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GROUP DECISION MAKING FOR SELECTION OF K-BEST ALTERNATIVES

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Abstract

The paper describes an approach for determination of k-best alternatives via group decision making. The aim is to reduce a given set of alternatives to k-best ones to be considered later in the final choice. For the purpose, an optimization model is proposed and used for determination of k-best alternatives by solution of a single optimization task. An algorithm is proposed for evaluation of the determined k-best alternatives in respect to their proximity to an ideal utopian alternative. The described approach is numerically tested and its applicability is demonstrated for selection of 3-best alternatives.

Key words: k-best alternatives, group decision making, multiple attributes, optimization model

1. Introduction. Decision making is a key factor to achieve success in different disciplines, especially in the field where large amounts of information and knowledge are to be managed. The decision making problems can be handled by formulating proper optimization models involving criteria, constraints and restrictions with active participation of decision maker (DM). The main difficulty in multi-criteria decision making is the trade-off in decision because of existing constraints and requirements and in general, the consideration of all of them leads to NP discrete combinatorial problems [1, 2]. The evaluation and selection of alternatives have multi-level and multi-factor features and can be found in different fields of business, engineering, and other areas of human activity. Some decisions require selecting the best or a most appropriate alternative among a given set. Such problems can be approached by multi-attribute decision making (MADM), where the number of decision criteria is finite and multiple decision

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alternatives are given explicitly [3]. Many techniques are proposed to tackle with the problems of MADM [4–7] as PROMETHEE based on pair wise comparisons [4, 5], ELECTRE aimed to rank a number of alternatives [6, 7], TOPSIS based on the concept for shortest geometric distance from the positive ideal solution and vice versa [8], etc. MADM problems can benefit from using combinatorial optimization ranking algorithm for sequentially solving of several multi-criteria optimization tasks [9]. Despite the variety of MADM approaches, there are no better or worse techniques, but some techniques suit better to particular decision problems than others. Due to the complexity of real problems, a group of experts with different skills, experience and knowledge relating to different aspects of the problem are to be involved [3]. Group decisions can benefit from multi-attribute utility theory (MAUT) by incorporating them into a group decision support system. MAUT as a structured methodology designed to handle the tradeoffs among multiple objectives could be used as an appropriate quantitative tool for group decision support [10]. In some cases, the information provided by DMs can be imprecise or uncertain due to a lack of data, limited time to get more precise data or limited information presented to the DMs, etc. To overcome the uncertainty of data for such problems, methods for fuzzy data processing can be used [11, 12].

The problem of finding the k-best solutions is the problem of combinatorial optimization with different applications for knapsack problems, network flows, etc. [13, 14]. In the paper, the MADM problems are considered as problem of combinatorial optimization aiming to find k-best alternatives. In contrast to the mentioned above MADM techniques, this paper describes an approach of group decision making for simultaneous determination of k-best alternatives. The idea of the described approach is to reduce a given set of alternatives to k-best alternatives according to the evaluations of involved DMs. For the purpose, an optimization model is formulated. A distinguishing feature of the described approach is the determination of k-best alternatives as solution of single run of optimization task. The determined k-best alternatives can be evaluated toward an ideal one by proposed algorithm.

2. Group decision making. The problem of group decision making assumes the existence of supra decision maker (SDM) responsible for organization of the group decision making process as shown in Fig. 1.

A group of DMs are involved in definition of criteria, alternatives and alternatives evaluations. Criteria are essential components of analysis since they form the basis for the evaluation of alternatives. Criteria should be meaningful as they facilitate a choice of alternative considering expertise of DMs. Each DM evaluates the alternatives against criteria by scores and estimates the importance of criteria by weighting coefficients. SDM coordinates this process of criteria and alternatives determination and participates in optimization task formulation. Thus, defined evaluation criteria and number of alternatives together with evaluations

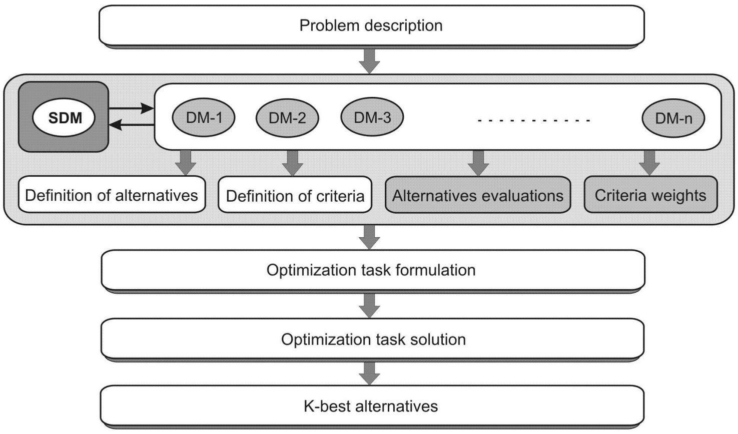

Fig. 1. Group decision making process

scores and corresponding weights for each criterion, accordingly DM expertise, are used to determine a decision matrix. It is assumed that all DMs are capable to evaluate all determined criteria.

During the process of multi-criteria group decision making (MCGDM), DMs use qualitative and/or quantitative measures to evaluate the performance of alternatives with respect to each criterion considering the relative importance of each criterion with regard to the overall goal. The information from created decision matrix can be used to formulate proper optimization task, which solution will determine the subset of k-best alternatives.

3. Optimization model for selection of k-best alternatives via group decision making. The problems of MADM can be described by a decision matrix consisting of finite number of alternatives and criteria [15,16]. The proposed approach of MCGDM is associated with selection of small subset (k-best) of the most satisfactory alternatives. Combining the specifics of MADM with described group decision making process in section 2, the following weighted group decision matrix can be stated (Table 1).

The typical usage of weights and scores approach can be described by a matrix where criteria are associated with weights using a scale 0 to 10 [17]. Each

T a b l e 1

Weighted group decision matrix

Criteria	Weights			Alternative 1			Alternative 2			...	Alternative J		
	DM^1	...	DM^N	DM^1	...	DM^N	DM^1	...	DM ^{N}	...	DM^N	...	DM^N
c_1	w_1^1	...	w_1^N	$e_{1,1}^1$	...	$e_{1,1}^N$	$e_{1,2}^1$	...	$e_{1,2}^N$	...	$e_{1,j}^1$	...	$e_{1,j}^N$
c_2	w_2^1	...	w_2^N	$e_{2,1}^1$	...	$e_{2,1}^N$	$e_{2,2}^1$	...	$e_{2,2}^N$	...	$e_{2,j}^1$	...	$e_{2,j}^N$
...	...			...			...			...	...		
c_M	w_M^1	...	w_M^N	$e_{M,1}^1$	...	$e_{M,1}^N$	$e_{M,2}^1$	...	$e_{M,2}^N$	...	$e_{M,j}^1$	...	$e_{M,j}^N$

alternative is scored against criteria, then alternative scores for each criterion are multiplied by the criterion weight and the sum gives a total alternative score. This result represents the overall preference for alternative performance. The notations in Table 1 are used to formulate a group decision making model for k-best alternatives selection as follows:

$$\text{maximize} \sum_{i=1}^{M} \sum_{n=1}^{N} w_i^n A_i^n \tag{1}$$

subject to

$$\forall i = 1, 2, \ldots, M : (\forall n = 1, 2, \ldots, K : A_i^n = \sum_{j=1}^{J} e_{i,j}^n x_j) \tag{2}$$

$$\sum_{n=1}^{N} w_i^n = 1, \ w_j \in [0, 1] \tag{3}$$

$$\sum_{i=1}^{J} x_j = k, \ x_j \in \{0, 1\} \tag{4}$$

$$1 \leq k < J, k \in Z, \tag{5}$$

where $i = 1, 2, \ldots, M$ are the evaluation criteria, $n = 1, 2, \ldots, N$ is the group of DMs, w_i^n are weighting coefficients representing relative importance of criteria evaluated by DMs, e_{ij}^k are evaluation scores of i-th DM for performance of j-th alternative ($j = 1, 2, \ldots, J$) against i-th criterion, x_j are binary integer variables assigned to each alternative and k is integer number of alternatives that are to be chosen. The number of alternatives is limited within the range of 1 (a single choice) and number k (multiple choices) where k is less than the number of alternatives to choose from.

4. Algorithm for k-best alternatives evaluation. Only one alternative has to be defined as a final decision. To assist the DM in the final decision, an algorithm for the evaluation of determined k-best alternatives to one “ideal utopian alternative” is proposed. This algorithm is composed of 3 stages. (1) Determination of an “ideal utopian alternative” with “ideal” parameters. This is alternative whose parameters have optimal values (maximum or minimum). For this ideal utopian alternative the value of objective function is calculated (f_{opt}). (2) Calculation of the objective functions values for each of the determined k-best alternatives (f_k). (3) Determination of difference ($\Delta = f_{\text{opt}} - f_k$) between the objective functions value for “ideal utopian alternative” and objective functions values for each of defined alternatives. The obtained result is represented in percentage to find the closeness of different multiple alternatives in regard to the “ideal utopian” one. As a result of algorithm implementation, the proximity for the derived k-best alternatives in respect to the ideal utopian alternative will be available.

5. Numerical testing. The numerical testing is done in 2 stages: 1) determination of k-best alternatives by group decision making, 2) evaluation of obtained k-best alternatives in respect to the ideal one according to the proposed algorithm in section 3.

5.1. Group decision making for selection of k-best alternatives. The numerical testing is based on a set of 10 alternatives with 10 criteria and 5 DMs. Each DM gives scores (within the range of 0 to 10) and higher scores values mean a better performance, which means that the final goals are to be maximized. The DM preferences are usually expressed as numbers on a certain interval [0, 1]. The input data for validation of the proposed optimization model (1)–(5) are shown in Table 2.

Taking into account the input data from Table 2 for solving the optimization problem (1) – (5) the following single-objective maximization task for 3-best alternatives is formulated as follows:

$$\text{maximize} \sum_{i=1}^{10} \sum_{n=1}^{5} w_i^n A_i^n \tag{6}$$

subject to

$$\forall i = 1, 2, \dots, 10 : \; (\forall n = 1, 2, 3, 4, 5 : \; A_i^n = \sum_{j=1}^{10} e_{i,j}^n x_j \tag{7}$$

$$\sum_{n=1}^{5} w_i^n = 1, \;\; w_j \in [0, 1] \tag{8}$$

$$\sum_{i=1}^{10} x_j = 3, \;\; x_j \in \{0, 1\}. \tag{9}$$

The result of optimization task (6)–(9) solution for input data from Table 2 determines 3-best alternatives as A2, A8 and A9.

5.2. Evaluation of defined best alternatives in respect to the ideal alternative. The evaluation of defined 3-best alternatives is done following the proposed algorithm in section 4. The objective function value for ideal utopian alternative is assumed to be 100%. The results for calculated differences between the objective functions values of each alternative and value of ideal utopian alternative that represents the proximity to the ideal utopian alternative are shown in Fig. 2.

6. Results, analysis and discussion. The goal of group decision making is to make a collective choice according to preferences of group of DMs. Instead of defining a single most preferable choice, the described problem aims to reduce the set of 10 alternatives to 3-best alternatives by a group of 5 different DMs. For the purpose 10 criteria are defined to make the alternatives evaluations. The formulated optimization task (6)–(9) defines 3-best alternatives by a single run

T a b l e 2

Weighted decision matrix

Criteria	Weighted coefficients					Alternative-1					Alternative-2					Alternative-3					Alternative-4					Alternative-5				
	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5
1	0.2	0.4	0.8	0.8	0.6	2	4	6	1	1	8	5	8	9	9	8	4	6	8	9	2	10	8	5	4	4	8	8	7	6
2	1	0.8	0.8	0.8	0.8	9	8	7	7	6	9	8	7	1	2	3	1	5	1	2	6	5	6	1	8	8	9	8	8	9
3	1	1	0.8	0.4	0.8	9	2	8	7	6	10	4	7	10	9	6	5	7	8	8	2	8	2	1	2	10	1	7	6	6
4	0.6	0.2	0.6	0.6	0.6	10	6	4	4	4	9	2	8	7	8	8	8	8	6	7	4	1	2	3	7	8	2	8	7	6
5	1	0.4	1	1	1	3	1	9	1	2	5	10	7	3	2	3	7	2	8	9	3	6	2	3	1	9	5	8	7	8
6	1	0.2	0.8	0.6	0.6	4	5	3	5	2	8	5	7	8	7	5	9	2	7	7	5	4	2	5	6	8	10	6	8	8
7	0.6	0.6	0.6	0.8	0.8	8	9	4	2	2	9	6	10	7	5	7	4	6	7	8	7	8	1	4	1	7	9	9	8	7
8	1	0.8	1	0.8	0.8	6	4	6	3	2	8	7	8	7	5	5	10	6	7	8	10	9	10	7	5	3	8	4	5	4
9	0.6	1	0.8	1	1	5	6	8	2	2	6	10	6	2	1	6	5	6	5	7	4	4	1	4	9	2	6	1	1	1
10	0.4	0.2	0.8	0.6	0.6	4	10	8	1	1	4	8	8	7	6	10	10	6	9	9	4	2	7	1	2	8	8	6	7	6

Criteria	Weighted coefficients					Alternative-6					Alternative-7					Alternative-8					Alternative-9					Alternative-10				
	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5
1	0.2	0.4	0.8	0.8	0.6	10	6	9	9	4	6	7	5	5	5	8	9	9	9	4	6	8	9	2	8	8	7	6	9	
2	1	0.8	0.8	0.8	0.8	7	4	2	7	8	8	7	7	6	7	8	7	1	2	10	1	5	10	2	10	9	8	8	9	7
3	1	1	0.8	0.4	0.8	6	6	5	6	6	2	8	7	6	7	4	7	5	9	10	5	7	8	7	9	1	7	6	6	8
4	0.6	0.2	0.6	0.6	0.6	5	10	5	7	6	6	4	4	4	3	3	8	7	8	8	8	5	6	7	2	2	8	7	6	9
5	1	0.4	1	1	1	4	4	2	6	6	7	9	6	2	6	7	7	3	2	7	7	2	8	9	9	5	8	7	8	10
6	1	0.2	0.8	0.6	0.6	4	8	2	8	7	5	3	1	2	1	5	7	8	7	7	9	2	4	7	7	10	6	8	8	10
7	0.6	0.6	0.6	0.8	0.8	7	10	1	7	8	9	4	2	2	8	6	10	7	5	10	4	6	9	8	5	9	9	8	7	9
8	1	0.8	1	0.8	0.8	5	9	2	8	8	4	6	2	2	2	7	8	7	5	10	9	6	7	4	3	8	4	5	4	5
9	0.6	1	0.8	1	1	1	10	1	1	2	6	8	2	2	8	10	6	2	2	5	5	6	5	7	9	6	1	1	1	1
10	0.4	0.2	0.8	0.6	0.6	3	8	2	6	5	10	8	1	1	1	8	8	7	6	9	10	6	9	9	9	8	6	7	6	8

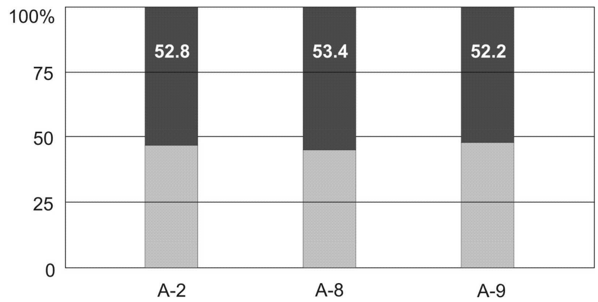

Fig. 2. Evaluations of 3-good alternatives

of solution process. The solution determines Alternative-2, Alternative-8 and Alternative-9 as alternatives to be considered in the final choice of the management or executive staff.

It should be noted that only one alternative can be determined if the restriction (9) is changed to $\sum_{i=1}^{10} x_j = 1$. In case when the managerial staff is interested how good the defined alternatives are, the proposed algorithm can be used. The proximity to "ideal utopian" alternative according to the group of experts' estimation is shown in Fig. 2. The choice of one of the defined alternatives can be done by some other subjective considerations but if the proximity to "ideal" alternative is preferred, the results shown in Fig. 2 can be used. According to these results the parameters of alternative Alternative-9 are close to ideal one, followed by alternative Alternative-2 then alternative Alternative-8.

The optimization task is solved by means of LINGO ver.12. The solution times for described optimization task took few seconds on PC with 2.93 GHz Intel i3 CPU and 4 GB RAM. There is no limitation to perform the proposed model for other optimization solvers.

The advantage of the proposed MCGDM approach is the determination of k-best alternatives as a solution of a single optimization task thus avoiding time consuming process of other proposed methods for MADM. The defined subset of k-best alternatives is assessed by the described algorithm that evaluates the alternatives toward an "ideal" alternative. Another possibility is choice among determined k-best alternatives by other subjective criteria. In case of imprecise or uncertain information due to lack of data, time pressure, or the DMs limited information, the proposed approach can be modified with fuzzy data to overcome such uncertainty.

7. Conclusion. The paper describes a combinatorial optimization model for multi-attribute group decision making for selection of k-best alternatives. This selection is the result of a solution to the problem of optimization by a single run of optimization task. The reducing of set of alternatives to k-best alternatives can be used by executive managers during the final selection of the best alternative.

The described modelling approach could be applied for any other kind of problems that can be represented with certain set of alternatives and given criteria. The formulated model for k-best alternatives and described algorithm for their evaluation can be used separately of each other. Numerical testing demonstrates the practical applicability of both of k-best alternatives selection model and of algorithm for their evaluation. Future development of the proposed approach is related with incorporation of fuzzy data in group decision making for selection of k-best alternative. Because of well structured decision matrix, the described model and algorithm can be formalized and coded as software tool for decision support to be used by managers without specific mathematical background.

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