

TECHNICAL SYSTEMS DESIGN BY COMBINATORIAL
OPTIMIZATION CHOICE OF ELEMENTS ON THE EXAMPLE
OF NIGHT VISION DEVICES DESIGN

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Abstract

The design process of the real technical systems sometimes needs a proper choice of their elements and/or subsystems. As a result of world globalisation and technologies development sets of standard (but with different parameters, quality and price) technical elements are available to choose from. The main idea developed in the current paper is to use sets of ready to use elements and modules by making some flexible intelligent choice reflecting different preliminary requirements to the designed system. The optimization combinatorial choice of the technical system elements decreases costs for the “trials and errors” choosing and for the multiple prototypes building and testing. The proposed approach is illustrated on the night vision devices design and some of its experimental numerical results are shown.

Key words: technical system design, elements choice, combinatorial optimization, optimal design approach, night vision devices design

1. Introduction. Often the design process of the real technical systems relies on the proper choice of their elements and/or subsystems. The designed technical systems usually have to satisfy many preliminary (and sometimes conflicting) requirements for the systems operational characteristics. The traditional approach to the design process is to make some intuitive choice of the needed components based on the experience, then to build a prototype and to test it against design goals. If the design goals are not satisfied a new choice is done, new prototype is built and tested. This “trials and errors” method continues until the preliminary design requirements are met. Those kinds of design processes based on the proper element choosing can be formalized as some combinatorial problems and some proper mathematical optimization methods can be used to reduce “trials and errors” time and costs in the technical systems design. Mathematical optimization will not only reduce the design errors but can also be used for creating CAD systems eliminating to some degree need of human-expert design solutions. Depending on functional description of the optimization problem, different optimization techniques can be used – linear programming, nonlinear programming,

discrete optimization, etc. [1-4]. When the design goal is formalized, the first decision to make is a choice between the single objective and multiobjective optimization. The multiobjective optimization seems to be the natural choice [5] but there are many practical problems, where the single objective optimization gives satisfactory results with less computational efforts. The numerical experiments and comparing of the results obtained by single objective or multiobjective optimization will answer the question what optimization method is best suited for each particular technical system design.

Various mathematical models have been proposed to optimize the technical systems. Most of them deal with elements optimization or with subsystems optimization based on their physical, technical, performance, etc. characteristics [6-9]. For some technical systems there exist sets of elements or modules which have been already optimised and have different performance characteristics reflecting in their price, quality and availability. The question is which of them to use to satisfy the given preliminary requirements of the designed system as a whole. The main idea developed in the current paper is to make some flexible intelligent choice of the needed elements or modules and to get as close as possible the desired characteristics of the designed system. The current paper proposes a mathematical combinatorial optimization approach for technical systems design by optimal choice of their elements and is illustrated on the example of night vision device design. The choice is discrete and mostly integer which requires using of the integer or mixed-integer combinatorial optimization tools to model and solve such decision-making problems [10]. If functional relations between chosen elements are known some designed system performance estimations could be done in advance prior to its prototype building and testing [11].

2. Technical systems design by combinatorial optimization choice of elements. For the goals of the proposed design approach as an optimal combinatorial choice of the technical system elements a generalized optimization model could be defined as

$$(1) \quad \max F(P) = (f_1(P), f_2(P), \dots, f_q(P))$$

subject to

$$(2) \quad P = \sum_{j_i=1}^{J_i} P_{j_i, k_i}^i x_{j_i}^i,$$

$$(3) \quad g(P) = (g_1(P), g_2(P), \dots, g_m(P)),$$

$$(4) \quad \sum_{j_i} x_{j_i}^i = 1, \quad x \in [0, 1]$$

$$(5) \quad P_{j_i, k_i}^{L_i} \leq P_{j_i, k_i}^i \leq P_{j_i, k_i}^{U_i}, \quad i = 1, \dots, n.$$

In this formulation $f_1(P), f_2(P), \dots, f_q(P)$ are the q objective functions (performance criteria) of the design variables vector (optimization parameters) $P = \{P_{j_i, k_i}^i \mid i = 1, \dots, n, j_i \in \{J_i\}, k_i \in \{K_i\}\} \in R^n$ of all elements used in design process. R^n is the parameter's space of n elements to choose from, j_i are the types number of i -th element, k_i are the parameters of the i -th element of type j_i and P_{j_i, k_i}^i is k_i -th parameter of i -th element of type j_i . $P' = \{P_{s,r}^t \mid t \in \{i\}, r \in \{k_i\}, s \in \{j_i\}\}$ is the solution vector of the chosen elements parameters as a result of optimal combinatorial choice. The optimal choice is done by using restrictions (2) based on some binary integer variables $X = \{x_{j_i}^i\}$, subject to (4). Realistic optimal design involves not only objective functions, but also constraints, which represent limitations in the design variables space. For

example, P_{j_i, k_i}^{Li} and P_{j_i, k_i}^{Ui} denote the lower and upper bounds on the design variables and the relation functions $(g_1(P), g_2(P), \dots, g_m(P))$ describe some specific physical, technical or user restrictions on the modelled system. For example, there exist technical systems where the choice of some element i of type j_i restricts the choice of the other elements types to some subsets of the common elements set.

3. Night vision devices design example of technical system design by choice of elements. Choosing a high value performing NVD that satisfies user's requirements and some cost restriction is a difficult task. The performance of the NVD is usually measured with some factors in mind as small size, weight and big working distance. For example, bigger working distance can be provided by a unit with larger lens which means bigger size and weight of the device [12].

The main NVD unit is called "optoelectronic channel" and is most essential subsystem for the NVD operational characteristics. It consists of an objective, an image intensifier tube (IIT) and an ocular [11]. There exists a number of different quality NVD optical systems (objectives, oculars) and different types of IIT to choose from [11, 13]. Using optimization approach the designer can get a preliminary theoretical estimation of the most practical NVD operational parameters as its working distance, weight, price, etc.

The choice of elements – objectives, IITs, oculars, etc., from the existing sets of elements with different parameters is essential for the designing process. Using mathematical modelling some optimization models can be formulated taking into account the NVD characteristics. A mathematical integer combinatorial optimization approach is used as a design tool to reduce number of the NVD prototypes building and testing. That approach reduces the design time and cost and guarantees to some degree the preliminary requirements to the NVD.

4. A night vision device optimal design problem. For some practical example the NVD performance could include its working range, weight, cost and the operational time duration. Other practical utility functions (as objective/ocular aberrations, NVD adjustment possibilities, etc.) could be considered also, but the listed above are quite adequate for a good NVD example of an optimization design by elements choice. The generalized NVD optimization choice model can be based on four NVD performance criteria or four objective functions

$$(6) \quad \max F(P) = (f_1(P), -f_2(P), -f_3(P), f_4(P)).$$

The NVD working range functional relation of the IIT parameters P_{j_1, k_1}^1 ($k_1 = 1, 2, 3, 4$), of the objective parameters P_{j_2, k_2}^2 ($k_2 = 1, 2, 3$) and of the external surveillance conditions E_e ($e = 1, 2, 3, 4$) is [14]

$$(7) \quad f_1(P) = \sqrt{\frac{0.07P_{j_2,1}^2 P_{j_2,2}^2 P_{j_2,3}^2 P_{j_1,1}^1 P_{j_1,2}^1 E_1 E_2 E_3 E_4}{P_{j_1,3}^1 P_{j_1,4}^1}}.$$

The NVD weight functional relation of the parameters corresponding to the weights of the j -th types of the IIT, objective, ocular, electrical battery power supply P_{j_1, w_1}^1 , P_{j_2, w_2}^2 , P_{j_3, w_3}^3 , P_{j_4, w_4}^4 is

$$(8) \quad f_2(P) = (P_{j_1, w_1}^1 + P_{j_2, w_2}^2 + P_{j_3, w_3}^3 + P_{j_4, w_4}^4).$$

The NVD cost function of the j -th types cost parameters P_{j_1, c_1}^1 , P_{j_2, c_2}^2 , P_{j_3, c_3}^3 , P_{j_4, c_4}^4 for the IIT, objective, ocular, and electrical battery power supply is

$$(9) \quad f_3(P) = (P_{j_1, c_1}^1 + P_{j_2, c_2}^2 + P_{j_3, c_3}^3 + P_{j_4, c_4}^4).$$

Operational time duration calculated on the basis of the chosen electrical battery type capacity P_{j_4, b_4}^4 and the chosen IIT supply current P_{j_1, I_1}^1 is

$$(10) \quad f_4(P) = \left(\frac{P_{j_4, b_4}^4}{P_{j_1, I_1}^1} \right).$$

The design variables are subject to the following restrictions:

$$(11) \quad P^1 = \sum_{j_1=1}^{J_1} P_{j_1, k_1}^1 x_{j_1}^1 - \text{single IIT choice,}$$

$$(12) \quad P^2 = \sum_{j_2=1}^{J_2} P_{j_2, k_2}^2 x_{j_2}^2 - \text{single objective choice,}$$

$$(13) \quad P^3 = \sum_{j_3=1}^{J_3} P_{j_3, k_3}^3 x_{j_3}^3 - \text{single ocular choice,}$$

$$(14) \quad P^4 = \sum_{j_4=1}^{J_4} P_{j_4, k_4}^4 x_{j_4}^4 - \text{battery type choice,}$$

$$(15) \quad g_1(P) = \sum_{j_2=1}^{J_2} P_{j_2, k_2}^2 x_{j_2}^2 - \alpha \sum_{j_3=1}^{J_3} P_{j_3, k_3}^3 x_{j_3}^3 = 0,$$

$$(16) \quad g_2(P) = \sum_{j_3=1}^{J_3} P_{j_3, k_3}^3 x_{j_3}^3 - \sum_{j_1=1}^{J_1} P_{j_1, k_1}^1 x_{j_1}^1 \geq 0.$$

Here (15) represents the needed optical system magnification α , depending on the objective P_{j_2, l_2}^2 and ocular P_{j_3, l_3}^3 focal length and (16) describes needed dependence between the IIT screen diameter P_{j_1, d_1}^1 and the ocular field of view, P_{j_3, d_3}^3 .

The optimal choice of single IIT, objective, ocular and electrical battery type is done by using of binary integer variables $x_{j_i}^i$ in (11)–(16)

$$(17) \quad \sum_{j_i} x_{j_i}^i = 1, \quad x \in [0, 1].$$

And finally the upper and lower boundaries about NVD parameters requirements exist

$$(18) \quad P_{j_i, k_i}^{L_i} \leq P_{j_i, k_i}^i \leq P_{j_i, k_i}^{U_i}.$$

5. Numerical results from an optimal night vision monocular goggles design. The proposed optimisation design approach is numerically illustrated on the example of monocular night vision goggles (MNVG) design by optimal combinatorial choice from the sets of five types of IITs, five types of objectives, five types of oculars and six electrical batteries types [11, 13]. A combinatorial mixed integer nonlinear optimization model (6)–(18) for MNVG can be defined as single or multiobjective problem.

The goal is to maximize the standing man detection range and the operational time duration, while minimizing the device weight and cost for the given observation external conditions. The target (standing man) is observed at the background that provides contrast with value 0.2 at ambient light of $1/4$ moon. The electrical battery operational time duration is chosen to be equal or more than fixed number of hours (for this example 100 hours) and the electrical battery weight is also limited to some figure (for this example $1/3$ of the sum weight of the other elements – IIT, objective and ocular). Depending on the functional description of the optimization problem, different optimization technique can be used for the solution. The single and multiobjective formulations are used to compare the results from the numerical experiments and trying to answer the question what formulation is best suited for the solved problem.

Starting with the single objective formulation the optimization problem (A) can be stated as

$$(19) \quad \max F(P) = (f_1(P) - f_2(P) - f_3(P) + f_4(P))$$

$$= \max \left\{ \sqrt{\frac{0.07 P_{j_2,1}^2 P_{j_2,2}^2 P_{j_2,3}^2 P_{j_1,1}^1 P_{j_1,2}^1 E_1 E_2 E_3 E_4}{P_{j_1,3}^1 P_{j_1,4}^1}} - (P_{j_1,w_1}^1 + P_{j_2,w_2}^2 + P_{j_3,w_3}^3 + P_{j_4,w_4}^4) \right. \\ \left. - (P_{j_1,c_1}^1 + P_{j_2,c_2}^2 + P_{j_3,c_3}^3 + P_{j_4,c_4}^4) + \left(\frac{P_{j_4,b_4}^4}{P_{j_1,I_1}^1} \right) \right\},$$

subject to

$$(20) \quad P^1 = \sum_{j_1=1}^{J_1} P_{j_1,k_1}^1 x_{j_1}^1 - \text{single IIT choice,}$$

$$(21) \quad P^2 = \sum_{j_2=1}^{J_2} P_{j_2,k_2}^2 x_{j_2}^2 - \text{single objective choice,}$$

$$(22) \quad P^3 = \sum_{j_3=1}^{J_3} P_{j_3,k_3}^3 x_{j_3}^3 - \text{single ocular choice,}$$

$$(23) \quad P^4 = \sum_{j_4=1}^{J_4} P_{j_4,k_4}^4 x_{j_4}^4 - \text{battery type choice,}$$

$$(24) \quad \sum_{j_2=1}^{J_2} P_{j_2,k_2}^2 x_{j_2}^2 - \sum_{j_3=1}^{J_3} P_{j_3,k_3}^3 x_{j_3}^3 = 0,$$

$$(25) \quad \sum_{j_3=1}^{J_3} P_{j_3,k_3}^3 x_{j_3}^3 - \sum_{j_1=1}^{J_1} P_{j_1,k_1}^1 x_{j_1}^1 \geq 0,$$

$$(26) \quad \sum_{j_i} x_{j_i}^i = 1, \quad x \in [0, 1],$$

$$(27) \quad \left(\frac{P_{j_4,b_4}^4}{P_{j_1,I_1}^1} \right) \geq 100 \text{ (more than 100 h operational time duration),}$$

Table 1

Experimental optimal solutions results

Parameter	Single objective problem	Multiobjective problems		
	(A)	(B)	(C)	(D)
Man detecting range [m]	347	651	319	319
Device operational time duration [h]	594	119	119	534
Weight [g]	334	291	245	303
Price [€]	1120	6454	1164	1213

$$(28) P_{j_4, w_4}^4 < \frac{P_{j_1, w_1}^1 + P_{j_2, w_2}^2 + P_{j_3, w_3}^3}{3} \text{ (less than 1/3 optoelectronic channel weight).}$$

Some based on the practice fixed external surveillance conditions values are taken as constants: $E_1 = 0.7$ for atmosphere transmittance, $E_2 = 0.01$ for ambient illumination, $E_3 = 0.2$ for contrast between the background and surveillance target and $E_4 = 0.7$ for target area. The single criteria nonlinear mixed integer problem is solved by means of the LINGO software system (<http://www.lindo.com>) and the solution results are shown in Table 1.

Another example of optimization formulation as a multiobjective problem is

$$(29) \max f_1(P) = \max \sqrt{\frac{0.07 P_{j_2, 1}^2 P_{j_2, 2}^2 P_{j_2, 3}^2 P_{j_1, 1}^1 P_{j_1, 2}^1 E_1 E_2 E_3 E_4}{P_{j_1, 3}^1 P_{j_1, 4}^1}}$$

$$(30) \min f_2(P) = \min(P_{j_1, w_1}^1 + P_{j_2, w_2}^2 + P_{j_3, w_3}^3 + P_{j_4, w_4}^4),$$

$$(31) \min f_3(P) = \min(P_{j_1, c_1}^1 + P_{j_2, c_2}^2 + P_{j_3, c_3}^3 + P_{j_4, c_4}^4),$$

$$(32) \max f_4(P) = \max \left(\frac{P_{j_4, b_4}^4}{P_{j_1, J_1}^1} \right),$$

subject to restrictions (20)–(28).

As a well-known and adequate for the current numerical experiments approach for solving the formulated multiobjective problem the “weighted sum” approach [15–17] is used. The weights reflect to the relative importance of criteria $f_1(P)$, $f_2(P)$, $f_3(P)$ and $f_4(P)$ on subjective basis. They can represent the opinion of a single decision maker or synthesize the opinion of a group of experts [18–20]. The used weight coefficients shown in Table 2 are empirically defined and reflect the most practical needs. The required for that approach transformed and normalized single objective function is

$$\max \left\{ k_1 \frac{\sqrt{\frac{0.07 P_{j_2, 1}^2 P_{j_2, 2}^2 P_{j_2, 3}^2 P_{j_1, 1}^1 P_{j_1, 2}^1 E_1 E_2 E_3 E_4}{P_{j_1, 3}^1 P_{j_1, 4}^1}}}{651 - 295} + k_2 \frac{334 - (P_{j_1, w_1}^1 + P_{j_2, w_2}^2 + P_{j_3, w_3}^3 + P_{j_4, w_4}^4)}{334 - 240.4} \right\}$$

$$\left. +k_3 \frac{6500 - (P_{j_1, c_1}^1 + P_{j_2, c_2}^2 + P_{j_3, c_3}^3 + P_{j_4, c_4}^4)}{6500 - 1053} + k_3 \left(\frac{P_{j_4, b_4}^4}{P_{j_1, t_1}^1} \right) - 100 \right\}$$

Coefficients k_1, k_2, k_3, k_4 are taken according to the values in Table 2.

T a b l e 2

Weight coefficients for the multiobjective problems

Problem	k_1	k_2	k_3	k_4
(B)	0.50	0.40	0.10	0.10
(C)	0.20	0.40	0.20	0.20
(D)	0.25	0.25	0.25	0.25

The problems (B), (C), and (D) are solved also by means of the LINGO software system and the corresponding optimal solutions are shown in the Table 1.

The problem (B) expresses the preferences of the NVD working range and device weight to the NVG price and operational time duration. The problem (C) represents the preferences of the NVG weight to the other criteria. The problem (D) has equal preferences about the all criteria. The results from the solution of the single objective problem (A) are close to those for the multiobjective problem (D). It means that a single objective problem can be used (with less computational efforts) when equal preferences are needed.

6. Conclusion. The numerical results of the NVD design by optimal choice from the sets of elements prove the possibility of using that design approach for the real technical systems design. This optimization design approach differs from the traditional design process where intuitive "trials and errors" approach is used and the final design goals are achieved by repetitively building and testing a series of prototypes. The optimization approach reduces design time and cost and gives some preliminary theoretical estimation of the designed system parameters. Another bonus of using optimization methods in the design process is the possibility to computerize that part of the design process in a CAD system. Similar optimization design approach can be used also to research and analyse the parameters of complex technical systems. Using the proposed approach for other technical systems design needs to take into account their specific physical and technical parameters and restrictions to develop an adequate mathematical optimization model.

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