# Interpolation of acoustic field from nearby located single source

Iurii Chyrka[1]

[1] Mathematical Methods for Sensor Information Processing Department of Institute of Information and Communication Technologies of Bulgarian Academy of Sciences,
25 A, Acad. G. Bonchev str., 1113 Sofia, Bulgaria
Yurasyk88@gmail.com

**Abstract.** The problem of performance increasing of a small measurement array in the task of single narrowband source localization in the nearfield is considered. Two different approaches for acoustic field interpolation before applying conventional acoustic holography are proposed. One of them, based on the autoregressive model of a sinewave, gives better resolution in localization of the source near the center of array. The second one, based on the estimation of instantaneous phases of signals at each microphone, decreases error of peak localization from sources near the boundaries of the measurement array.

## 1 Introduction

Near-field acoustic holography (NAH) [1] is a technique that deals with noise source reconstruction problem. Its original idea comes from Fourier acoustics and provide, based on the 2D recording of sound waves, reconstruction of the 3D sound field between the source boundary and the measurement (hologram) plane [1-4].

Modern NAH approaches [5-8] do not require location of microphones on specific positions and usually works with measurements that are made over a limited surface and by relatively low amount of microphones.

Precision of reconstruction and source localization depends mostly on microphone aperture (area, microphone number and spacing distance), sound frequency and distance between source and measurement plane. Papers [9], [10] are focused on the numerical enlarging the measurement surface tangentially. The work [10] deals with an analytic continuation of a coherent pressure field located over the surface of a vibrator. This analytic continuation is an extrapolation of the measured field into a region outside the original finite sheet, and is based on the Green's functions.

When number of available measurement channel is limited, precision of reconstruction and method resolution can be improved by acoustic field interpolation and extrapolation also known as a virtual microphone technique [1]. In contraposition to mentioned works, the present one deals mostly with calculation of the sound field inside the measurement array aperture and adding virtual microphones. In addition, this paper is focused on the specific problem of single source localization, which allows simplifying of acoustic field model and its interpolation.

## 2 Statistically Optimized Holography (SONAH)

Based on features of the available measurement equipment and built-in software research in this work is focused on the SONAH.

SONAH is based on the representation of the pressure at the point $\mathbf{r} = (x, y, z)$ in the space as a weighted sum of pressures measured at *N* microphone positions:

$$p(\mathbf{r}) = \sum_{n=1}^{N} \mathbf{c}(\mathbf{r})\, p(\mathbf{r}_{h,n}) = \mathbf{p}_h^{\mathrm{T}} \mathbf{c}(\mathbf{r}), \quad (1)$$

where weighting matrix $\mathbf{c}(\mathbf{r})$ depends only on the point coordinate $\mathbf{r}$. It can be obtained as a least square solution for an infinite set of elementary waves

$$\mathbf{c}(\mathbf{r}) = \left(\mathbf{A}^{\mathrm{H}}\mathbf{A} - \theta^2 \mathbf{I}\right)^{-1} \mathbf{A}^{\mathrm{H}} \boldsymbol{\alpha}, \quad (2)$$

where $\mathbf{A}^{\mathrm{H}}\mathbf{A}$ is a square matrix that depends on *N* positions on the hologram plane; $\mathbf{A}^{\mathrm{H}}\boldsymbol{\alpha}$ is a square matrix that depends on the *N* positions on the prediction plane and *N* positions on the prediction plane; **I** is the identity matrix; and θ is a regularization parameter. In view of unlimited number *M* of elementary waves all elements of matrices $\mathbf{A}^{\mathrm{H}}\mathbf{A}$ and $\mathbf{A}^{\mathrm{H}}\boldsymbol{\alpha}$ actually are integrals and must be calculated numerically via Gauss and Gauss-Laguerre quadratures [5].

## 3 Acoustic field interpolation approaches

Two approaches for interpolation of acoustic fields, generated by the single point source, are considered. One of them is based on the sinewave model of the field and the second one works with the instantaneous phases of signals at each microphone.

It is well known that any sinewave signal can be represented by the auto-regression model of the second order [11]. After simple transformations it can be written as

$$s_{\mathrm{i}} = \alpha s_{\mathrm{i}-1} - s_{\mathrm{i}-2}, \qquad \alpha = 2\cos(\omega \Delta t) \ . \quad (3)$$

Having signal values at any two of three points, time difference $\Delta t$ between them and the signal angular frequency $\omega$, we can calculate the interpolated signal value placed inside (interpolation) or outside (extrapolate) the segment of pair of real microphones.

The model (3) can be used for signal interpolation only after assumptions about narrowband nature of the signal (that can be achieved by preliminary band filtering of measured signals) and minor difference in the signal power at two real microphones that can be neglected in calculations. The interpolated signal can be calculated as

$$s_v = (s_1 + s_2)/\alpha = (s_1 + s_2)/2\cos(\omega \Delta t), \quad (4)$$

where $s_1$ and $s_2$ are signal values from the first and second microphones in an arbitrary pair. Time difference and frequency give us a phase growth between two points of the signal. The angular frequency is obtained from known signal frequency *F* as $\omega = 2\pi F$. The time difference $\Delta t$ for the equation (4) can be estimated from known coordinates of real microphones and estimated position of the sound source as

$$\Delta t = |l_1 - l_2| / 2\lambda F \, , \qquad \textbf{(5)}$$

where $\lambda$ is a wavelength, $l_1$, $l_2$ are distances from the source to corresponding real microphones. This formula is derived from the next considerations.

In order to assure the equal phase shift $\omega\Delta t$ of the interpolated signal $s_v$ to two other signals in the equation the virtual microphone must be placed in the position where the distance to the signal source is a middle between distances from real microphones to the source. The source coordinates are estimated as argmax of a sound pressure map, calculated by some conventional holography method.

Equation (4) has one substantial limitation: it requires continuous growth of phase between two real microphones, that can be achieved only if projection of the source point lies outside the segment between these microphones. Therefore, not every microphone pair can be used in such a manner. In practical cases, this technique can be applied only for sources that are located near to the center of the array when generated virtual microphone aperture is almost symmetrical. In other cases it becomes highly asymmetrical and destroys a reconstruction of an acoustic field.

The second approach is rather the extrapolation than interpolation, because it allows calculation of the signal value in the any arbitrary point on the measurement plane. In the case of the single source, the acoustic field on some distance around each real microphone can be easily predicted, if position of the source, its frequency, power and instantaneous phase of the signal from the microphone is known.

The idea of signal generation at interpolation points is based on the model of the spherical wave [1]. Because the initial phase and amplitude of the source are unknown, the interpolated signal should be calculated by the modified equation

$$p(r) = \frac{\hat{A}e^{-j(\hat{\varphi}+\omega\Delta t)}}{r} \, , \qquad \textbf{(6)}$$

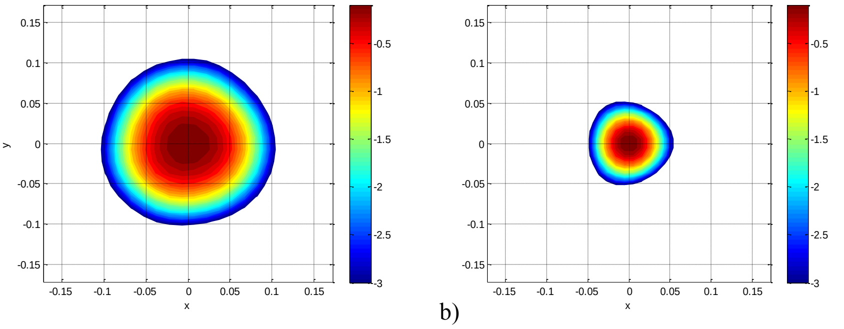
where $\hat{A}$, $\hat{\varphi}$ are estimates of the signal amplitude and instantaneous phase at the closest microphone, time difference between the microphone and the interpolation point is calculated as $\Delta t = \Delta r / (\lambda F)$, $\Delta r$ is a difference of distance from source for the microphone and the interpolation point. As in the previous case, it requires rough estimate of source position and additionally estimates of signal amplitude and initial phase at each microphone, that can be obtained from the likelihood equations [12].

## 4 Simulation Results

The developed software enables simulation of the acoustic pressure data from one or several point sources and obtaining of sound pressure maps, calculated as root-mean-squares (RMS) of pressure at any desired distance and signal frequency. It uses a matrix-form implementation of the SONAH algorithm with proposed modifications.

Signals were simulated for a Brüel & Kjær acoustic camera equipped with non-uniform slice wheel microphone array type WA-1558-W-021 that contains 18 microphones [13]. The signal measured by each microphone was generated by using the already mentioned spherical wave model and additionally polluted by an additive white Gaussian noise with SNR=20 dB and without cross-channel correlation.

The carried out research has shown that this technique allows increasing of SONAH localization resolution of sources with frequency bigger than 2 kHz. Examples of reconstructed acoustic maps for frequency 2.5 kHz are shown in the Fig. 1.



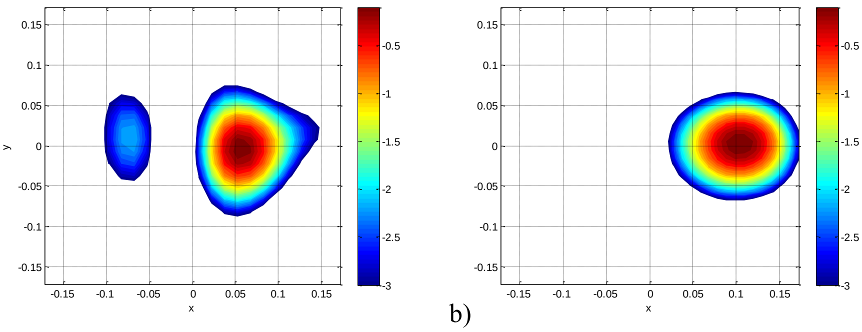

a) b)

**Fig. 1.** Reconstructed by SONAH acoustic map of point source of a single-tone signal on the distance 10 cm: a) without interpolation b) with interpolation.

It is well visible, that the proposed approach allows up to 2 times improvement in resolution in comparison to conventional SONAH.

For the second type of interpolation the preliminary research has shown that the most preferable and optimal by computational load is a regular geometry of virtual microphones with spacing distance between them equal half of a wavelength.

The research results have shown that this interpolation technique increases precision of source localization by the SONAH. Examples of reconstructed acoustic maps for frequency 2.5 kHz are shown in the Fig. 2.




a) b)

**Fig. 2.** Reconstructed by SONAH acoustic map of point source of the point source located in the point (0.1, 0) on the distance 10 cm: a) without interpolation b) with interpolation.

As one can see, ordinary SONAH can give a significant shift in the pressure peak localization, when source is located far from the center of the array. The proposed technique gives much better precision of the source localization, even with a coarse initial estimate of the source position.

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