One-dimensional cutting stock model for joinery manufacturing

Ivan C. Mustakerov and Daniela I. Borissova

***Abstract*—**The current paper describes one-dimensional cutting stock model for joinery manufacturing. The joinery elements differ in size and number that are specific for each particular project. The goal is to determine the optimal length of blanks (which are usually ordered with equal size in large quantities) in order to satisfy the demand for all joinery elements. Along with this, it is necessary to find the optimal cutting patterns that minimize the overall trim waste. For the goal, one-dimensional cutting stock model for joinery manufacturing using combinatorial optimization is proposed. Numerical example of real-life problem is presented to illustrate the applicability of the proposed approach.

***Keywords*—**Combinatorial optimization model, joinery manufacturing, linear programming model, one dimensional cutting stock problem.

I. Introduction

THE cutting-stock problem has many applications in industry. This problem arises when the available material has to be cut to fulfill certain goals as cutting patterns with minimal material waste and cost efficient production, higher customer satisfaction, etc. In general, cutting stock problems consist in cutting large pieces (*blanks*), available in stock, into a set of smaller pieces (*elements*) accordingly to the given requirements, while optimizing a certain objective function. These problems are relevant in the production planning of many industries such as the metallurgy, plastics, paper, glass, furniture, textile, joinery manufacturing, etc. In the last four decades cutting stock problems have been studied by an increasing number of researchers [1]-[5]. The interest in these problems is provoked by the many practical applications and the challenge they provide to researchers. On the first glance they are simple to formulate, but in the same time they are computationally difficult to solve. It could be summarized that: cutting and packing problems [6] belong to the class of NP-hard problems; solution of these problems extensively uses mathematical programming and combinatorial methods; many real-life problems are computationally hard and can be formalized only as NP-hard problems. The continuous growth of the prices of the materials and of the energy requires minimization of the production expenses for every element.

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Most materials used in the industry are supplied of standard forms and lengths, and direct use of such forms is most cases are impossible. They should be cut in advance to some size, expected to be optimal in the sense of trim waste. This can be done using various methods of cutting planning. The problem of optimal cutting is that different size elements have to be manufactured using blanks of single standard size. This demands developing of methods for optimal cutting of source material. The one-dimensional cutting stock problem (1D-CSP) is one of the crucial issues in production systems, which involve cutting processes. The classical 1D-CSP addresses the problem of cutting stock materials of length in order to satisfy the demand of smaller pieces while minimizing the overall trim loss. Kantorovich first formulates 1D-CSP [7], [8] and Gilmore and Gomory [9], [10] propose the first solution methodology for the cutting stock problems.

In most cases, cutting stock problem is formulated as an integer linear programming optimization problem that minimizes the total waste while satisfying the given demand [11]. In [12] a review of some linear programming formulations for the 1D-CSP and bin packing problems, both for problems with identical and non-identical large objects, is presented. It is investigated haw different ways of defining the variables and structure of the models affect the solvability of problems. Because of NP-hard nature of cutting stock problems finding an optimal solution in reasonable time is essentially difficult and often researchers turn to heuristic algorithms to deal with this kind of complex and large-sized problem [3], [13]. Some researchers look for solutions of 1D-CSP in which the non-used material in the cutting patterns may be used in the future, if large enough [4] . A two-stage decomposition approach for 1D-CSP is proposed in [14]. In the first stage is performed calculation of the total number of patterns that will be cut and generation of the cutting patterns through a heuristic procedure. On the second stage optimal cutting plan is determined. In [15] a new approach to cutting stock problem is proposed where a ‘‘good’’ solution is seeking for consecutive time periods. It is adjusted to situations where useful stock remainders can be returned to the warehouse between time periods and used lately for other orders. A similar problem for wood industry is described in [16]. It is stated that cutting problems from the practice usually have its own specificity that do not allow the application of known models and solution algorithms. In many cases, proper modifications are needed or even completely new methods have to be developed on order to cope with real word requirements.

The current paper proposes new approach for optimization of real-life 1D-CSP from the joinery manufacturing practice. A combinatorial optimization task is formulated to determine the optimal length of the blanks and optimal cutting patterns in sense of minimal waste. In contrast to other 1D-CSPs, the optimal length of the blanks and optimal cutting patterns are defined simultaneously as a result of single optimization task solution. A proper algorithm for practical application of the proposed approach is defined and numerically tested using real-life data.

II. Problem Description

Aluminum or PVC blanks usually are supplied from the factory with fixed length of 6 meters. These blanks are used to cut out different elements of joinery. The joinery elements differ in size and number that are specific for each particular project. The goal is to determine the optimal length of blanks (which are usually ordered with equal size in large quantities) in order to satisfy the demand for all joinery elements. Along with this, it is necessary to find the optimal cutting patterns minimizing the waste. In [17] an in-depth investigation of joinery modules used in a wide range of buildings is performed. It was found that the number of joinery types in the apartments could be reduced to a certain number of unified modules. For example, in case of a middle size flat, these modules involve four modules:

- Module 1 is used for 4 doors with dimensions 2200 mm x 730 mm
- Module 2 is used for 2 doors with dimensions 2000 mm x 650 mm
- Module 3 is used for 1 window with dimensions 1400 mm x 1400 mm
- Module 4 is used for 2 windows with dimensions 1700 mm x 2100 mm

The investigated cutting stock problem can be narrowed down to definition of optimal length of blanks and optimal cutting patterns for modules used in an apartment. The problem can be described as follows: a factory has to fulfill order of blanks with certain length needed to assemble a given number of modules, consisting of elements with known length and number. For the sake of simplicity of the presentation only casement elements for the modules in the example above are summarized as a manufacturing order shown in Table I.

In practice, all PVC and aluminum profiles for doors and windows come with fixed length of 6 meters. However, this is not mandatory requirement and it is possible to order blanks with different length. There are no obstacles to order to the manufacturing company to produce a number of blanks with different length than standard 6 meters – for example any length between 5 and 7 meters. When the optimal length of blanks is determined, the next step is to define the optimal cutting patterns of joinery elements for each blank.

TABLE I
Joinery Elements Length and Demand

Element *j*	**Length l_i, mm**	**Demand $k_{i,j}$**
1	$l_1 = 650$	4
2	$l_2 = 730$	8
3	$l_3 = 1400$	4
4	$l_4 = 1700$	4
5	$l_5 = 2000$	4
6	$l_6 = 2100$	4
7	$l_7 = 2200$	8

The problem of optimal joinery manufacturing can be investigated as 1D-CSP by means of proper mathematical modeling.

III. Mathematical Model Formulation

The described one-dimensional cutting stock problem for joinery is formalized via combinatorial optimization model. In contrast to other similar models it allows determining optimal length of blanks and optimal cutting patterns minimizing the trim loss, accordingly given demands of joinery elements. This type of functionality of the model requires introducing of inequalities for each of blanks. That means there is a necessity of knowing in advance the number N of the blanks. Number N can be calculated as overall demand of joinery elements divided by the length L of the blanks. On the other hand, the length L of the blanks is to be determined after solution of the optimization task. This "recursive" type of problem can be overcome taking into account that length L will have some value close to the standard length of 6 meters. Having this in mind, number of blanks N can be calculated as overall demand of joinery elements divided by the length of 6 meters, rounded to integer value. Then this value of N can be used to formulate the optimization task as:

$$\min \rightarrow \sum_{i=1}^{N} (L - L_i), i = 1,...,N \tag{1}$$

subject to

$$\forall i : L_i = \sum_{j=1}^{J} x_{ij} l_j , j = 1,..., J \tag{2}$$

$$\forall i : L_i \leq L \tag{3}$$

$$\forall j : \sum_{i=1}^{N} x_{ij} = k_{ij} \tag{4}$$

$$(6 - \Delta_{\min}) \leq L \leq (6 + \Delta_{\max}) \tag{5}$$

$$\forall j :\ x_{ij} = \begin{cases} \textit{binary integer}\ 0\ \textit{or}\ 1,\ \text{if}\ N\ \leq k_{ij} \\ \textit{integer},\ \ \text{otherwise} \end{cases} \tag{6}$$

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where N is number of blanks; L is length of blanks; L_i is the utilized length of each blank; l_j is length of joinery elements; x_{ij} are decision variables assigned to each element for particular blank; k_{ij} represents the demand of each element.

The objective function (1) minimizes the sum of trim loss for each blank. The optimal cutting pattern for each of the blanks is defined by decision variables x_{ij} in (2). Depending on the given particular joinery project, the decision variables (6) could be binary integer variables or integer variables. For example, if the number of the blanks is less than the maximum demand of some element, then the decision variables x_{ij} are to be considered as integers. This statement allows the model to allocate more than 1 element within cutting pattern in the blank to satisfy the elements demand. This elements demand is satisfied by (4). Deviation from the standard length of 6 meters is represented by $\Delta_{\min}$ and $\Delta_{\max}$ both approximately in the range of 1 meter.

IV. Numerical Illustration

The demand of elements for the example of joinery manufacturing order from Table I is illustrated in Fig. 1.

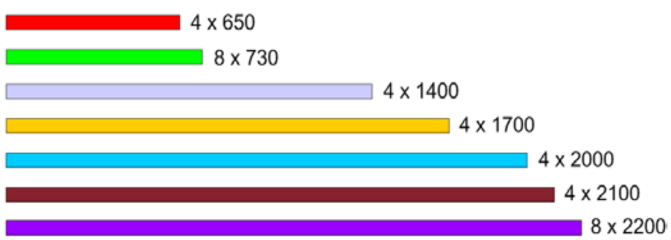

Fig. 1. Joinery elements and demand

Using the input data from Table I the following steps are performed:

1) Determination of total length of all elements considering their demand $L_{sum} = 54840$ mm;

2) Determination number of blanks N as rounded to integer result of the total elements length 54840 mm divided by 6000 mm as 54840/6000 = 9.14 => $N = 9$ and setting of deviations $\Delta_{\min} = \Delta_{\max} = 1000$ mm.

3) Formulation of optimization task:

$$\min \{(L - L_1) + (L - L_2) + (L - L_3) + (L - L_4) + (L - L_5) + (L - L_6) + (L - L_7) + (L - L_8) + (L - L_9)\} \quad (7)$$

subject to

$$x_{11}l_1 + x_{12}l_2 + x_{13}l_3 + x_{14}l_4 + x_{15}l_5 + x_{16}l_6 + x_{17}l_7 = L_1 \quad (8a)$$

$$x_{21}l_1 + x_{22}l_2 + x_{23}l_3 + x_{24}l_4 + x_{25}l_5 + x_{26}l_6 + x_{27}l_7 = L_2 \quad (8b)$$

$$x_{31}l_1 + x_{32}l_2 + x_{33}l_3 + x_{34}l_4 + x_{35}l_5 + x_{36}l_6 + x_{37}l_7 = L_3 \quad (8c)$$

$$x_{41}l_1 + x_{42}l_2 + x_{43}l_3 + x_{44}l_4 + x_{45}l_5 + x_{46}l_6 + x_{47}l_7 = L_4 \quad (8d)$$

$$x_{51}l_1 + x_{52}l_2 + x_{53}l_3 + x_{54}l_4 + x_{55}l_5 + x_{56}l_6 + x_{57}l_7 = L_5 \quad (8e)$$

$$x_{61}l_1 + x_{62}l_2 + x_{63}l_3 + x_{64}l_4 + x_{65}l_5 + x_{66}l_6 + x_{67}l_7 = L_6 \quad (8f)$$

$$x_{71}l_1 + x_{72}l_2 + x_{73}l_3 + x_{74}l_4 + x_{75}l_5 + x_{76}l_6 + x_{77}l_7 = L_7 \quad (8g)$$

$$x_{81}l_1 + x_{82}l_2 + x_{83}l_3 + x_{84}l_4 + x_{85}l_5 + x_{86}l_6 + x_{87}l_7 = L_8 \quad (8h)$$

$$x_{91}l_1 + x_{92}l_2 + x_{93}l_3 + x_{94}l_4 + x_{95}l_5 + x_{96}l_6 + x_{97}l_7 = L_9 \quad (8i)$$

$$L_1 \le L \quad (9a)$$

$$L_2 \le L \quad (9b)$$

$$L_3 \le L \quad (9c)$$

$$L_4 \le L \quad (9d)$$

$$L_5 \le L \quad (9e)$$

$$L_6 \le L \quad (9f)$$

$$L_7 \le L \quad (9g)$$

$$L_8 \le L \quad (9h)$$

$$L_9 \le L \quad (9i)$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} + x_{71} + x_{81} + x_{91} = 4 \quad (10a)$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} + x_{72} + x_{82} + x_{92} = 8 \quad (10b)$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} + x_{73} + x_{83} + x_{93} = 4 \quad (10c)$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} + x_{74} + x_{84} + x_{94} = 4 \quad (10d)$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} + x_{75} + x_{85} + x_{95} = 4 \quad (10e)$$

$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} + x_{76} + x_{86} + x_{96} = 4 \quad (10f)$$

$$x_{17} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} + x_{77} + x_{87} + x_{97} = 8 \quad (10g)$$

$$5 \le L \le 7 \quad (11)$$

$$x_{ij} \text{ – binary integer: 0 or 1} \quad (12)$$

The relations (8) in combination with inequalities (9) define optimal cutting patterns for each particular blank. The optimal cutting patterns are defined not to exceed the length of the blanks and to satisfy the requested demand of elements expressed by (10). The objective function (7) seeks for solution that minimizes the waste of all blanks. The optimal length of blanks is to be defined within interval of 5 to 7 meters (11). In this example the decision variables for optimal cutting patterns are binary integer variables (12).

The solution the formulated mixed integer optimization task (7) – (12) determines the optimal length of blanks; total waste; waste for each blank; and used length of each blank, as shown in Table II.

TABLE II

Optimal Solution Results

Optimal length of blanks *L*, mm	**Total waste for order, mm**	**Used length of each blank, mm**	**Waste for each blank, mm**
		$L_1 = 6330$	220
		$L_2 = 6330$	220
		$L_3 = 6030$	520
		$L_4 = 6030$	520
6550	4110	$L_5 = 5680$	870
		$L_6 = 5680$	870
		$L_7 = 5680$	870
		$L_8 = 6530$	20
		$L_9 = 6550$	0

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The optimal cutting patterns defined by the values of the binary integer variables for each blank are shown in Table III.

TABLE III
OPTIMAL CUTTING PATTERNS FOR EACH BLANK

	Element1	Element2	Element3	Element4	Element5	Element6	Element7
L_1	0	1	1	0	1	0	1
L_2	0	1	1	0	1	0	1
L_3	0	1	1	1	0	0	1
L_4	0	1	1	1	0	0	1
L_5	1	1	0	0	0	1	1
L_6	1	1	0	0	0	1	1
L_7	1	1	0	0	0	1	1
L_8	0	1	0	1	1	1	0
L_9	1	0	0	1	1	0	1

V. RESULT ANALYSIS AND DISCUSSION

The defined optimal length of blanks to fulfill the order is 6550 mm and the overall minimum waste is 4110 mm. The graphical illustration of optimal cutting patterns for each of the blanks is shown in Fig. 2.

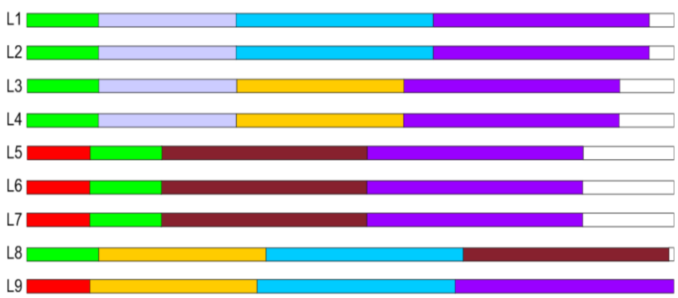
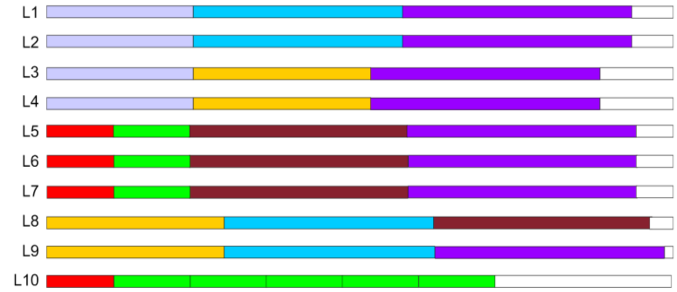


Fig. 2. Optimal cutting patterns for blanks
(L = 6550 mm, waste = 4110 mm)

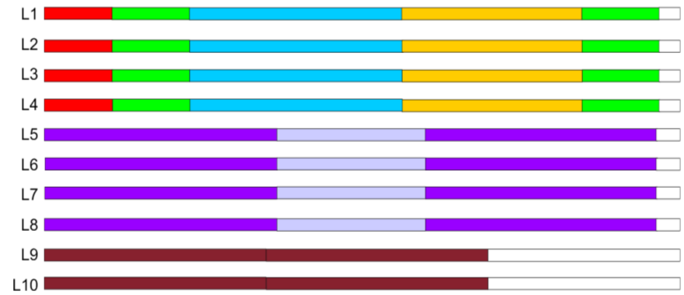
It is compared with cutting patterns combinations defined by experienced practitioners for standard length of blanks equal to 6000 mm. The comparison shows that without optimization the trim loss is bigger as shown on Fig. 3.

The proposed optimization approach determines the optimal length of blanks that is increased toward standard length with 550 mm. This reduces number of needed blanks to fulfill the requested order and waste and costs as compared to the case of standard length using. Using of standard length of 6 m not only increases the trim loss but also increases the number of required blanks to execute the order. That is important for large joinery work projects in means of increasing of transportation costs.

Due to NP-hard nature of considered problems, the computational time increases essentially with increasing the number of decision variables. The formulated mixed integer linear optimization task (7) – (12) is solved by LINGO solver using branch-and-bound method [18].



a) waste = 5160 mm



b) waste = 5160 mm

Fig. 3. Cutting patterns for standard blank length L = 6000 mm

The solution time for the described example with 64 integer variables amounts to 1 hour, 23 minutes and 50 seconds on PC with 2.93 GHz Intel i3 CPU and 4 GB RAM. The task solution report is shown in Fig. 4.

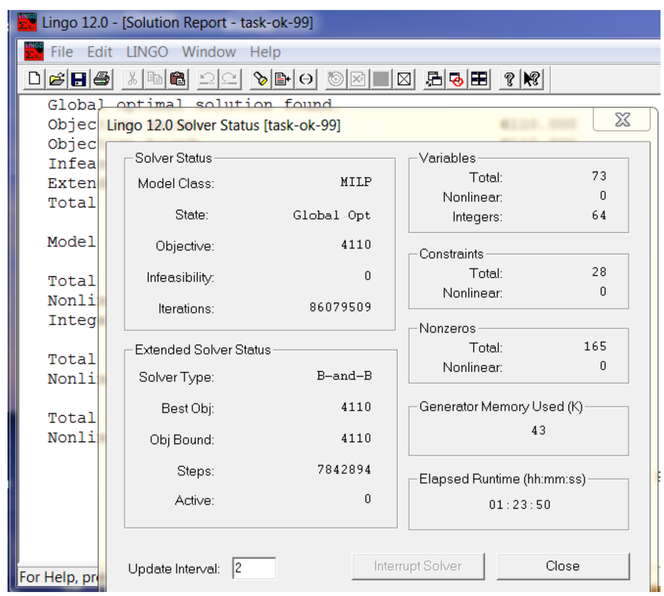


Fig. 4. Task solution report

VI. CONCLUSION

In the paper, joinery work manufacturing problem is investigated as one-dimensional cutting stock problem by means of combinatorial optimization. The advantage of the proposed approach is the possibility to determine simultaneously the optimal length of the blanks and optimal cutting patterns for each blank. In contrast to heuristic approaches to this type of problems the described approach defines solution as a global optimum. The reduction of cutting trim loss is one of the main problems in joinery manufacturing. This problem turns to be important especially for large scale projects where the joinery work for a whole building or for several buildings has to be done. The described approach can contribute not only to reduce the trim loss via optimization of length of the blanks and cutting patterns, but also could decrease the overall production time and costs.

Future investigations are to be done with different large scale problems to determine the computational difficulties. Implementation of the proposed approach in a software tool for joinery work design will help the practitioners to reduce costs and will contribute to their competitiveness.

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REFERENCES

[1] A. Mobasher and A. Ekici, "Solution approaches for the cutting stock problem with setup cost". *Computers & Operations Research*, vol. 40, 2013, pp. 225-235.

[2] A. C. Dikili, E. Sarioz and N. A. Pek, "A successive elimination method for one-dimensional stock cutting problems in ship production". *Ocean Engineering*, vol. 34, 2007, pp. 1841-1849.

[3] Y. Cui and Y. Lu, "Heuristic algorithm for a cutting stock problem in the steel bridge construction". *Computers & Operations Research*, vol. 36, 2009, pp. 612-622.

[4] C. Cherri, M. N. Arenales and H. H. Yanasse, "The one-dimensional cutting stock problem with usable leftover – A heuristic approach". *European Journal of Operational Research*, vol. 196, 2009, pp. 897-908.

[5] A. C. Dikili, A. C. Takinaci and N. A. Pek, "A new heuristic approach to one-dimensional stock-cutting problems", *Ocean Engineering*, vol. 35, no.7, 2008, pp. 637-645.

[6] E. A. Mukhacheva and A. S. Mukhacheva. "L. V. Kantorovich and Cutting-packing problems: New approaches to combinatorial problems of linear cutting and rectangular packing". *Journal of Mathematical Sciences*, vol. 133, no. 4, 2006, pp. 1504-1512.

[7] L. V. Kantorovich, Mathematical methods of organizing and planningproduction. *Management Science,* vol. 6, 1960, pp. 366-422.

[8] L. V. Kantorovich and V. A. Zalgaller, *Rational Cutting of Stock* [in Russian], Nauka, Novosibirsk, 1971.

[9] P. Gilmore and R. Gomory, "A linear programming approach to the cutting stock problem". *Operations Research,* vol. 9, no. 6, 1961, pp. 848-859.

[10] P. Gilmore and R. Gomory, "A linear programming approach to the cutting stock problem, part II". *Operations Research*, vol. 11, 1963, pp. 863-888.

[11] S. M. A. Suliman, "Pattern generating procedure for the cutting stock problem". *Int. Journal of Production Economics*, vol. 74, 2001, pp. 293-301.

[12] J. M. Valerio de Carvalho, "LP models for bin packing and cutting stock problems". *European Journal of Operational Research*, vol. 141, 2002, pp. 253-273.

[13] M. HMA Jahromi, R. Tavakkoli-Moghaddam, A. Makui and A. Shamsi, "Solving an one-dimensional cutting stock problem by simulated annealing and tabu search". *Journal of Industrial Engineering International*, vol. 8, no. 24, 2012, doi:10.1186/2251-712X-8-24.

[14] T. Aktin and R. G. Ozdemir. "An integrated approach to the one-dimensional cutting stock problem in coronary stent manufacturing". *European Journal of Operational Research*, vol. 196, 2009, pp. 737-743.

[15] P. Trkman and M. Gradisar. "One-dimensional cutting stock optimization in consecutive time periods", *European Journal of Operational Research*, vol. 179 , 2007, pp. 291-301.

[16] S. Koch, S. Konig and G. Wascher. "Linear Programming for a Cutting Problem in the Wood Processing Industry – A Case Study". *FEMM Working Paper* No 14, 2008.

[17] Ch. Korsemov, Hr. Toshev, I. Mustakerov, D. Borissova and V. Grigorova. "An optimal approach to design of joinery for renovation of panel buildings". *International Journal of Science and Engineering Investigations*, vol. 2, no. 18, 2013, pp. 123-128.

[18] Lindo Systems ver. 12, http://www.lindo.com