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A COMBINATORIAL OPTIMIZATION RANKING ALGORITHM FOR REASONABLE DECISION MAKING

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Abstract

The focus of the paper is on a quantitative decision making strategy that could underline the decision making process. A decision making model based on multicriteria optimization and devices ranking algorithm are developed. This approach could be used to support the reasonable choice making in many real life problems when a variety of alternatives and conflicting user requirements exist. The algorithm is numerically illustrated and its applicability is demonstrated on the example of real mobile phones data for three different scenarios of user preferences.

Key words: decision making, multicriteria combinatorial optimization, ranking algorithm

1. Introduction. The successful competition market's factor is the design and component innovation [1]. This results in products with so many options that the choice of particular device taking into account all of these options is far from trivial. An easy to understand example used in the paper as case study is the choice of the mobile (cell) phone – one of the most popular consumer devices today. The mobile phone choice example can be considered as a complex choice making problem that could be used to model the reasonable decision making process.

The first key challenge of the modelling is to identify the object's attributes that are important for the choice. One possible approach is to evaluate them in respect to the users' preferences by using specific questionnairies [2]. Selection of device with optimal features can be viewed as an optimization problem to search for optimal features combination [3]. The selection process is based on evaluation of the alternatives with respect to the set of relevant criteria [4, 5]. The dominant perspective on choosing behaviour suggests that users tend to rely on objective criteria when making product choice decisions [6]. In theory, the decision maker (DM) somehow estimates the relative value of each alternative in the choice set and then makes the optimal choice in a systematic manner [7]. A special but widely applied class of discrete choice models is the attraction choice model [8]. With respect to choice modelling a large number of empirical studies has been successfully carried out to formally capture user choice behaviour as a function of any relevant variables in discrete choice models [9].

In contrast to popular choice models, current paper proposes multicriteria combinatorial optimization approach. It is realized in a device ranking algorithm to give the user different strategies in choice decision making.

2. Combinatorial optimization model. The simulation of choice behaviour is based on multicriteria optimization. To formulate multicriteria optimization tasks, the device choice process is modelled as a combinatorial choice from a discrete set of N devices, where decision variables x_i, $(i = 1, \ldots, N)$ are associated to each of the devices. The choice of k-th device $(1 \leq k \leq N)$ means $x_k = 1$ while the rest of variables x_i for $i \neq k$ are equal to zero, i.e. the variables x_i are defined as binary integer variables. If some of the device attributes are taken as maximization criteria f^j and other as minimization criteria φ^k, then the corresponding multicriteria task formulation is

$$\left\{ \begin{array}{l} \max\left(f^1, f^2, \ldots, f^J\right) \\ \min\left(\varphi^1, \varphi^2, \ldots, \varphi^K\right) \end{array} \right. \tag{1}$$

subject to

$$f^j = \sum_{i=1}^{N} f_i^j x_i, \ \forall j = 1, 2, \ldots, J, \tag{2}$$

$$\varphi^k = \sum_{i=1}^{N} \varphi_i^k x_i, \ \forall k = 1, 2, \ldots, K, \tag{3}$$

$$\sum_{i=1}^{N} x_i = 1, \ \forall x_i \in \{0, 1\}, \tag{4}$$

where j and k are indexes of maximized and minimized device features.

Any choice problem, and in particular the multicriteria one, is not formed without the DM participation. If DM is familiar with more or less different multicriteria methods, his/her choice for a particular method may depend on the specific problem [10, 11]. The main goal of the paper is to define ranking lists of all devices from a given set accordingly different user preferences in the process of decision making and assisting the DM to make the more appropriate decision.

3. Device ranking algorithm considering DM preferences. Generally, optimal device selection is reduced to search for a single device with optimal features combination [12]. When a large number of features has to be considered, the decision space becomes very complex [3]. The basic idea of the proposed ranking algorithm is to solve sequentially n multicriteria optimization tasks. The first task solution gives the best Pareto-optimal device that is included on the top of the device ranking list. Then that device is excluded from the optimization task formulation, i.e. its corresponding restriction matrix column is removed and the modified task is solved again to define the second Pareto-optimal device in the list. The procedure is repeated as many times as is the number of devices. Apparently, on the last cycle step when a single device is left its choice is evident but the task solution will provide the information about the objective function value. The objective function values are used as estimation of how far each device from the best one is. The optimization based ranking algorithm is graphically illustrated in Fig. 1.

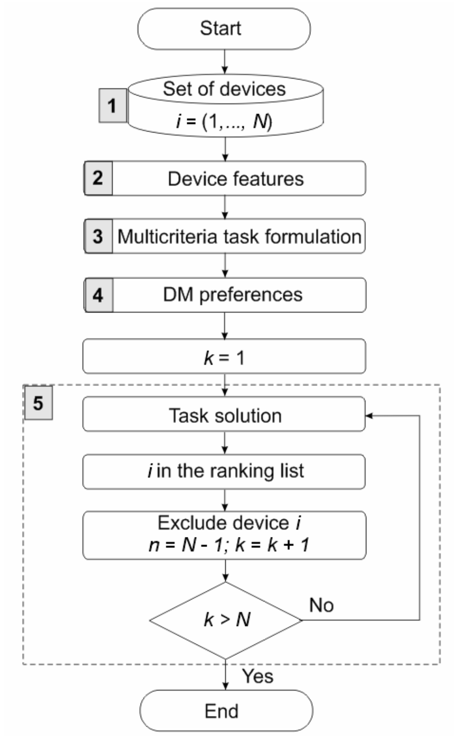

Fig. 1. Device ranking algorithm

On the 1st step a set of N different devices to choose from is defined. Each device is described by a set of features. On the 2nd step, the DM point of view about the maximized and minimized features is expressed by defining the criteria sets f_j^i, φ_k^i.

These sets are used to formulate a proper multicriteria task on step 3. On the 4th step the DM decides on the relative importance between optimization criteria (according to the chosen multicriteria solving method). At step 5 the device ranking list is created by multicriteria task solving following the procedures:

a) Set a counter $k = 1$.

b) Solve the multicriteria task and include the defined Pareto optimal device number (index i for $x_i = 1$) on the top of the ranking list.

c) Remove the corresponding i-th matrix column, thus excluding that device from the decision ($n = N - 1$) and increment the counter $k = k + 1$;

d) Check if ($k > N$) and if NO – go to b) or if YES – go to END.

The ranking device list is available on exit of step 5.

If the DM is not satisfied with ranking in the list the algorithm can be adjusted by modifying the input data. Three possible scenarios for adjusting the algorithm are: 1) add or remove some device features, and/or, 2) change the preferences, and/or, 3) define another set of devices. Because of the fact that multicriteria optimization is a core step of the algorithm, the choosing of different method for multicriteria task solving can be considered too. Thus, the proposed algorithm can be used as a based on Pareto optimization simulation tool for reasonable decision making.

4. Illustrative numerical examples. To demonstrate the applicability of the proposed algorithm, the numerical illustration is done based on real data of the features of 15 mobile phones as shown in Table 1.

Using the data from Table 1, the multicriteria optimization task (1)–(4) can be formulated as

$$\begin{cases} \max\left(f^1, f^2, f^3, f^4, f^5, f^6\right) \\ \min\left(\varphi^1, \varphi^2\right) \end{cases} \tag{5}$$

subject to

$$f^1 = \sum_{i=1}^{15} f_i^1 x_i; \tag{6}$$

$$f^2 = \sum_{i=1}^{15} f_i^2 x_i; \tag{7}$$

$$f^3 = \sum_{i=1}^{15} f_i^3 x_i; \tag{8}$$

T a b l e 1

Real data of mobile phone features

No	Cell phonemodel	Camera resolution, MP	Integrated memory, GB	Card slot capacity, GB	Stand-by time, h	GPS yes/no	Talk time, h	Weight, gr	Price, BGL
		(f^1)	(f^2)	(f^3)	(f^4)	(f^5)	(f^6)	(ϕ^1)	(ϕ^2)
1	BlackBerry 9000 Bold	2.0	1.0	32	310	1 (yes)	5	133	949
2	Samsung Omnia HD	8.0	16.0	32	430	1 (yes)	10	148	919
3	Sony-Ericsson U1i Satio	12.0	0.128	32	350	1 (yes)	10	126	879
4	Apple iPhone 3G	2.0	8.0	0	250	1 (yes)	5	133	825
5	Nokia N-97 Mini	5.0	8.0	16	310	1 (yes)	6	138	785
6	HTC Google G1	3.15	0.192	16	400	1 (yes)	5	158	659
7	Sony-Ericsson W-995	8.1	0.118	8	370	1 (yes)	9	113	605
8	LG GD900 Crystal	8.0	1.5	32	300	0 (no)	4	127	599
9	Nokia E-75	3.15	0.050	16	250	1 (yes)	4	139	539
10	Samsung S8300	8.0	0.080	16	350	1 (yes)	4	105	485
11	BlackBerry Curve 8520	2.0	0.256	32	408	0 (no)	4.5	106	480
12	Nokia 5530	3.15	0.070	16	336	0 (no)	5	107	459
13	Sony-Ericsson C-903	5.0	0.105	8	350	1 (yes)	4	96	435
14	LG KM-900 Arena	5.0	8.0	16	300	1 (yes)	3.3	105	435
15	Samsung M-7600	3.15	0.050	16	350	0 (no)	5	99.7	419

Note: Data are collected by Internet at the time of manuscript preparing

$$f^4 = \sum_{i=1}^{15} f_i^4 x_i; \tag{9}$$

$$f^5 = \sum_{i=1}^{15} f_i^5 x_i; \tag{10}$$

$$f^6 = \sum_{i=1}^{15} f_i^6 x_i; \tag{11}$$

$$\varphi^1 = \sum_{i=1}^{15} \varphi_i^1 x_i; \tag{12}$$

$$\varphi^2 = \sum_{i=1}^{15} \varphi_i^2 x_i; \tag{13}$$

$$\sum_{i=1}^{15} x_i = 1, \ \forall x_i \in \{0, 1\}. \tag{14}$$

A typical approach to solving a multicriteria problem is to transform it to a problem with a single scalar evaluation criterion [10, 13]. After proper normalization the widely used weighted sum method can be performed to solve the task (5) – (14). The normalization is done by the normalization scheme [14]

$$f^* = \frac{f(x) - f_{\min}}{f_{\max} - f_{\min}} \text{ – for maximizing objective;} \tag{15}$$

$$\varphi^* = \frac{\varphi_{\max} - \varphi(x)}{\varphi_{\max} - \varphi_{\min}} \text{ – for minimizing objective.} \tag{16}$$

The weighed sum method transforms multiple objectives into an aggregated objective function by using normalized objectives and weighting coefficient for each objective function. The aggregated objective function is used to formulate scalar optimization task

$$\begin{aligned} &\max(w_1(f^1)^* + w_2(f^2)^* + w_3(f^3)^* + w_4(f^4)^* \\ &\quad + w_5(f^5)^* + w_6(f^6)^* + w_7(\varphi^1)^* + w_8(\varphi^2)^*) \end{aligned} \tag{17}$$

subject to (6)–(14) and

$$\sum_{i=1}^{8} w_i = 1, \tag{18}$$

where $w_1, \ldots, w_8$ are non-negative weight coefficients and $(f^1)^*$, $(f^2)^*$, $(f^3)^*$, $(f^4)^*$, $(f^5)^*$, $(f^6)^*$, $(\varphi^1)^*$, $(\varphi^2)^*$ are the normalized criteria.

The weight coefficients w_i (Table 2) reflect a priory preference information of the DM point of view about the relative importance of the device features considered as optimization criteria.

Set (a) of the weight coefficients corresponds to equivalent importance of the device features. Set (b) of the weight coefficients illustrates the dominant importance of devices price and weight toward other devices features. Set (c) reflects the DM strong preferences about the devices price, camera resolution and standby duration time.

As a result of algorithm execution three device ranking lists for three sets of weight coefficients are defined as shown on Table 3.

T a b l e 2

Criteria's weight coefficients

Criteria	$(f^1)^*$	$(f^2)^*$	$(f^3)^*$	$(f^4)^*$	$(f^5)^*$	$(f^6)^*$	$(\varphi^1)^*$	$(\varphi^2)^*$
w_i	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
set (a)	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
set (b)	0.1	0.1	0.1	0.1	0.1	0.1	0.15	0.25
set (c)	0.15	0.1	0.1	0.15	0.05	0.05	0.05	0.35

5. Results analysis and discussion. There exist different multicriteria optimization software systems to be used for formulated tasks solution [15, 16]. Most of them require knowledge on multicriteria optimization specifics. For numeric illustration weighted sum method and LINGO ver. 12 solver are used. The solution times for the described example take few seconds on PC with 2.93 GHz Intel i3 CPU and 4 GB RAM. The advantage of the proposed approach is that it requires only attributes weighting. Another advantage is reducing of the task dimension on every cycle on step 5 of algorithm execution. Further numerical testing with different tasks dimensions is needed to get the functional dependence between solution times and tasks dimensions.

As it is seen from Table 3 the different DM preferences lead to different ranking lists of devices. Two main application scenarios of the proposed algorithm for reasonable decision making would be suggested: 1) the DM makes choice of a particular device using the defined ranking list of devices or 2) the DM evaluates a preliminary chosen device accordingly to its position in the ranking list. In the first case, the ranking list of devices reflecting the DM preferences will supply knowledge not only for the first best device but also for all devices situated in relation to the best one. Using that information the DM can choose a different then the first in the list device accordingly to some other subjective preferences. For example, if the DM is a "fan" of particular brand of mobile phones, the ranking lists can help to choose such device that is close enough to the best device. For example, Sony-Ericsson "fan" could choose the second device (# 3 from Table 1) for set (a) of weight coefficients or third device of the lists (# 13 from Table 1) for the sets (b) and (c). The second type of application of the proposed algorithm is the possibility to evaluate some preliminary chosen device. In most cases, the users can make some intuitive choice of some device and would like to know how far their choice from the best one for the given preferences and criteria is. Using information from the ranking lists the DM could change the preliminary intuitive choice with other device that is located closer to the best one. By changing the DM preferences it is possible to get different ranking lists of devices. It is interesting to mention that different preferences expressed by different weighting coefficients sets change the devices ranking but does not change significantly the

Table 3

Device ranking lists as results of numerical testing

Rank	Set (a) Objective function value	Set (a) Device from Table 1 #	Set (a) Device from Table 1 name	Set (b) Objective function value	Set (b) Device from Table 1 #	Set (b) Device from Table 1 name	Set (c) Objective function value	Set (c) Device from Table 1 #	Set (c) Device from Table 1 name
1	0.7272	2	Samsung Omnia	0.6283	14	LG KM-900 Arena	0.6279	10	Samsung S8300
2	0.6511	3	Sony-Ericsson U1i Satio	0.6233	10	Samsung S8300	0.6187	14	LG KM-900 Arena
3	0.5946	7	Sony-Ericsson W-995	0.6138	13	Sony-Ericsson C-903	0.5983	13	Sony-Ericsson C-903
4	0.5615	10	Samsung S8300	0.6093	7	Sony-Ericsson W-995	0.5936	11	BlackBerry Curve 8520
5	0.5501	14	LG KM-900 Arena	0.5983	2	Samsung Omnia	0.5729	7	Sony-Ericsson W-995
6	0.5229	13	Sony-Ericsson C-903	0.5665	3	Sony-Ericsson U1i Satio	0.5679	2	Samsung Omnia
7	0.4742	11	BlackBerry Curve 8520	0.5540	11	BlackBerry Curve 8520	0.5603	15	Samsung M-7600
8	0.4583	5	Nokia N-97 Mini	0.5335	15	Samsung M-7600	0.5164	12	Nokia 5530
9	0.4206	15	Samsung M-7600	0.4893	12	Nokia 5530	0.5059	3	Sony-Ericsson U1i Satio
10	0.4073	6	HTC Google G1	0.4474	8	LG GD900 Crystal	0.5021	8	LG GD900 Crystal
11	0.4042	8	LG GD900 Crystal	0.4292	5	Nokia N-97 Mini	0.4473	6	HTC Google G1
12	0.3869	12	Nokia 5530	0.4113	9	Nokia E-75	0.4086	9	Nokia E-75
13	0.3812	1	BlackBerry 9000 Bold	0.4079	6	HTC Google G1	0.3894	5	Nokia N-97 Mini
14	0.3499	9	Nokia E-75	0.3251	1	BlackBerry 9000 Bold	0.2388	1	BlackBerry 9000 Bold
15	0.3012	4	Apple iPhone 3G	0.2962	4	Apple iPhone 3G	0.2176	4	Apple iPhone 3G

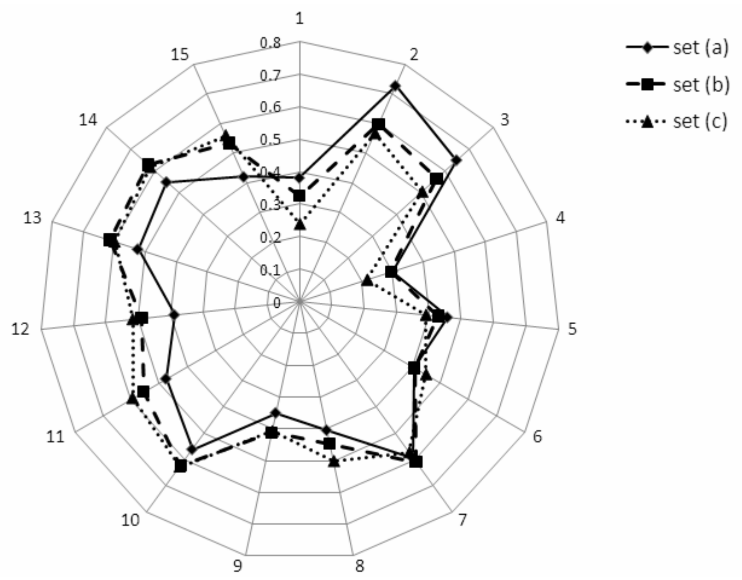

Fig. 2. Graphical representation of the objective function values

corresponding graphical representation of the objective function values (Fig. 2). Further investigation is needed to explain that phenomenon.

Experimental study indicates that the proposed algorithm is more informatively efficient in respect of reasonable choice problems than single solution methods. User perception or preference is the critical input variable, in the form of multivariate product information and the proposed approach is consistent with this.

6. Conclusion. The paper describes a combinatorial optimization approach to modelling choice decision making behaviour when numerous and conflicting evaluations of a set of devices should be done. A ranking algorithm based on repetitive multicriteria optimization problem solving using a priori aggregation of preference information is developed. The final result of its execution is a ranking list where the devices are ordered accordingly to the given user objectives and preferences. The defined ranking list of devices can be used to assists the user to make a reasonable choice. The proposed approach is numerically illustrated on the example of real mobile phones data for three different scenarios of user preferences. The results of numerical testing show its applicability for: 1) assisting the user to make a reasonable choice of a particular device using the ranking list of devices and 2) evaluating a preliminary chosen device how far it is from the "best" device. The proposed combinatorial optimization ranking approach can be used for choice decision making of different kind of objects, characterized with many options – from consumer goods to complex systems as manufacturing tools, machines, robots, cars, aircrafts, etc.

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