

Criteria		Weighted coefficients for criteria importance and DMs' competences			Generated scores								
					A1 Dell U2724DE			A2 Samsung U32J590			A3 MSI MEG 342C		
		DM-1: Financial Officer	DM-2: Human Resources	DM-3: Software Engineer	DM-1	DM-2	DM-3	DM-1	DM-2	DM-3	DM-1	DM-2	DM-3
Group 1: Financial criteria		0,62	0,21	0,17	0,224			0,179			0,136		
C1	Price	0,19	0,07	0,03	0,633	0,633	0,367	0,789	0,789	0,211	0,094	0,094	0,906
C2	Warranty	0,16	0,12	0,01	0,640	0,640	0,640	0,426	0,426	0,426	0,640	0,640	0,640
C3	Power Consumption	0,11	0,07	0,01	0,681	0,681	0,681	0,230	0,230	0,230	0,448	0,448	0,448
Group 2: Health care and...		0,26	0,40	0,34	0,168			0,130			0,194		
C4	Eye Care Technologies	0,10	0,19	0,15	0,400	0,500	0,700	0,300	0,400	0,500	0,600	0,750	0,950
C5	Tilt Angle	0,04	0,07	0,05	0,348	0,652	0,652	0,574	0,426	0,426	0,373	0,627	0,627
C6	Portrait Mode	0,02	0,09	0,15	0,000	0,000	1,000	1,000	1,000	0,000	1,000	1,000	0,000
Group 3: Technical criteria		0,30	0,15	0,55	0,139			0,154			0,177		
C7	Screen size	0,09	0,08	0,20	0,267	0,200	0,067	0,267	0,200	0,333	0,133	0,200	0,200
C8	Panel Type	0,08	0,06	0,10	0,267	0,267	0,133	0,267	0,200	0,067	0,133	0,133	0,333
C9	Resolution	0,09	0,06	0,10	0,267	0,200	0,133	0,133	0,333	0,133	0,067	0,133	0,333
C10	Refresh Rate	0,04	0,06	0,05	0,456	0,544	0,544	0,728	0,272	0,272	0,206	0,794	0,794
C11	Brightness	0,04	0,06	0,05	0,311	0,311	0,689	0,468	0,468	0,532	0,508	0,508	0,492
C12	Color Gamut by sRGB	0,04	0,07	0,10	0,495	0,505	0,505	0,499	0,501	0,501	0,297	0,703	0,703

Fig. 4.21. Decision matrix for Scenario 3. Including aggregated scores by criteria groups. View from the DSS prototype.

The trends observed in the initial Scenario 2 (the aggregated score of A1 increasing at the expense of A2 and A3) are confirmed and even amplified in Scenario 3, leading to a change in the ranking, with A1 surpassing A3, as shown in Fig. 4.22.

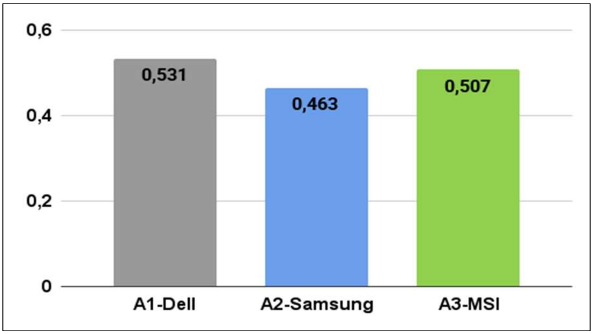


Fig. 4.22. Ranking of monitors in Scenario 3, using combined model of WSM.

The two modifications combine and complement each other successfully, as they do not interfere with each other, being executed at different steps of the algorithm.

4.3. Numerical testing of the modification formalizing the generation of coefficients giving advantage in the overall performance of the alternatives

The applicability of the proposed modification of WSM and WPM, which formalizes the generation of weighting coefficients for the alternatives (see Chapter 2, Section 2.3), was tested in a task for selecting a software framework for web development.

4.3.1. Testing scenarios

To test the applicability of the proposed modification on both methods, as well as on the generalized WSM model, three scenarios are developed. Scenario 1 and Scenario 2 are set in the context of individual decision-making and aim to explore the effect of the newly introduced type of coefficient when using the WSM and WPM methods. For comparison, the results from the traditional methods are used. The difference between Scenario 1 and Scenario 2 lies in the change of preferences for the options in the key criterion used to give an advantage to the alternatives, as well as in the specified importance coefficients for the remaining criteria. The scenario used to test the applicability of the generalized model (Scenario 3) is set in the context of group decision-making. The task parameters are expanded by adding two additional criteria — one matrix-represented and one subjective.

4.3.2. Case study

The task of selecting a framework for web application development is sized in the first two scenarios with the following parameters: 6 alternatives, 1 decision-maker, and a total of 3 criteria, 2 of which are used for evaluation and the other for giving an advantage to the alternatives, decomposed into 3 options. In Scenario 3, the alternatives remain the same, but the number of decision-makers increases to two. The number of criteria becomes 5 in order to cover all types of allowed evaluations in the generalized algorithm. All the input data is summarized in Table 4.9.

Table 4.9. Types of criteria: for evaluation and for prioritization of alternatives. Criteria importance coefficients determined by the DMs for the three scenarios.

Criteria		Type of criteria according to their purpose and according to the way the evaluation is formed	Coefficients for criteria importance			
			Scenario 1	Scenario 2	Scenario 3	
			DM-1	DM -2	DM -1	DM -2
C1	Programming language	For advantage of alternatives / Matrix-represented	-	-	-	-
C2	GitHub rating	For evaluation / Numerically represented	0,60	0,27	0,26	0,10
C3	StackOverflow rating	For evaluation / Numerically represented	0,40	0,73	0,17	0,32
C4	Software layers coverage	For evaluation / Matrix-represented	x	x	0,41	0,16
C5	Ease of use	For evaluation / Subjective	x	x	0,16	0,42

Since criteria C1 and C4 are defined as matrix-represented criteria, they are decomposed into options as shown in Table 4.10.

Table 4.10. Matrix-represented criteria (decomposable to options) used to generate a score or to generate a coefficient for the advantage of alternatives.

Criteria and the available options			Option values						Coefficients / evaluation points for options importance			
									Scenario 1	Scenario 2	Scenario 3	
			A1	A2	A3	A4	A5	A6	DM-1	DM-2	DM-1	DM-2
C1	Programming language											
	O1	JavaScript	1	1	0	0	0	0	0,46	0,22	0,46	0,22
	O2	Python	0	0	1	1	0	0	0,33	0,32	0,33	0,32
	O3	PHP	0	0	0	0	1	1	0,21	0,46	0,21	0,46
C4	Software layers coverage											
	O4	User interface	1	1	1	1	1	1	x	x	4	5
	O5	Client-side logic	1	1	0	0	0	0	x	x	5	2
	O6	Server-side logic	0	0	1	1	1	1	x	x	2	3
	O7	Database interaction	0	0	1	0	1	1	x	x	2	1

Based on the objective data from the framework ratings on GitHub and StackOverflow, scores can be generated for Scenario 1 and Scenario 2. In Scenario 3, the corresponding scores are generated through expressing preferences regarding their benefits or drawbacks. The data for both criteria (C2 and C3) is taken from the specialized website https://hotframeworks.com, which provides comparisons between different web development frameworks, and are presented in Table 4.11.

Table 4.11. Numerically presented criteria, values for alternatives and preferences of the DMs for their usefulness in Scenario 3.

Criteria	Values of the alternatives						Cost / Benefit
	A1 React	A2 Vue.js	A3 Django	A4 Flask	A5 Laravel	A6 Symfony	
C2: GitHub rating	99	100	89	89	90	81	benefit
C3: StackOverflow rating	97	87	97	83	93	86	benefit

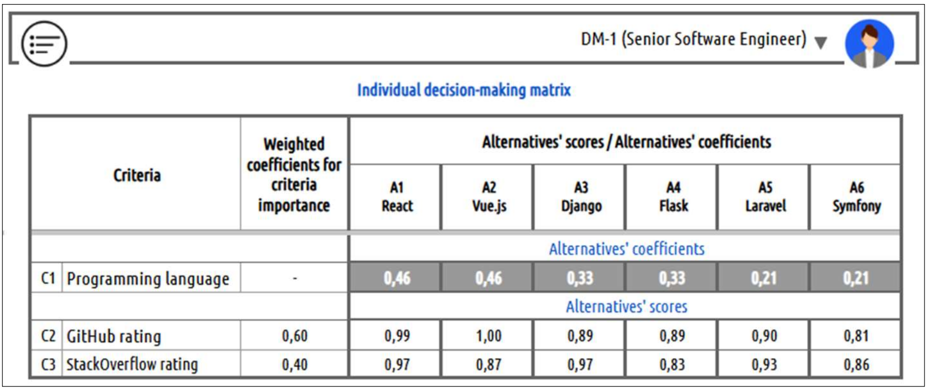
The competence weights assigned to the two decision makers in Scenario 3 are shown in Table 4.12.

Table 4.12. Weighting coeficients for DMs' competence, in accordance to the individual criteria used in Scenario 3.

Criteria	Weighting coefficients for the DMs competence	
	DM-1	DM-2
C1: Programming language	-	-
C2: GitHub rating	0,70	0,30
C3: StackOverflow rating	0,45	0,55
C4: Software layers coverage	0,80	0,20
C5: Ease of use	0,50	0,50

4.3.3. Results of numerical testing of the modified WSM

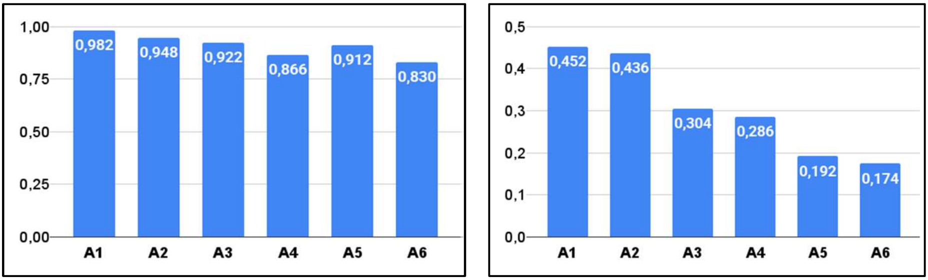
The numerical testing of the modification formalizing the generation of weight coefficients for the Weighted Sum Model (Scenario 1) is carried out in the DSS prototype. The aggregated individual decision matrix on which the experiment is conducted is shown in Fig. 4.23.



Criteria		Weighted coefficients for criteria importance	Alternatives' scores / Alternatives' coefficients					
			A1 React	A2 Vue.js	A3 Django	A4 Flask	A5 Laravel	A6 Symfony
			Alternatives' coefficients					
C1	Programming language	-	0,46	0,46	0,33	0,33	0,21	0,21
			Alternatives' scores					
C2	GitHub rating	0,60	0,99	1,00	0,89	0,89	0,90	0,81
C3	StackOverflow rating	0,40	0,97	0,87	0,97	0,83	0,93	0,86

Fig. 4.23. Individual decision matrix in Scenario 1. View from the DSS prototype.

To demonstrate the effect of the newly introduced coefficients, the results of the alternatives against criteria C2 and C3 are calculated using the classical WSM (1.3). The final ranking for both methods is presented in Fig. 4.24 (a, b).



a) Classical WSM b) Modified WSM

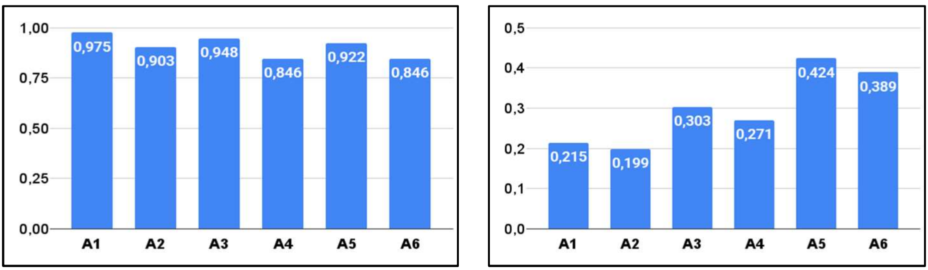
Fig. 4.24. Ranking of web development frameworks in Scenario 1, using modified WSM.

In the modified WSM, the results are adjusted by the newly introduced coefficient, reflecting a higher preference for the Python programming language, on which Flask is based, at the expense of PHP, used in Laravel.

4.3.4. Results of numerical testing of the modified WPM

In Scenario 2, designed to test the WPM, the decision maker is a junior software engineer. This justifies a change in the weighting coefficients for the importance of the criteria, increasing the weight of criterion C3 – the StackOverflow ranking, since this platform is particularly useful for novice developers. Additionally, preferences regarding the used programming language also change, resulting in new values of ψ_i^k.

In order to determine the impact of the newly introduced coefficients on the modified WPM, the results of the alternatives are also calculated using the classical Weighted Product Model (1.7). The comparison between the final rankings of the alternatives when used both methods is illustrated in Fig. 4.26 (a, b).



a) Classical WPM b) Modified WPM

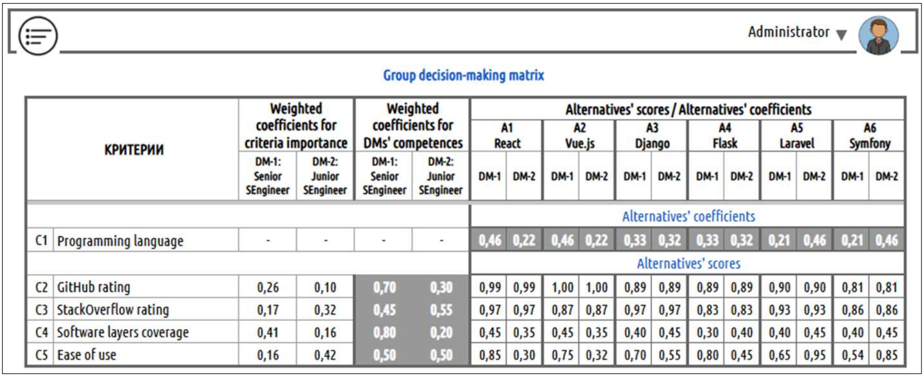
Фиг. 4.26. Ranking of web development frameworks in Scenario 2, using modified WPM.

A similarity in results is observed between frameworks using the same programming language (and thus having the same coefficients), as well as distinct differences among frameworks based on different languages. These results confirm the effectiveness and applicability of the modification to the Weighted Product Method as well.

4.3.5. Results of numerical testing of the generalized model of the WSM

Scenario 3 aims to test the applicability of the newly proposed coefficient for an advantage of alternatives in combination with the other two modifications, which build up the so-called in this dissertation – generalized model of the WSM.

The data from the completed individual forms, together with the importance coefficients of the criteria and the competence coefficients assigned by the DMs, is structured into a consolidated decision matrix. A view of the generated matrix from the DSS prototype is shown in Fig. 4.29.



КРИТЕРИИ		Weighted coefficients for criteria importance		Weighted coefficients for DMs' competences		Alternatives' scores / Alternatives' coefficients											
						A1 React		A2 Vue.js		A3 Django		A4 Flask		A5 Laravel		A6 Symfony	
		DM-1: Senior SEngineer	DM-2: Junior SEngineer	DM-1: Senior SEngineer	DM-2: Junior SEngineer	DM-1	DM-2	DM-1	DM-2	DM-1	DM-2	DM-1	DM-2	DM-1	DM-2	DM-1	DM-2
						Alternatives' coefficients											
C1	Programming language	-	-	-	-	0,46	0,22	0,46	0,22	0,33	0,32	0,33	0,32	0,21	0,46	0,21	0,46
						Alternatives' scores											
C2	GitHub rating	0,26	0,10	0,70	0,30	0,99	0,99	1,00	1,00	0,89	0,89	0,89	0,89	0,90	0,90	0,81	0,81
C3	StackOverflow rating	0,17	0,32	0,45	0,55	0,97	0,97	0,87	0,87	0,97	0,97	0,83	0,83	0,93	0,93	0,86	0,86
C4	Software layers coverage	0,41	0,16	0,80	0,20	0,45	0,35	0,45	0,35	0,40	0,45	0,30	0,40	0,40	0,45	0,40	0,45
C5	Ease of use	0,16	0,42	0,50	0,50	0,85	0,30	0,75	0,32	0,70	0,55	0,80	0,45	0,65	0,95	0,54	0,85

Fig. 4.29. Consolidated decision matrix for Scenario 3. View from the prototype decision support system.

The ranking of the alternatives after applying the generalized model (3.3) is shown in Fig. 4.30.

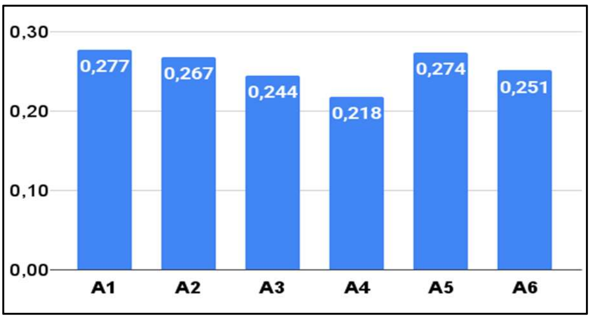


Fig. 4.30. Ranking of web development frameworks in Scenario 3, using generalized WSM.

In group decision-making, the effect of the adjustment coefficients can be balanced by opposing preferences among the DMs. However, the inclusion of the modification that accounts for competence by criteria allows greater weight to be given to the opinions of more competent DMs, which ultimately influences the final decision.

Combining all the proposed modifications into a single generalized algorithm is both feasible and successful. This is because each modification affects different steps of the algorithm and does not introduce mathematical constraints that would prevent the application of the others.

CONCLUSION

Thanks to digitalization, today more than ever before, it is possible to use a wide variety of data. To form an informed decision, this data must be processed appropriately so that structured information can be obtained, providing the basis for selecting the most suitable decision. In this context, the present study focuses on decision-making methods and models, including the formation of group decisions.

The objects of research in this dissertation are the Weighted Sum Method, the Weighted Product Method, and the software architectures through which they can be integrated into a decision support system. These two methods are among the most popular and fundamental evaluation techniques in multicriteria decision-making. Over time, they have been developed through various modifications. Their evolution shows that they have not remained static methods but have dynamically adapted to new practical needs. As a result, they have become increasingly flexible, widely applicable, and suitable for integration into decision support systems. As a software tool for rational decision-making, a DSS must be designed with consideration of the functional requirements imposed by the algorithms and mathematical models. The software architecture is determined by the combination of different styles and patterns, in accordance with the organization's non-functional requirements, such as infrastructure capabilities, the technological expertise of the team, the expected scale of system usage, and others.

This dissertation describes the proposed modifications to the Weighted Sum Model and the Weighted Product Model. The first modification introduces competence coefficients for each DM for every individual criterion, in this way refining the influence of each DM in the group decision-making. The second modification formalizes the generation of evaluations by combining the preferences of the DMs with objective data, thus reducing the subjectivity and facilitating the assessment of alternatives. The third modification introduces the generation of a new type of coefficients that allow the suppression or amplification of the aggregated results of the alternatives, and in this way meeting critical requirements of the decision task. The results of the conducted experiments confirm that all proposed modifications can be successfully combined within the Weighted Sum Model into a single generalized algorithm. The different combinations of modifications allow the results to be aggregated through one of 13 distinct models: 9 based on the Weighted Sum Model and 4 based on the Weighted Product Model.

Two different software architectures for the Decision Support System have been proposed and designed, addressing conflicting non-functional requirements of the system. The proposed three-tier architecture follows the classical three-tier architectural pattern. A monolithic architectural style with synchronously managed event flow is chosen. This architecture is suitable for organizations that maintain their own server infrastructure. The proposed serverless architecture employs the distributed function architectural pattern with asynchronous event-flow management. This approach provides high scalability, minimal need for infrastructure management, and very high adaptability to dynamic workloads. These characteristics make the serverless architecture well suited for organizations that rely on cloud services and must support a large number of users, such as corporate or public institutions. Based on the proposed and designed architectures, DSS prototypes were developed to test the applicability of both the proposed modifications and the architectures themselves. The prototypes provide centralized data management, access from any location, automation of routine operations, and a module for linear analysis of intermediate results.

Based on the results obtained from the conducted testing of both the proposed modifications and the proposed architectures for implementing the decision-support models, their practical applicability can be unambiguously confirmed through the solution of specific real-world tasks.

As a future development of the research related to the modifications of the decision-making methods, the following activities are planned: (1) searching a solution to overcome the limitation of applying the modification that formalizes the generation of evaluations for alternatives to the Weighted Product Model; (2) combining all modifications for both methods; (3) adapting the proposed modifications to other similar decision-making methods.

The planned activities for the future elaboration of the developed prototypes of the DSS are related to: (1) developing them as a fully functional application for academic purposes such as experiments, demonstrations and training; (2) implementing or integrating with an external service for upgrading network security with authorization and accountability (AAA , Authentication, Authorization, Accounting); (3) integration of artificial intelligence agents as a virtual expert and automation of part of the administrative tasks through it.

The results of the presented research have been reported at 4 international conferences. Most of the results have been published in a total of 5 scientific publications, as 1 of them is in an international scientific journal. All 5 publications are reffered and indexed in the globally recognized scientific database Scopus, with three of them published in journals with SJR.

CONTRIBUTIONS

Based on the obtained results, the following contributions can be formulated:

1) A modification of the algorithm and the mathematical models of the Weighted Sum Model and the Weighted Product Model has been proposed in the context of group decision-making. The modification considers the different areas of expertise of the decision-makers by introducing competence weight coefficients for each DM with in accordance to each individual criterion.
2) A modification of the algorithm for implementing the Weighted Sum Model has been proposed, formalizing the generation of evaluations for the alternatives. Instead of

relying on subjectively assigned scores, a mechanism is introduced that combines objective data about the alternatives with the preferences of the decision-makers through the decomposition of criteria into options. The modification is applicable to both group and individual decision-making.

3) A modification of the algorithm and the mathematical models of the Weighted Sum Model and the Weighted Product Model has been proposed, formalizing the generation of coefficients that provide an advantage in the overall performance of the alternatives. These coefficients are generated with respect to key criteria, using the same mechanism for decomposing criteria into options and combining objective data about the alternatives with the preferences of the decision-makers. The modification is applicable to both group and individual decision-making.
4) A generalized algorithm and combined mathematical models have been proposed, integrating the proposed modifications of the two methods. The different combinations allow the results to be aggregated by 13 different models - 9 using the Weighted Sum Model and 4 using the Weighted Product Model.
5) Two software architectures (three-tier and serverless) have been proposed for implementing a decision support system, meeting different non-functional requirements. The two architectures demonstrate different architectural styles for implementing the proposed generalized algorithm and the combined mathematical models of the modified Weighted Sum Model and Weighted Product Model.
6) Two prototypes of a decision support system have been developed in accordance with the proposed software architectures, including a module for interpreting the results. The prototypes are used for numerical testing of the proposed modifications to the Weighted Sum Model and the Weighted Product Model, convincingly demonstrating their applicability.

LIST OF PUBLICATIONS ON THE DISSERTATION

1. **Dimitrova, Z.**, Borissova, D., & Dimitrov, V. (**2024, a**). Web application based on serverless architecture to support group decision-making by scoring models. In: *Proceedings of the 2024 5th International Conference on Communications, Information, Electronic and Energy Systems*, 365-369, E-ISBN: 979-8-3503-5286-3; USB ISBN: 979-8-3503-5285-6; PoD ISBN: 979-8-3503-5287-0. https://doi.org/10.1109/CIEES62939.2024.10811190 (Scopus)
2. **Dimitrova, Z.**, Borissova, D., & Dimitrov, V. (**2024, б**). A methodology for alternatives ranking by estimations forming based on values from criteria decomposition into options. *WSEAS Transactions on Business and Economics, 21*, 2136-2144, Print ISSN: 1109-9526, E-ISSN: 2224-2899. http://dx.doi.org/10.37394/23207.2024.21.176 (**SJR 2024 = 0.192, Q4**) (Scopus)
3. **Dimitrova, Z.**, Borissova, D., Mikhov, R., & Dimitrov, V. (**2023**). Group decision-making involving competence of experts in relation to evaluation criteria: Case study for e-commerce platform selection. In: *Modelling and Development of Intelligent Systems* (Simian, D., Stoica, L.F., eds.), 42–53. Communications in Computer and Information Science, 1761, Springer, Cham, Print ISBN: 978-3-031-27033-8; Online ISBN: 978-3-031-27034-5. https://doi.org/10.1007/978-3-031-27034-5_3 (**SJR 2023= 0.203 Q4**) (Scopus)
4. **Dimitrova Z.**, Borissova D., & Dimitrov V. (**2021**). Design of Web Application with Dynamic Generation of Forms for Group Decision-Making. In: *Computer Information Systems and Industrial Management* (Saeed K., Dvorsky J., eds.), 112-123. Lecture Notes in Computer Science, 12883, Springer, Cham, Print ISBN: 978-3-030-84339-7; Online ISBN: 978-3-030-84340-3. https://doi.org/10.1007/978-3-030-84340-3_9 (**SJR 2021= 0.407 Q2**) (Scopus)
5. Borissova, D., **Dimitrova, Z.**, Dimitrov, V., Yoshinov, R., Garvanova, M., & Garvanov, I. (**2021**). Multi-attribute decision-making model for ranking of web development frameworks. In: 25th International Conference on Circuits, Systems, Communications and Computers, 3-8, E-ISBN: 978-1-6654-2749-4; (PoD) ISBN: 978-1-6654-2750-0. https://doi.org/10.1109/CSCC53858.2021.00009 (Scopus)

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