



**BULGARIAN ACADEMY OF SCIENCES
INSTITUTE OF INFORMATION AND COMMUNICATION
TECHNOLOGIES**

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**METHODS, ALGORITHMS AND SOFTWARE SYSTEMS
FOR DECISION SUPPORT**

A B S T R A C T

Of thesis for awarding educational and scientific degree PhD

Scientific Specialty: „Informatics“

Professional area: „4.6. Computer sciences“

Scientific supervisor:

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Sofia, 2020

The thesis contains:

- 174 pages
- 43 figures,
- 16 tables
- 20 pages of bibliography that includes 181 titles

INTRODUCTION

1. Relevance of the topic

Various tasks of planning, control and analysis in production [Kirilov, Gulishaki et al. (2016)], transport, ecology [Jaszkiewicz, Slowinski, (1997)], education and other fields can be defined as multi-criteria decision-making tasks [White, (1990)]. Depending on their formal formulation, these tasks can be divided into two distinct classes [Vincke, (1992)]. In the first class of tasks, a finite number of alternatives are presented in tabular form. These are called multicriteria decision-making problems with discrete alternatives or multicriteria analysis (MA) tasks [Dyer, (2004)]. In the second class of problems, a finite number of explicitly defined constraints in form of functions determine an infinite number of acceptable alternatives [Sawaragi, Nakayama et al., (1985)]. These are called multicriteria optimization (MO) problems. This class is subject of more detailed consideration in this thesis.

In multicriteria optimization and multicriteria analysis problems, several criteria are optimized simultaneously in a given set of acceptable alternatives. In general, there is no alternative that is optimal for all criteria, but there are many alternatives that have the following common property: any improvement in the value of one of the criteria leads to a deterioration in the value of at least one of the other criteria. This property was first discovered in 1896 by the Italian mathematician-economist Vilfredo Pareto and therefore, in 1951, the set was named after him - a set of non-dominated (Pareto-optimal) alternatives [solutions] [Collette, Siarry, (2013)]. From a purely mathematical point of view, any alternative to the Pareto set can be a solution to a multicriteria problem. In the end, in order to choose an alternative, additional information is required, which is determined by the so-called. "Decision maker (DM)". The information provided by the HLS reflects his personal preferences with regard to the qualities of the most preferred alternative sought.

Mathematically, solving this kind of problem is a long interactive process involving complex and large-scale calculations. This automatically necessitates the use in practice of

assistive software systems designed specifically to assist in solving this type of task [Sierksma, (2001)].

2. Multicriteria analysis problems.

In multicriteria analysis problems, a finite number of alternatives are presented in tabular form. Multicriteria analysis is also called multiattribute analysis or multi-criteria decision making with given discrete alternatives. In general, the ideal alternative does not exist. The final goal is to obtain a full or partial order of alternatives from best to worst.

3. Multicriteria optimization problems

3.1. Formal formulation of multicriteria optimization problems

Multicriteria optimization problems can be both linear and nonlinear. There are approaches and methods for solving both types of problems [Miettinen, Kirilov (2005)], but the focus of this thesis is on linear problems for multicriteria optimization.

In the general case, two types of static mathematical models are used.

The first model is as follows:

$$\max z = f(x)$$

(I) subject to:

$$g_j(x) \leq b_j, j=1, \dots, m,$$

where:

– The symbol “max” means that all criteria (objective functions) must be maximized simultaneously;

– $f(x)$ is an objective function (criterion);

– $x = (x_1, \dots, x_n)^T$ is a variables vector;

– $g_j(x), j=1, \dots, m$, are problem subjective functions;

The functions $g_j(x), j=1, \dots, m$, and the objective function $f(x)$ are real functions: $g_j: \mathbb{R}^n \rightarrow \mathbb{R}, j=1, \dots, m$, и $f: \mathbb{R}^n \rightarrow \mathbb{R}$, where \mathbb{R} is the set of the real numbers, \mathbb{R}^n is the n -sized euclidain space.

These functions determine the so-called set of acceptable alternatives. We denote this set $S, S \subset \mathbb{R}^n$.

The other mathematical model, which is often more relevant to certain applied problem and will be considered, is the following:

$$\text{“max” } \{z_1 = f_1(x), z_2 = f_2(x), \dots, z_k = f_k(x)\}$$

(II) subject to:

$$g_j(x) \leq b_j, j=1, \dots, m,$$

where:

– “max” means that all objective functions are maximized simultaneously;

– $f_i(x), i=1, \dots, k$, are the objective functions (criteria). These are real functions: $f_i: \mathbb{R}^n \rightarrow \mathbb{R}, i=1, \dots, k$. What is special about them is that they are usually contradictory and incommensurable.

– $f(x) = (f_1(x), \dots, f_k(x))$ or $z = (z_1, \dots, z_k)$, is the objective vector;

– $k \geq 2$;

– $g_j(x), j=1, \dots, m$, are the problem subjective functions and are real function: $g_j: \mathbb{R}^n \rightarrow \mathbb{R}$, $j=1, \dots, m$

These functions determine the allowable set of solutions (variables). We will denote it by S и $S \subset \mathbb{R}^n$.

– We denote with $f(S) = Z$ the projection of the set S in the criterion space \mathbb{R}^k , $Z \subset \mathbb{R}^k$. Z is called the admissible set in the criterion space.

– We denote with $z=(z_1, \dots, z_k)^T$, where $z_i=f_i(x)$, $i=1, \dots, k$. z is the criteria vector and $z \in Z$.

3.2. Methods for solving multicriteria optimization problems.

There are two main approaches to solving multicriteria optimization problems: the scalarization approach [Miettinen (1999)] and the approximation approach [Ehrgott and Wiecek (2004)]. The main representatives of the scalarization approach are the interactive algorithms. In these algorithms, multicriteria optimization problems are considered as decision-making problems, and the emphasis is on the real participation of the DM in the process of problem solving.

The interactive algorithms are the most used. They have the following basic characteristics:

- A small fraction of Pareto-optimal solutions needs to be generated and evaluated by the DM.
- In the process of solving a multicriteria problem, the DM can be trained in the specifics of the task.
- The DM feels more confident in the final result.

By the final solution of problem (II) we will consider a Pareto-optimal solution that best satisfies the preferences and requirements of the DM. This solution is also called the most preferred solution to the problem.

3.3. Scalarization approach for transforming a multicriteria problems into single criteria optimization problems

Scalarization is the process of transformation of multicriteria optimization problems into one or more single-criterion optimization problems with a real objective function, which usually depends on one or more parameters. This transformation allows us to use the theory and results of single-criteria optimization. Each of the interactive algorithms for solving different classes of multicriteria optimization problems, developed so far, has its advantages and disadvantages, mainly related to the type of information provided by the DM and reflecting its global and local preferences, as well as the way this information is extracted from him; the type and manner of solving the scalarizing task. Interactive algorithms are especially suitable for solving linear and convex nonlinear problems of multicriteria optimization in which the time to generate a new solution is not of great importance.

CHAPTER I. CLASSIFICATION-ORIENTED SCALARIZING PROBLEMS AND ALGORITHMS

The chapter describes scalarizing tasks and algorithms for solving multicriteria optimization problems, developed in cooperation with a team of scientists at BAS. As a result, an interactive algorithm based on the GENS-IM method was developed and implemented, and serves as the basis for the computational modules in the developed multicriteria optimization application systems MKO-2.1. and WebOptim. Some of the results are presented in publication #5.

2.1. Classification-oriented scalarization problems DAL

The name of the scalarization problems DAL [Vassileva (2004)] (also called the desired and acceptable level tasks) is derived from the first letters of the English words: Desired, Acceptable, and Level.

2.2. Classification-oriented scalarization problems DALDI

The name of the scalarization problems DALDI-1 [Vassilev, Genova et al. (2004)] is derived from the first letters of the English words: Desired, Acceptable, Level, Direction, and Interval.

2.3. Generic scalarization problems

In order to obtain a weak Pareto-optimal solution, starting directly or indirectly from the current weak Pareto-optimal solution, a generic scalarization problem GENWS was developed. By changing its parameters, a large part of other famous scalarization problems can be generated.

2.4. Generic interactive algorithm based on the GENS-IM method for solving linear and linear-integer problems of multicriteria optimization

This section describes a generalized interactive algorithm with variable scalarization and parameterization, based on the GENWS and GENS scalarizing problems, which has the following characteristics:

- The DM can set its preferences by criteria weights; by the ε constraints; by desired and acceptable levels of change in the criteria values; by desired and acceptable directions for changing the criteria values; by setting desired and acceptable levels; by setting directions and intervals of change of criteria values;
- In the process of solving multicriteria problems, the DM can change the way it sets its preferences

The algorithm serves as a design basis two software systems for solving multicriteria optimization problems - MKO-2.1 and WebOptim.

CHAPTER II. SOFTWARE SYSTEM MKO-2.1

The chapter describes the development of the MKO-2.1. This includes a syntax for defining multicriteria optimization problems; control module and optimization module. A detailed description of the system operations has also been made. The results are presented in publications #7 and #8.

2.1 Purpose of MKO-2.1. system

The MKO-2.1 software system is designed to support solving linear and linear-integer problems for multicriteria optimization. A generalized scalarization problem module is used to generate 12 well-known scalarization problems of different type and their corresponding 12 interactive algorithms.

2.2. Syntax for defining multicriteria optimization problems.

In order to present the data electronically to the computational modules in the MKO-2.1 system, it is necessary to translate the data from an appropriate, human-readable syntax for describing this type of problems. For this purpose, a formal grammar was developed with the corresponding parser, which accepts the description of the problem in text form. The grammar consists of a set of components and rules for arranging them.

2.3. Main modules in MKO-2.1 system.

The MKO-2.1 system consists of three main groups of modules: control module, optimization modules and interface modules.

The control module is an integrated software environment for creating, processing and storing system-associated files (with the extension "* .mlp"), as well as for connecting and executing various types of software modules.

The interface modules provide the dialogue between the DM and the system during the input and correction of the input data.

The optimization modules implement 12 interactive multicriteria optimization algorithms, as well as 2 linear and linear single-integer optimization algorithms.

2.4. Working with MKO-2.1 system

The MKO-2.1 system runs under MS Windows operating system. On startup, the “MKO-2.1 Main window” window opens, containing a bar with six main menus - File, Settings, Edit, View, Windows and Help and a second bar with shortcuts - New, Open, Save , Print, Settings and Graphics.

The command "New" opens a window for entering a new task, the data for which is saved by the system in a file with the extension "* .mlp". File saving is done with the “Save” or |”Save As” commands. If the task data contained in this file is not fully entered or the process for solving this task is not started, the "MKO-2.1 Editor" window opens with the command "Open". Otherwise, the MKO-2.1 “Solution” window opens.

2.4.1. Problem definition

The definition and correction of the criteria and subjective functions of the MO problem is done in two separate fields of the window “MKO-2.1 Editor” (Fig. 2).

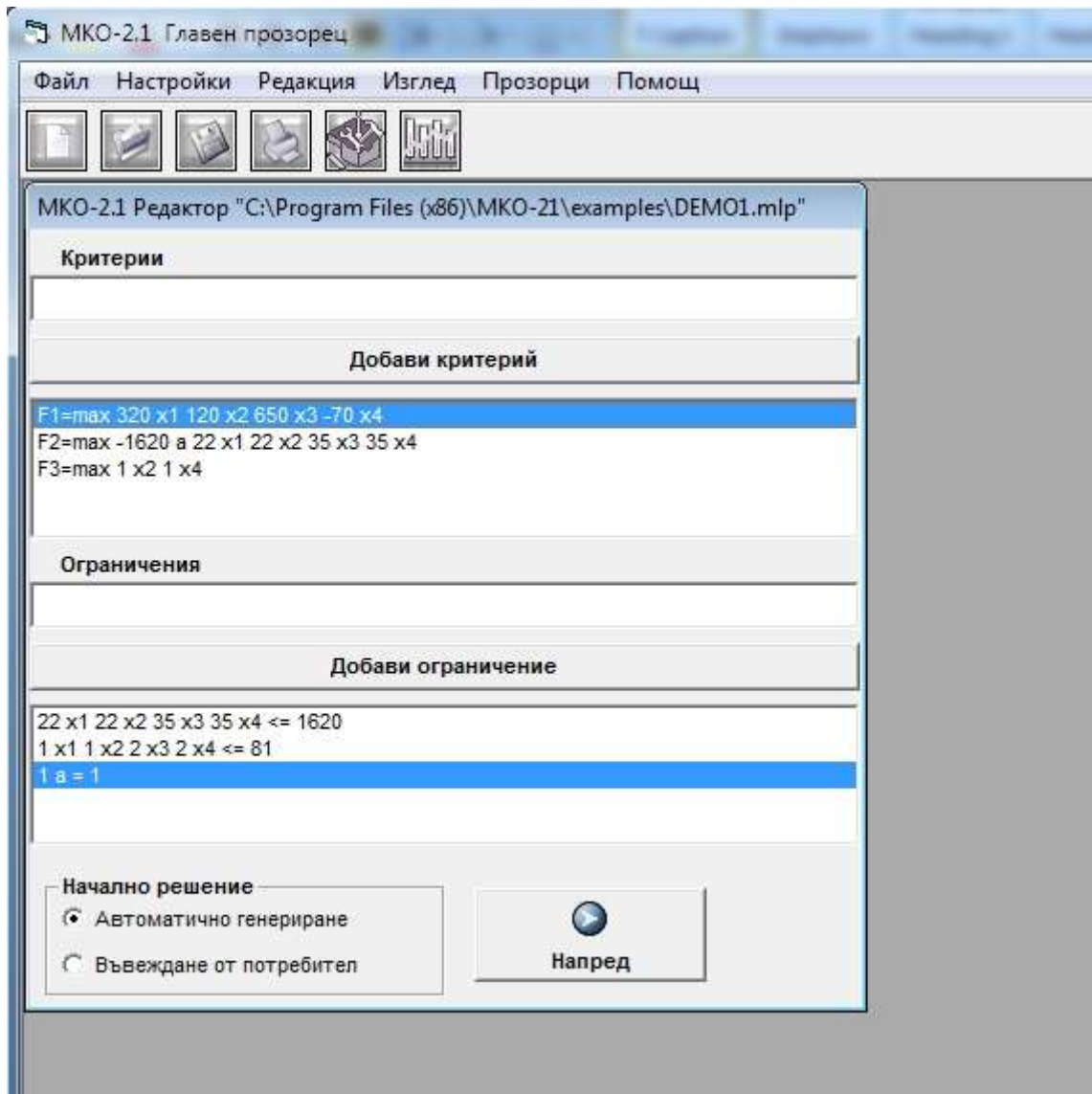


Figure 2. MKO-2.1. - Editor.

The button “Next” opens the “Variables Information” window, which sets information about the variables type and boundaries of changing.

The following is an interface for specifying how the DM preferences will be set in the process of problem solving. The ways of setting preferences are separated into two main groups - "Select preferences only" and "Select preferences and method".

2.4.2. Solving the multicriteria optimization problem

The process of solving linear and linear-integer multicriteria optimization problems is assisted by 12 additional windows in the interface module (Fig. 7). Each one of them servers to enter specific data for one of the 12 interactive algorithms.

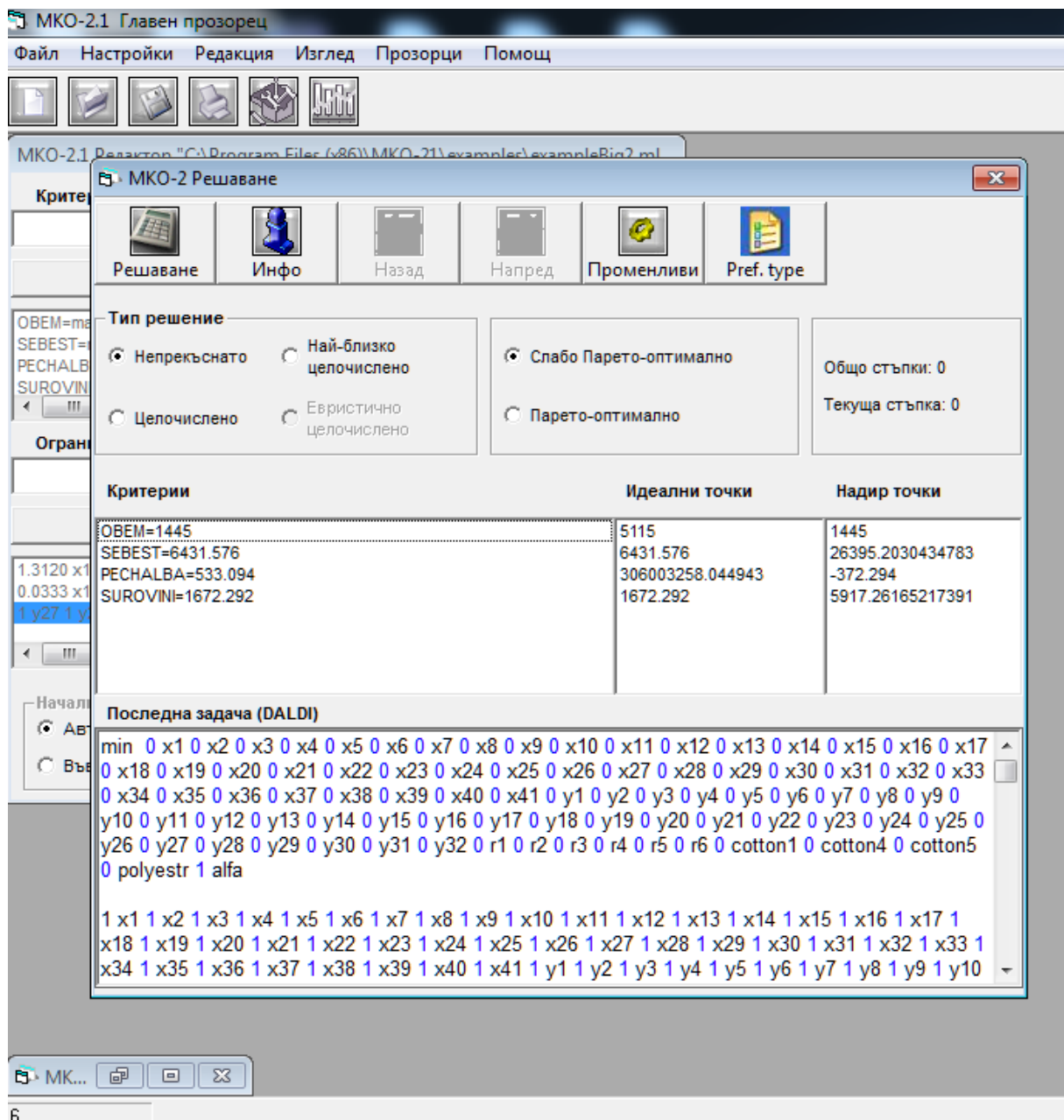


Figure 7. MKO-2.1. – Problem solving process.

At the top of the main window is a button bar that realizes the main functions of the process of interactive solving of linear and linear-integer problems for multicriteria optimization.

2.4.3. Settings

System settings of the MKO-2.1 system can be changed using the “Settings” menu, which contains three commands - "Language", "Global Variables" and "File Associations".

Graphics: Selecting the “Graphics” command allows two types of graphical information to be displayed during the process of problem solving (Fig. 10). The graph bar above can visually compare the solutions found in two iterations, selected by the fields below it. The graph below can visually track the changes in the values of the individual criteria at different iterations.

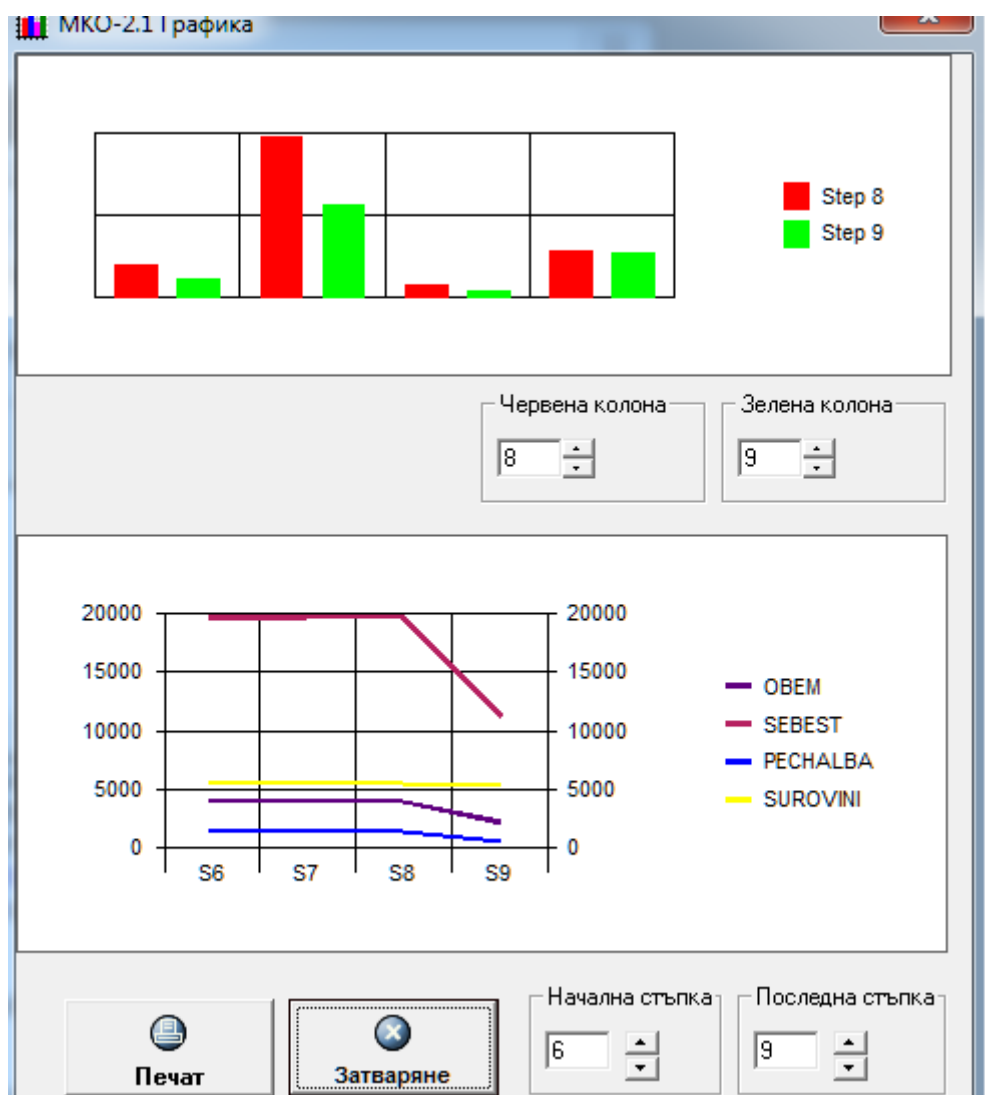


Figure 10. MKO-2.1. – Graphical representation of the solving process.

CHAPTER III. SOFTWARE SYSTEM WEBOPTIM

The chapter describes the development of the web-based decision support system WebOptim. This includes: the overall software architecture of the system; database architecture; interface modules; control module; module security and user management; module for management and maintenance of computing sub-modules (solvers); Intermodal communication system; public API module for connection and data exchange with third party systems. The results are presented in publications #2, #4 and #6.

3.1. Purpose of WebOptim system

The WebOptim software system is a natural successor to the MKO-2.1 system and was designed to be one contemporary and fully web based. Because of this, it provides easy and free access to as many users as possible. The other two main goals in system design are to be able to easily add new computing modules that implement different algorithms and to implement a communication link for data exchange with other independent external systems.

3.2. Structure of WebOptim system

WebOptim is modular system and contains the following basic modules:

- Database
- Interface modules
- Security and user management module
- Computational submodules (solvers)
- Module for management and support of computing sub-modules (solvers)
- Intermediate-module communication system.
- Public API module for data exchange with external independent software systems.

The system has been developed using only Microsoft technologies:

- MS SQL Server

- MS.NET framework
- MS Visual Studio version 10

A relational database MS SQL Server is used to store all user information, problem definitions and solution data, metadata concerning different methods and solvers.

3.3. Main components of WebOptim

- User management and security module
- Module for management and maintenance of calculation sub-modules (solvers)
- Inter-module communication system

Public API module for data exchange with external independent software systems.

3.4. Working with WebOptim system

Because the system is fully web-based, it requires nothing but internet connectivity and a web browser.

The first step is to register a personal user profile. This is possible through a standard web form interface for entering user credentials - email address, username and password. After successful logon, the user is provided with an interface containing a list of problems, whose owners have chosen to make them publicly available (Figure 15).

Home Members area Browse public problems Solvers Syntaxes Public API About							
My Problems My MC Problems		LIST OF PUBLIC PROBLEMS.					
	Title	Created on	Status	Sent to solver on	Syntax	Use solver	
Preview	LP test big	6/6/2011 4:42:06 PM	Solution obtained	3/23/2012 8:26:23 PM	lp	LP Solve	
Preview	MPS Test	6/9/2011 9:41:24 AM	Solution obtained	3/23/2012 8:26:21 PM	mps	LP Solve	
Preview	CPLEX test	6/11/2011 1:21:11 AM	Solution obtained	3/23/2012 8:26:19 PM	lpt	LP Solve	
Preview	Планиране производството на играчки	8/24/2011 2:50:53 PM	Solution obtained	8/26/2011 1:22:26 PM	lp	LP Solve	
Preview	Задача за работен график	8/25/2011 4:07:37 PM	Solution obtained	8/26/2011 1:38:30 PM	lp	LP Solve	
Preview	Задача за работен график	8/25/2011 4:12:00 PM	New		lp	LP Solve	
Preview	Краткосрочно финансово планиране	8/26/2011 2:18:14 PM	Solution obtained	4/3/2012 2:41:36 PM	lp	LP Solve	
Preview	Задача за смеси	8/30/2011 11:18:32 AM	Solution obtained	8/31/2011 11:51:47 AM	lp	LP Solve	
Preview	Задача за тримесечно планиране	8/31/2011 2:46:02 PM	Solution obtained	5/7/2012 11:51:19 AM	lp	LP Solve	
Preview	Задача за инвестиционна стратегия	9/2/2011 4:00:36 PM	Solution obtained	9/2/2011 4:26:59 PM	lp	LP Solve	

Figure 15. WebOptim – List of publically available problems.

This gives the opportunity to review and study these problems, which is a huge benefit for new users of the system and those who do not yet have sufficient experience in working with the system or solving optimization problems.

The user part of the system consists of two main interfaces for solving single or multi-criteria optimization problems respectively.

3.4.1. Solving single-criteria optimization problems

Single-criteria optimization problems in WebOptim are presented in the "My problems" section. There is a list of user defined problems and some of their more important attributes such as creation date, problem status (new, pending, solved, error) etc. (Fig. 16).

MY LP PROBLEMS

	ID	Title	Created on	Status	Sent to solver on	Syntax	Use solver	
Select <input type="button" value="Send to solver"/>	113	Test MO 3 Predachen cech	4/13/2016 1:22:15 PM	Solution obtained	4/13/2016 1:22:29 PM	lp	GENS-IM	<input type="button" value="Delete"/>
Select <input type="button" value="Send to solver"/>	92	Test MO 3 Predachen cech	7/13/2015 7:11:27 PM	Solution obtained	7/13/2015 7:13:12 PM	lp	GENS-IM	<input type="button" value="Delete"/>
Select <input type="button" value="Send to solver"/>	41	cplex	9/12/2011 5:32:19 PM	Solution obtained	11/22/2012 3:49:11 PM	lpt	GENS-IM	<input type="button" value="Delete"/>
Select <input type="button" value="Send to solver"/>	19	CPLEX test	6/12/2011 2:29:02 PM	Solution obtained	3/23/2012 8:26:15 PM	lpt	GENS-IM	<input type="button" value="Delete"/>
Select <input type="button" value="Send to solver"/>	18	CPLEX test	6/12/2011 2:23:58 PM	Solution obtained	3/23/2012 8:26:17 PM	lpt	GENS-IM	<input type="button" value="Delete"/>
Select <input type="button" value="Send to solver"/>	16	CPLEX test	6/11/2011 1:21:11 AM	Solution obtained	3/23/2012 8:26:19 PM	lpt	GENS-IM	<input type="button" value="Delete"/>

Figure 16. WebOptim – List of single criteria optimization problems

Creating a new problem or editing an existing one is done through single-criterion optimization problems defining interface (Fig. 17).

problem_id	19
Title	CPLEX test
created_on	6/12/2011 2:29:02 PM
Status	Solution obtained
Sent to solver on	3/23/2012 8:26:15 PM
Description	CPLEX test problem
Public problem	<input type="checkbox"/>
Syntax	LP - CPLEX
Solver	GENS-IM
Definition	<pre> Maximize obj: x1 + 2 x2 + 3 x3 + x4 Subject To c1: - x1 + x2 + x3 + 10 x4 <= 20 c2: x1 - 3 x2 + x3 <= 30 c3: x2 - 3.5 x4 = 0 Bounds 0 <= x1 <= 40 2 <= x4 <= 3 General x4 </pre>

Figure 17. WebOptim – single criterion optimization problem definition.

After the problem is defined and saved, it is sent to the calculation module. The information about the calculated solution is presented in two parts - basic and extended. The main part contains the values of the variables and the objective function, and the expanded part contains detailed information regarding the process of solving itself - the method used, number of iterations, intermediate solutions, etc. (Fig. 19).

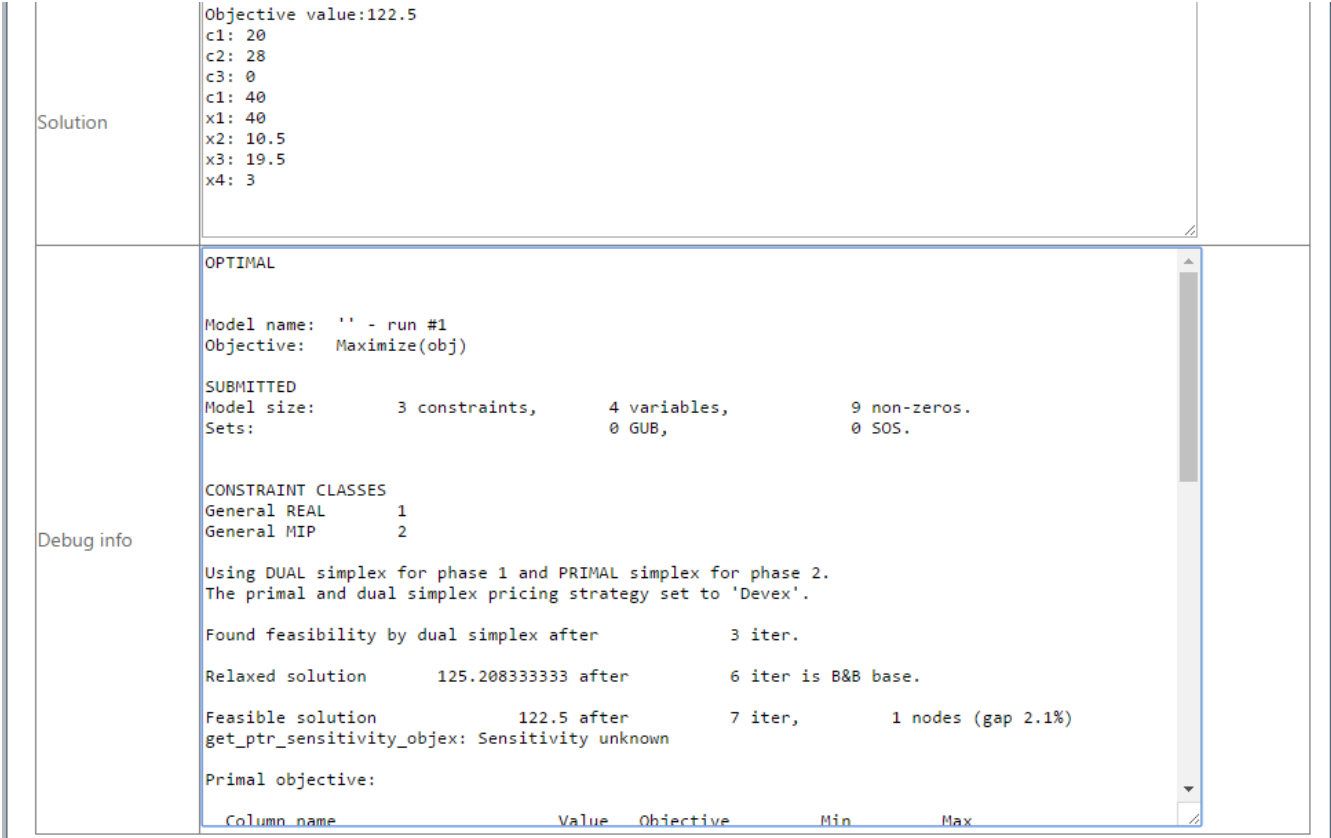


Figure 19. WebOptim – single-criterion problem solution presentation.

3.4.2. Multicriteria optimization problem solving

Solving multicriteria optimization problems in WebOptim system is performed in "My MCP problems" section. It is very similar to the single-criterion optimization problems solving interface and contains the same attributes (Fig. 21).

Select 1 test 1		6/19/2013 5:18:16 PM		New		lp		GENS-IM		Delete	
		Update		Cancel							
problem_id	19										
Title	Производство на фотоапарати										
created_on	7/15/2016 4:11:04 PM										
Status	New										
Sent to solver on											
Description	<p>Фабрика за фотоапарати произвежда 2 модела фотоапарати - стандартен и луксозен. Продукцията им е предназначена както за вътрешния, така и за международния пазар. Производствения процес определя съответни зависимости между човешката работната сила и машинното време за даден период.</p> <p>Целта е да се оптимизира производствения процес, имайки предвид, че марката стартира промоционална компания с цел да наложи името си на международния пазар.</p>										
Problem Definition	<pre> max Z1 = 320 x1 120 x2 650 x3 70 x4 min Z2 = 1620 a 22 x1 22 x2 35 x3 + 35 x4 max Z3 = x2 x4 22 x1 + 22 x2 + 35 x3 + 35 x4 <= 1620 x1 + x2 + 2 x3 + 2 x4 <= 81 a = 1 x1 >= 1 x2 >= 1 x3 >= 1 x4 >= 1 </pre>										
Public problem	<input checked="" type="checkbox"/>										
Syntax	LP - LPSolve										
Solver	GENS-IM										

Figure 21. WebOptim – Multicriteria optimization problems definition

Similar to the MKO-2.1 system, WebOptim implements 10 “aposteriori” methods for solving multicriteria optimization problems. The process continues until the DM has made one of all solutions final. This part of the system provides the user interface needed for the interactive process of searching for a solution (Fig. 34).

Select preference type

WS - scalarising problem of weighted sum
 Input parameters: The DM sets weights for each objective function
 Output parameters: one supported Pareto optimal solution.

Help

Requirements

Step: 4/4

Min/Max	x1	x2	x3	x4	a	Ideal	Nadir	Step1	Preference	Step2	Preference	Step3	Preference	Step4	Preference
Z1 = Max:	320	120	650	70	0	25535	1160	25535	Improve	25530	Improve	25530	Worse	22930	undefined
Z2 = Min:	22	22	35	35	1620	114	1619.999	1426.5	Worse with 5.0000	1431	Worse with 4.0000	1431	Interval -1.0000,1,300.0000	1291	undefined
Z3 = Max:	0	1	0	1	0	70.4545	2	2	Worse to 1.0000	2	Worse with 3.0000	2		2	undefined
a = Max:	0	0	0	0	0	0	0	0	Worse to 1.0000	0	Worse with 3.0000	0	Worse	0	undefined

Last Error:

Result

```

An optimal solution was obtained.
x1=2.0000 x2=1.0000 x3=34.0000 x4=1.0000 a=0.0000
Scalarized problem: (DALDI/Pure Integer)
Min: Alfa
320 x1 +120 x2 +650 x3 +70 x4 +25530 Alfa >= 25530
22 x1 +22 x2 +35 x3 +35 x4 +1620 a >= -1
22 x1 +22 x2 +35 x3 +35 x4 +1620 a <= 1300
0.01 Alfa >= 0
22 x1 +22 x2 +35 x3 +35 x4 <= 1620
x1 + x2 +2 x3 +2 x4 <= 81
x1 >= 1
x2 >= 1
x3 >= 1
x4 >= 1
a >= 0
          
```

Figure 34. WebOptim – Main interface for generating Pareto-optimal solutions

3.4.3. Public API module for data exchange with external independent software systems

WebOptim makes possible the communication and data exchange with other external systems through so-called web services.

The benefit from this is that the workflow is no longer tied to the local user interface of the system. Web services technology provides machine access to the whole system functionality. All the necessary information to communicate with WebOptim system is given in the "Public API" section.

CHAPTER IV. EXPERIMENTAL RESEARCH - SOLVING THE PROBLEM OF OPERATIONAL PLANNING USING MULTICRITERIA OPTIMIZATION

Purpose of this chapter is to test and validate the developed software systems and their multicriteria optimization algorithms. Systems performance is proven by solving a real-world example of multicriteria optimization problem and comparing results with those obtained by solving the same problem with another independent similar system. The results of the experimental design are described in publication #3.

4.1. Problem description

A camera factory produces two camera models - standard and luxury. Their products are intended for both domestic and international markets.

The production process determines the corresponding relationships between the labor workforce and machine time over a given period. These dependencies are presented in Table 2.

<i>Parameter</i>	<i>Labor force hours</i>	<i>Machine resource hours</i>	<i>Domestic market selling price</i>	<i>International market selling price</i>
<i>Standard model</i>	22	1	1000	800
<i>Luxury model</i>	35	0.5	1710	1130
<i>Available resources</i>	1620	81		
<i>Resource price</i>	30	20		

Table 2. Problem parameters

Goal: Production process optimization, taking into an account that the brand launches a promotional company in order to establish its name in the international market.

The problem is solved with the help of the MKO-2.1 software system and the DALDI method.

4.2. Building the mathematical model

Parameters (variables):

- x_1 : number of standard models produced for the domestic market
- x_2 : number of standard models produced for export
- x_3 : number of luxury models produced for the internal market
- x_4 : number of luxury models produced for export

Objective functions (criteria):

Maximize profit (production cost - cost):

$$\text{Max } Z_1 = 320 \cdot x_1 + 120 \cdot x_2 + 650 \cdot x_3 + 70 \cdot x_4$$

Where the coefficients before each x represent the difference between the selling price and the cost of production. It is obtained by the following formula:

$$x_1: 1000 - (30 \cdot 22 + 20 \cdot 1) = 320$$

$$x_2: 800 - (30 \cdot 22 + 20 \cdot 1) = 120$$

$$x_3: 1710 - (30 \cdot 35 + 20 \cdot 0.5) = 650$$

$$x_4: 1130 - (30 \cdot 35 + 20 \cdot 0.5) = 70$$

Minimize unused hours during which factory staff is idle. Represents minimizing the difference between the available resources in labor hours and the sum of labor hours required for production:

$$\text{Min } Z_2 = 22 \cdot x_1 + 22 \cdot x_2 + 35 \cdot x_3 + 35 \cdot x_4 - 1620$$

Since there is one free coefficient (1620) in this function, but the syntaxes for entering a task in MKO-2.1 do not allow such coefficient, it is necessary to add an additional variable equal to 1: $a = 1$.

Maximize the number of cameras produced for export by both models:

$$\text{Max } Z3 = x2 + x4$$

Subjective functions (constraints):

Labor hours resources constraint:

$$22*x1 + 22*x2 + 35*x3 + 35*x4 \leq 1620$$

Machine hours constraint:

$$x1 + x2 + 2*x3 + 2*x4 \leq 81$$

Requirements about integer nature of the variables:

$$x1, x2, x3, x4: \text{integer numbers, } \geq 1$$

4.3. Problem solving

The problem is solved with the help of MKO-2.1 system. After defining the problem and starting the solving process we get an initial solution with the following data about "ideal" and "nadir" vectors (Table 3):

<i>Objective function</i>	<i>Ideal vector</i>	<i>Nadir vector</i>
<i>Z1 (max)</i>	25535	1160
<i>Z2 (min)</i>	-1506	-4.547
<i>Z3 (max)</i>	70.4545	2

Table 3. Ideal and nadir vector values

The system automatically generates initial non-dominant solution and after selecting the closest integer solution, we obtain the following values of the criteria and variables (Table 4 and Table 5):

Z1	Z2	Z3
8960	-76	67

Table 4. Criteria values

x1	x2	x3	x4
1	66	1	1

Table 5. Variables values

This initial solution is our starting point, and its interpretation is that 66 pieces of standard and 1 piece of luxury model should be manufactured for export; 1 standard and 1 luxury model for domestic market. Economically this is not satisfactory for us as it gives too much priority only to the standard model for export. Therefore, we continue to look for a new Pareto-optimal solution, deciding to freely improve the profit criterion (Z1) at the expense of the criterion that maximizes exports (Z3). Considering the ideal point (70.4545) and the point above (2) of the criterion that will deteriorate, we decide to allow it to deteriorate to a maximum of level 30. The criterion Z2 - minimizing factory staff idling hours, at this step we set it to change freely.

We start solving the problem with the new preferences and get the following new results (Table 6 and Table 7):

Z1	Z2	Z3
19790	-63	30

Table 6. Criteria values

x1	x2	x3	x4
2	29	24	1

Table 7. Variables values

After two more steps, we finally decide that, in the context of the task, the solution obtained on step 2 seems most suitable for our final solution:

- 2 pieces of the standard model for domestic market

- 29 pieces of the standard model for export
- 24 pieces of the luxury model for domestic market
- 1 piece of luxury model for export

With this distribution of production, the solution to our problem guarantees mathematically a balanced strategy in terms of profit, minimizing factory staff idling hours and fulfillment of the condition for increased export production.

4.4. Comparative analysis of the results

For the purpose of the comparative analysis, the problem was solved with the popular web-based system of the University of Jyväskylä, Finland - WWW NIMBUS (wwwnimbus.it.jyu.fi).

The results of solving the problem with the two software systems are summarized in Table 16.

<i>Step</i>	<i>System</i>	<i>X1</i>	<i>X2</i>	<i>X3</i>	<i>X4</i>	<i>Z1</i>	<i>Z2</i>	<i>Z3</i>	<i>Chosen final solution</i>
1	MKO-2.1	1	66	1	1	8960	-76	67	
1	Nimbus	1	27	11	1	10780	-584	28	*
2	MKO-2.1	2	29	24	1	19790	-63	30	*
2	Nimbus	2	1	38	1	25530	-189	2	
3	MKO-2.1	2	19	26	1	19890	-213	20	
3	Nimbus	51	19	1	1	19320	-10	2	
4	MKO-2.1	1	23	23	1	18100	-252	24	
4	Nimbus	1	1	38	1	25210	-211	2	

Table 16. Comparison of results obtained by MKO-2.1 and NIMBUS

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SUMMARY OF RESULTS ACHIEVED

The results described in this thesis can be summarized in the following scientific and applied contributions:

1. Multiple methods for solving multicriteria optimization problems have been systematized and some of them have been selected for algorithmic and software implementation.
2. A syntax and corresponding software parser have been developed for defining linear and linear-integer problems for multicriteria optimization.
3. Control and calculation modules of the MKO-21 system have been designed and developed for operating under WINDOWS operating system.
4. The common architecture, functional abilities and user interface of the web-based system WebOptim have been designed and implemented.
5. Communication modules for electronic information exchange with third party systems have been developed for the purposes of the web-based system WebOptim.
6. Experimental studies have been conducted in order to prove the operability of the developed systems.

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