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MIXED-INTEGER MODEL FOR PLACEMENT OF OBJECTS AVOIDING FORBIDDEN ZONES

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Abstract

The design, planning and building of effective layouts are real-world engineering problems arising in many different applications. The aim of this paper is to provide a model for objects placement considering the existence of forbidden zones. For this purpose, an optimization mixed-integer model is proposed for determination of the most appropriate layout design corresponding to the restrictions of given input data. The described mathematical programming model is numerically tested for designing of offshore wind farm layout taking into account the presence of forbidden zones. The results show the practical applicability of the proposed model by supplying a strong optimal solution for considerably less computing time.

Key words: mixed-integer optimization model, layout design model, forbidden zones

1. Introduction. Many real-world engineering problems are related to layout design of some objects within given area where there exist some forbidden for objects placement zones. Locations of stations of mobile communication, placement of modules on reconfigurable resources under the constraints of field-programmable gate array (FPGA), wireless sensor networks (for example of forest fires detection), architectural floor plan layout design, placement of wind turbines, etc., are just some examples that could be mentioned. Due the complexity of those types of problems, the researchers propose different approaches, models and algorithms to tackle with fuzziness in relations and restrictions [1–3] or apply different approaches of soft computing [4]. For layout design in case of FPGA Ouni and Mtibaa [5] use fast modules fitter framework algorithm based on Cartesian-coordinate system. Other authors rely on polynomial algorithm for convex forbidden region [6]. The gradient-based algorithms and evolutionary algorithms contribute to make discrete decisions by global search [7]. In case of polygonal region with forbidden zones, algorithm based on Euclidean distances of a finite set of points to which an optimal solution must belong is determined [8].

The reasonable decision making for any technical system design relies on selection of important parameters that influence on the final choice and different preliminary requirements need to be considered [9, 10]. In case of layout design problem, the restrictions are related with given dimensions of area, integer number of objects for placement and forbidden for placement zones. In many real problems, there exist also restrictions for the distance between objects that is an important specific of layout design problems. For example, construction site layout planning requires determination of reasonable approximations to the path followed by workers, machinery, etc., between existing facilities [11]. Typical example for applying of restrictions about the distance between objects is wind farm layout design where wake effect between turbines should be considered [12, 13]. The wind farm design is essential in contemporary context of renewable energy usage. The existence of forbidden zones in wind site area is an important problem of wind farms optimal layout design. Such zones could be a result of physical obstacles due to archaeological ruins, natural areas, visual impact, etc. Different approaches to such type of layout problem include using of evolutive algorithm [14]; particle swarm optimization algorithm with multiple adaptive methods [15]; Monte Carlo simulation [16], etc. The heuristic approaches do not guarantee finding of an optimal solution. The evolutive algorithms rely on some fitness function and if it is defined imprecisely, the algorithm may be unable to find a solution. The idea of Monte Carlo simulation relies on repeated random sampling to obtain numerical results. The metaheuristic of particle swarm optimization makes few or no assumptions about the optimized problem and can search very large spaces of candidate solutions without guaranteeing an optimal solution is ever found.

In the paper, a generalized mathematical programming model leading to formulation of mixed-integer optimization task is proposed for objects layout design, considering the existence of forbidden zones. The use of mixed-integer optimization as a tool for modelling has the advantage to provide strong optimal solution. This is in contrast to other published evolutionary algorithms for similar problems which do not provide evidence that obtained solution is optimal.

2. A mixed-integer programming model for objects layout design considering forbidden zones. The basic problem discussed in the paper is to determine the optimal layout placement of certain objects within area with predefined dimensions and with existence of forbidden for placement zones with different shapes. Two different shapes of forbidden zones are considered – trapeze shape (A) and circular shape (B) as shown in Fig. 1 [15].

The zone A can be described by two equations of a straight line: $y_1 = ax + C_1$ and $y_2 = ax + C_2$, where a, C_1 and C_2 are constants for each line description. The circular shape of zone B is described by the equation $(x - x_c)^2 + (y - y_c)^2 = R^2$,

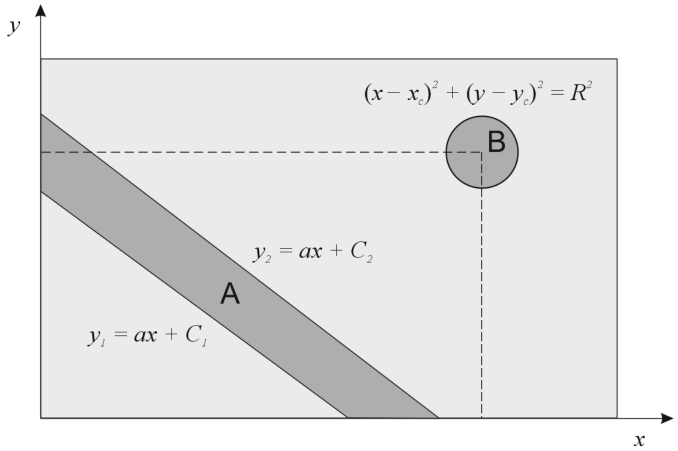

Fig. 1. Area with two forbidden zones A and B

where R is radius of the circle and x_c, y_c are centre coordinates.

An important feature of the described modelling approach is using of relative coordinates i and j for each object O_{ij} defined by means of required inter-objects distances d_x and d_y as $i = x/d_x$ and $j = y/d_y$.

The proposed generalized mixed-integer programming model for placement of objects O_{ij} avoiding forbidden zones is:

$$\max F(N^O) \tag{1}$$

subject to

$$N^O = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} O_{ij} \tag{2}$$

$$N_x = (L_x/d_x) - 1, \quad \text{integer} \tag{3}$$

$$N_y = (L_y/d_y) - 1, \quad \text{integer} \tag{4}$$

$$\forall i \in \{1, \ldots, N_x\} : J_1 = ai + C_1; \tag{5}$$

$$\forall i \in \{1, \ldots, N_x\} : J_2 = ai + C_2; \tag{6}$$

$$\forall i \in \{1, \ldots, N_x\} : Z_1 = \frac{2j_c - \sqrt{(2j_c)^2 - 4(R^2 - i^2 - i2i_c + i_c^2)}}{2} \tag{7}$$

$$\forall i \in \{1, \ldots, N_x\} : Z_2 = \frac{2j_c + \sqrt{(2j_c)^2 - 4(R^2 - i^2 - i2i_c + i_c^2)}}{2} \tag{8}$$

$$\forall i \in \{1, \ldots, N_x\} : O_{ij} = \begin{cases} 0, \forall i \in \{1, \ldots, N_y\} : J_1 \le j \le J_2 \text{ or } Z_1 \le j \le Z_2 \\ 1, \text{otherwise} \end{cases}. \tag{9}$$

In most cases, objective function (1) aims to maximize some function depending on the number of objects N^O. The inter-objects distances (d_x and d_y) define number of nodes and their relative coordinates excluding nodes on the boundaries by relations (3) and (4). The site area dimensions, where objects are to be placed are denoted by L_x and L_y, respectively. The forbidden zones are described by relations of a straight line (5) and (6) for zone A, and (7) and (8) for zone B. These equations also do not allow placement of objects on the area boundaries. The relation (9) expresses the objects O_{ij} that are considered as binary integer variables equal to 1 if an object is placed at optimal location outside forbidden zones, or 0 if it is placed in the areas of forbidden zones (including their boundaries). The optimization model (1)–(9) allows formulation of proper optimization tasks for determination of optimal number and placement of objects while taking account of forbidden zones.

3. Numerical testing. The applicability of the proposed model for optimal layout design avoiding forbidden zones is numerically tested for design of offshore wind farm. This problem is adopted from HOU et al. [15] where the number of certain type of wind turbine with rated power $P = 10$ MW and rotor diameter $D_R = 0.1783$ km has to be placed within rectangular area $L_x = 10.5$ km and $L_y = 13$ km. The prevailing wind direction (east to west) is given together with shapes and dimensions of forbidden zones. The coefficients for equations describing zone (A) are: $a = -1$; $C_1 = 6.5$; $C_2 = 7.8$; and for zone (B): $x_c = 8$ km, $y_c = 6$ km, $R = 0.7$ km. The examples for forbidden zones that are to be avoided for placement of wind turbines could be: (A) – installed Gas Pipe (or Marine Traffic Line) and (B) – Oil Well availability (Fig. 2).

The generalized model (1)–(9) is used to formulate an optimization task for solution of the problem of offshore wind turbines layout design avoiding forbidden zones. The used utility function expresses the ratio of annual energy per costs as function of number of turbines [16] and their rated power [13]. The mixed-integer optimization task for determination of number and placement of given type of turbines and considering forbidden zones is formulated as:

$$\max \left(\frac{0.3 * 8760 * N^O * 10}{N^0 \left(\frac{2}{3} + \frac{1}{3} \exp(-0.00174 (N^O)^2) \right)} \right) \tag{10}$$

subject to

$$N^O = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} O_{ij} \tag{11}$$

$$N_x = (10.5/d_x) - 1, \quad \text{integer} \tag{12}$$

$$N_y = (13/d_y) - 1, \quad \text{integer} \tag{13}$$

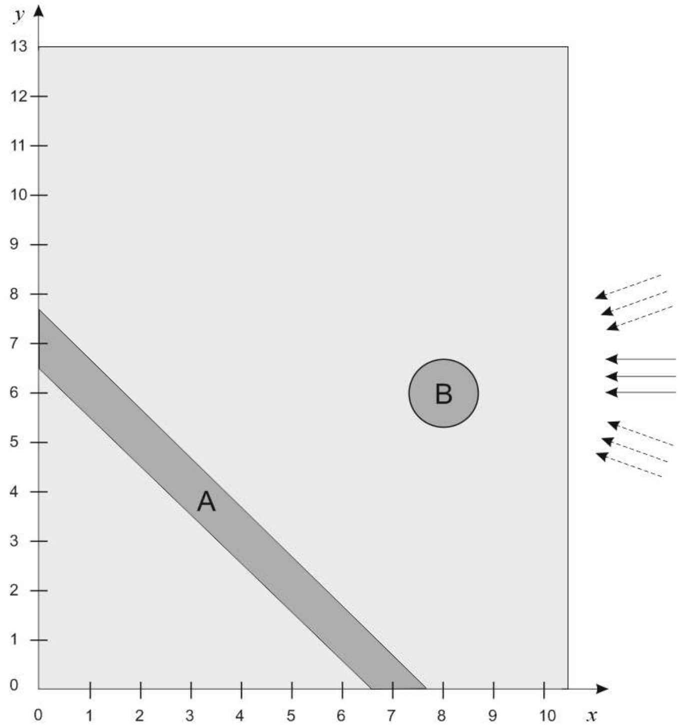

Fig. 2. Offshore wind farm area with forbidden zones A and B and prevailing wind direction

$$d_x = 0.1783k_x \tag{14}$$

$$d_y = 0.1783k_y \tag{15}$$

$$9 \leq kx \leq 11 \tag{16}$$

$$1.5 \leq ky \leq 3 \tag{17}$$

$$\forall i \in \{1, 2, \ldots, N_x\} : J_1 = -i + (6.5/d_x) \tag{18}$$

$$\forall i \in \{1, 2, \ldots, N_x\} : J_2 = -i + (7.8/d_y) \tag{19}$$

$$\forall i \in \{1, 2, \ldots, N_x\} : Z_1 = \frac{2j_c - \sqrt{(2j_c)^2 - 4(R^2 - i^2 - i2i_c + i_c^2)}}{2} \tag{20}$$

$$\forall i \in \{1, \ldots, N_x\} : Z_2 = \frac{2j_c + \sqrt{(2j_c)^2 - 4(R^2 - i^2 - i2i_c + i_c^2)}}{2} \tag{21}$$

$$i_c = 8/(0.1783k_x) \tag{22}$$

$$j_c = 6/(0.1783k_y) \tag{23}$$

$$(24)\quad \forall i \in \{1,2,\ldots,N_x\}: O_{ij} = \begin{cases} 0, \forall i \in \{1,2,\ldots,N_y\}: J_1 \le j \le J_2 \text{ or } Z_1 \le j \le Z_2 \\ 1, \text{otherwise} \end{cases}.$$

The wind farm layout design is characterized by the necessity to take into account the negative influence of wake effect by proper inter-turbines distances. For this purpose separation distances (defined as number of turbine's rotor diameter) are used to define inter-turbines distances according to the directions of wind as recommended in [13, 16, 17]. The used separation coefficients k_x and k_y given by the limits (16) and (17) correspond to the wind direction used in [15]. The distances between turbines are defined by means of separation coefficients as $d_x = k_x D_R$ and $d_y = k_y D_R$.

4. Results analysis and discussion. The optimization task was solved by means of LNGO software on PC with Intel Core i3 @ 2.93 GHz under MS Windows. The branch and bound algorithm is used and solution is found after 127 iterations for about a few seconds. The heuristic algorithms take more solution time without guarantee to get to optimal solution. For example, 1180 iterations are needed to get stable (not optimal) solution in [15]. The results of optimization task define 160 turbines of given type to be installed outside of forbidden zones using grid layout with separation distances $d_x = 1.75$ km and $d_y = 0.361$ km. The energy output per unit of costs for particular offshore wind farm example is equal to 32850.00 MW/h. The result of optimization task solution is graphically illustrated in Fig. 3.

The obtained values for separation distances between turbines satisfy the requirement to overcome the wake effect and guarantee the optimal placement of turbines even in the presence of small displacement of predominant wind direction shown in Fig. 3 as (b) and (c). The advantage of the proposed mixed-integer optimization model in contrast to the heuristic optimization method proposed in [15] is the optimal and compact utilization of given area by usage of grid layout for turbines placement. The solution of optimization task defines considerably larger number of turbines, which reflect in increasing of installed wind farm capacity and, respectively, in extracted output energy. Other advantage is computational time of order of seconds that is far better than heuristic approaches.

The integer variables for number of turbines combined with narrow limits for separations coefficients could result in infeasible solution for some fixed dimensions of wind farm area. This typical drawback for mixed-integer optimization can be tackled by widening limits for separation coefficients.

The essence in the described modelling approach is using of grid mesh relative coordinates of optimal objects locations instead of metric units coordinates defined by dimensions of the site. This reduces the number of locations that have to be checked if they belong to forbidden zones.

The described model can be used with different objective functions and different shapes of forbidden zones. These zones could be taken into account by decomposing them to quadrangles and circles (with some approximations) and

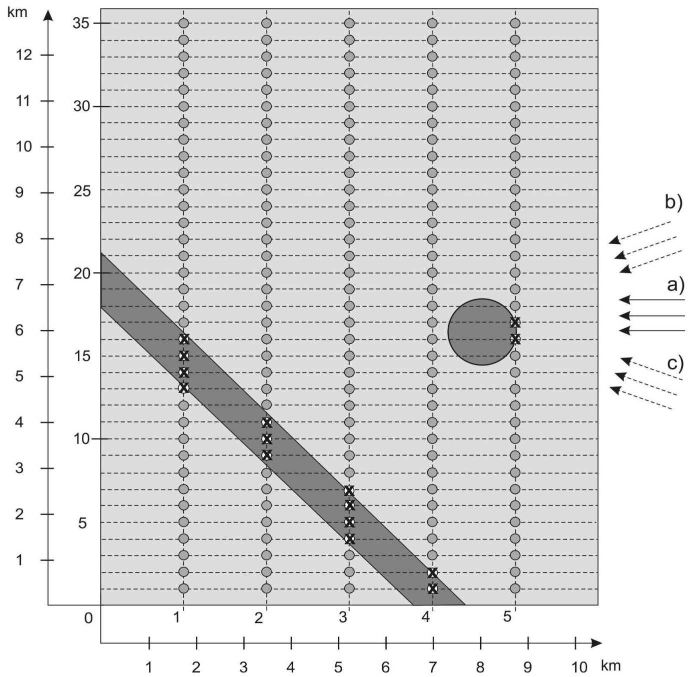

Fig. 3. Offshore wind farm optimal layout avoiding forbidden zones

using proper equations to describe them. The applied approximations for complex shapes of forbidden zones are not critical for the most practical applications.

5. Conclusion. In this paper, a mixed-integer optimization model is proposed for determination of optimal layout for objects while avoiding given forbidden zones. In contrast to the described in the introduction metaheuristic techniques, the mathematical modelling approach in the paper allows determining of strong optimal placement of a larger number of objects considering forbidden zones. It could be adjusted to take into account the required distances between objects in a way to better utilize of the available site area to increase the layout efficiency.

The applicability of the proposed approach is numerically tested for offshore wind farm layout design avoiding given forbidden zones.

Future developments will be related to investigation of applicability of the proposed model for other applications with complex shapes of forbidden zones.

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