PREFERENCES AND MODELING OF THE COMPLEX SYSTEM „HUMAN-PROCESS": A MACHINE LEARNING APPROACH

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INTRODUCTION

There are two main approaches to reduce the complex system to simpler form. The first is to accumulate sufficient information in order to achieve structural and parametric al clarity. The second is to seek interpretation and consolidation and expression of the different aspects of the complex system through expert analysis and description of the expert's preferences and concepts as an element of the system itself. In this sense a complex system is a system with active or decisive participation of the human in the determination of the objective, the description and the choice of the final decision, moreover, as an element of the system itself. In the context of the system analysis this is the system 'human-process'.

Machine learning focuses on prediction, based on known properties learned from training data. It explores the construction and study of algorithms that can learn from teaching excerpts and make predictions. According to some opinions the machine learning and pattern recognition can be viewed as two facets of one and the same problem. As scientific field Machine learning is an area of computer science that is evolved from the study of pattern recognition and computational learning theory. It is closely related to and often overlaps with computational statistics - a discipline that also specializes in prediction-making.

The decision making process is also an iterative process who includes learning as an essential part of its realization. The presentation of human preferences analytically using utility functions is a good possible approach to mathematical description of human being. It is the first step during realization of a human-centered value-driven design process and decision making, whose objective is to avoid the contradictions in human decisions and to permit mathematical calculations in these fields.

The combination of previous theories and approaches allows using flexible iterative mutual learning process inconstructionof mathematical models of Complex system like „human- process" and build mathematically well-founded control solution. The human thinking, notions and preferences have of cardinal significance and the machine learning in mathematical meaning need analytical representation. These are two contradiction tendencies with their proper requirements. The latter impose the need of new approaches and algorithms for inclusion of the machine learning in the decision making process in a harmonious way.

The decision making is based on the preferences and starting from this position the incorporation of human preferences in complex systems is a contemporary trend in scientific investigations. The objective of the paper is to present a strict logical mathematical approach for modeling and estimation of human preferences as mashine learning in the process of building of mathematical models of complex systems with human participation.

MATHEMATICAL FORMULATION, PREFERENCES, UTILITY FUNCTION

The productive merger of the mathematical exactness with the empirical uncertainty in the human notions is the main challenge in the problems to solve. People's preferences contain uncertainty due to the cardinal type of the empirical expert information. This uncertainty is of subjective and probability nature. Probability theory and expected utility theory address decision making and machine-learning under these conditions (P. Fishburn, 1970; R. Keeney, 1978; R. Keeney, H. Raiffa, 1993; V. Terzieva, Y. Pavlov, R. Andreev, 2007). As approach for solution of these problems we choose stochastic programming. The uncertainty of the subjective preferences could be taken as an additive noise that could be eliminated, as is typical in the stochastic approximation procedures and machine-learning (M. Aizerman, E.Braverman , L. Rozonoer 1970; Y. Pavlov, 2005).

The machine learning evaluation method presented here rest upon the achievements of the theory of measurement (scaling), utility theory and, statistic programming. The difficulties that come from the mathematical approach are due to the probability, and subjective uncertainty of the DM expression and the cardinal character of the expressed human preferences. The so called normative (axiomatic) approach considers the conditions for existence of utility function. The mathematical description follows. Let $\boldsymbol{X}$ be the set of alternatives and $\boldsymbol{P}$ be a set of probability distributions over $\boldsymbol{X}$. A utility function $u(.)$ will be any function for which the following is fulfilled:

$$(p \succ q \ , (p,q) \in \boldsymbol{P}^2) \Leftrightarrow (\int u(.)dp > \int u(.)dq).$$

In live with Von Neumann and Morgenstern (P. Fishburn, 1970) the interpretation of the above formula is that the integral of the utility function $u(.)$ is a measure for comparison of the probability distributions p and q defined over $\boldsymbol{X}$. The notation ($\succ$) expresses the preferences of DM over $\boldsymbol{P}$ including those over $\boldsymbol{X}$($\boldsymbol{X} \subseteq \boldsymbol{P}$). There are different systems of mathematical axioms that give satisfactory conditions for the existence of a utility function. The most famous of them is the system of *von Neumann and Morgenstern*:

(A.1) The *preference* relation ($\succ$) is transitive, i.e. if ($p \succ q$) and ($q \succ r$) then ($p \succ r$) for all $p,q,r \in \boldsymbol{P}$**;**

(A.2) *Archimedean Axiom*: for all $p,q,r \in \boldsymbol{P}$ such that ($p \succ q \succ r$), there is an $\alpha,\beta \in (0,1)$ such that $((\alpha p + (1-\alpha)r) \succ q)$ and $(q \succ (\beta p + (1-\beta)r))$;

(A.3) *Independence Axiom*: for all $p,q,r \in \boldsymbol{P}$ and any $\alpha \in [0, 1]$, then ($p \succ q$) if and only if $((\alpha p + (1- \alpha)r) \succ (\alpha q + (1- \alpha)r))$.

Axioms (A1) and (A3) are insufficient for the solution of the utility function existence. Axioms (A1), (A2) and (A3) give a solution in the interval scale (precision up to an affine transformation): $((p \succ q) \Leftrightarrow (\int v(x)\mathrm{d}p \succ \int v(x)\mathrm{d}q) \Leftrightarrow (v(x)= \mathrm{a}u(x)+\mathrm{b},\ \mathrm{a,b} \in \boldsymbol{R},\ \mathrm{a}>0))$. The *"indifference"* relation ($\approx$) based on ($\succ$) is defined in (P. Fishburn, 1970):

$$((x \approx y) \Leftrightarrow \neg((x \succ y) \vee (x \prec y))).$$

The presumption of existence of a utility function $u(.)$ leads to the "*negatively transitive*" $(\neg(p \succ t) \wedge \neg(t \succ q)) \Rightarrow \neg(p \succ q))$ and "*asymmetric*" relation ($\succ$). From these properties *leads to the existence of: asymmetry* $((x \prec y) \Rightarrow (\neg(x \succ y)))$*, transitivity* $((x \succ y) \wedge (y \succ z) \Rightarrow (x \succ z))$ *and transitivity of the "indifference" relation* ($\approx$).The transitivity of the relations ($\succ$) and ($\approx$) is violated most often in practice. The violation of the transitivity of the relation ($\succ$) could be interpreted as a lack of information, or as a DM's subjective mistake. The violation of the transitivity of the relation ($\approx$) is due to the natural "uncertainty" of the human's preference and the qualitative nature of expressions of the subjective notions and evaluations (M. Cohen, J-Y. Jaffray, 1988; D. Kahneman, H.Tversky 1979; H. Raiffa, 1968).

There is different utility evaluation methods, all of them based on the "lottery" approach (gambling approach). A "lottery" is called every discrete probability distribution over X. We mark as $\langle x,y,\alpha \rangle$ the lottery: here α is the probability of the appearance of the alternative x and (1-α) - the probability of the alternative y. The most used evaluation approach is the assessment: $z \approx \langle x,y,\alpha \rangle$, where $(x,y,z) \in \boldsymbol{X}^3$, $(x \succ z \succ y)$ and $\alpha \in [0,1]$ (Farquhar P., (1984; Jaffray J-Y, 1988). Weak points of this approach are the violations of the transitivity of the relations and the so called "certainty effect" and "probability distortion", identified by Kahneman and Tversky (D. Kahneman, A. Tversky, 1979) and discussed in Choen and Jaffray (M. Cohen, J-Y. Jaffray, 1988). Additionally, it is very difficult to determine the alternatives x (*the best*) and y (*the worst*) on condition that $(x \succ z \succ y)$, where z is the analyzed alternative. *Therefore, the problem of utility function evaluation on the grounds of expert preferences is a important one*.

The measurement scale of the utility function $u(.)$ originates from the previous mathematical formulation of the relations ($\succ$) and ($\approx$). It is accepted that ($\boldsymbol{X} \subseteq \boldsymbol{P}$) and that $\boldsymbol{P}$ is a convex set $((q,p) \in \boldsymbol{P}^2 \Rightarrow (\alpha q+(1-\alpha)p) \in \boldsymbol{P}$, for $\forall \alpha \in [0,1])$. Then the utility function $u(.)$ over $\boldsymbol{X}$ is determined with the accuracy of an affine transformation (i.e. interval scale) (P. Fishburn, 1970):

Proposition1. If $((x\in X \wedge p(x)=1) \Rightarrow p\in \boldsymbol{P})$ and $(((q,\ p)\in \boldsymbol{P}^2)\ \Rightarrow\ ((\alpha p+(1-\alpha)q)\in \boldsymbol{P},\ \alpha\in[0,1]))$ then the utility function $u(.)$ is defined with precision up to an affine transformation $(u_1(.) \approx u_2(.)) \Leftrightarrow (u_1(.)=au_2(.)+b\ ,\ a>0)$ (in the case of utility function existence).

The first condition in the proposition 1 can be interpreted as an opportunity of the DM to imagine one alternative independently of all the others. The second condition is the opportunity of the DM to report on the uncertainty of the results. This proposition reveals that the utility measurement scale of the utility function is equivalent to the temperature scale (interval scale).

MACHINE LEARNING, UTILITY AND STOCHASTIC APPROXIMATION

Starting from the properties of the preference relation ($\succ$) and indifference relation ($\approx$) and from the weak points of the "lottery approach" we propose the following stochastic approximation procedure for evaluation of the utility function. In correspondence with **Proposition 1** it is assumed that $(\boldsymbol{X}\subseteq\boldsymbol{P})$, $((q,p)\in \boldsymbol{P}^2 \Rightarrow (\alpha q+(1-\alpha)p)\in \boldsymbol{P}$, for $\forall\alpha\in[0,1])$ and that the utility function $u(.)$ exists. We define two sets:

$\boldsymbol{A}_{u^*}=\{(\alpha,x,y,z)/(\alpha u^*(x)+(1-\alpha)u^*(y))>u^*(z)\}$, $\boldsymbol{B}_{u^*}=\{(\alpha,x,y,z)/(\alpha u^*(x)+(1-\alpha)u^*(y))>u^*(z)\}$.

The notation $u^*(.)$ is the DM's empirical utility assessment. The following proposition is fundamental for the stochastic approximation approach:

Proposition2. We denote $\boldsymbol{A}_u=\{(\alpha,x,y,z)/(\alpha u(x)+(1-\alpha)u(y))>u(z)\}$. If $\boldsymbol{A}_{u1}=\boldsymbol{A}_{u2}$ and $u_1(.)$ and $u_2(.)$ are continuous functions, then is true $(u_1(.)=au_2(.)+b,\ a>0)$ (Y. Pavlov, 2005)**.**

The approximation of the utility function is constructed by pattern recognition of the set $\boldsymbol{A}_u$ (M. Aizerman, E. Braverman, L. Rozonoer 1970; Y. Pavlov 2005). The proposed assessment process is a machine-learning approach based on the DM's preferences. The machine learning is a probabilistic pattern recognition procedure because $(\boldsymbol{A}_{u^*}\cap\boldsymbol{B}_{u^*}\neq\emptyset)$ and the utility evaluation is a stochastic approximation with noise (uncertainty) elimination**.** Key element in this solution is Proposition 2.

The following presents the evaluation procedure: *DM compares the "lottery" $<x,y,\alpha>$ with the simple alternative z, $z\in\boldsymbol{Z}$ ("better-$\succ$, $f(x,y,z,\alpha)=1$", "worse-$\prec$, $f(x,y,z,\alpha)=(-1)$" or "can't answer or equivalent- $\sim$, $f(x,y,z,\alpha)=0$", $f(.)$ denotes the qualitative DM's answer). This determine a learning point $((x,y,z,\alpha), f(x,y,z,\alpha))$. The following recurrent stochastic algorithm constructs the utility polynomial approximation* $u(x)=\sum_i c_i\Phi_i(x)$:

$$c_i^{n+1}=c_i^{\,n}+\gamma_n\left[f(t^{n+1})-\overline{(c^n,\Psi\ (t^{n+1}))}\right]\Psi_i(t^{n+1}),\quad \sum_n \gamma_n=+\infty\ ,\ \sum_n \gamma_n^2<+\infty\ ,\forall n,\gamma_n>0\ .$$

In Equation (1) are used the following notations (based on $\boldsymbol{A}_u$):$\boldsymbol{t}=(x,y,z,\alpha)$, $\psi_i(t)=\psi_i(x,y,z,\alpha)=\alpha\Phi_i(x)+(1-\alpha)\Phi_i(y)-\Phi_i(z)$, where $(\Phi_i(x))$ is a family of polynomials. The line above the scalar product $\overline{v=(c^n,\Psi(t))}$ means: $(\overline{v}=1)$, if $(v>1)$, $(\overline{v}=-1)$ if $(v<-1)$, and $(\overline{v}=v)$ if $(-1<v<1)$. The coefficients c_i^n take part in the polynomial representation $g^n(x)=\sum_{i=1}^{n}c_i^n\Phi_i(x)$ and $(c^n,\Psi(t))=\alpha g^n(x)+(1-\alpha)g^n(y)-g^n(z)=G^n(x,y,z,\alpha)$ is a scalar product. The learning points are set with a pseudo random sequence.

The mathematical procedure describes the following assessment process:

The expert relates intuitively the "learning point" $(x,y,z,\alpha))$ to the set $\boldsymbol{A}_{u^*}$with probability $D_1(x,y,z,\alpha)$ or to the set $\boldsymbol{B}_{u^*}$ with probability $D_2(x,y,z,\alpha)$. The probabilities $D_1(x,y,z,\alpha)$ and $D_2(x,y,z,\alpha)$ are mathematical expectations of $f(.)$ over $\boldsymbol{A}_{u^*}$and $\boldsymbol{B}_{u^*}$ respectively, $(D_1(x,y,z,\alpha)=M(f/x,y,z,\alpha))$ if $(M(f/x,y,z,\alpha)>0)$, $(D_2(x,y,z,\alpha)=(-)M(f/\ x,y,z,\alpha))$ if $(M(f/x,y,z,\alpha)<0)$. Let $D'(x,y,z,\alpha)$ is the random value: $D'(x,y,z,\alpha)=D_1(x,y,z,\alpha)$ if $(M(f/x,y,z,\alpha)>0)$; $D'(x,y,z,\alpha)=(-D_2(x,y,z,\alpha))$ if $(M(f/x,y,z,\alpha)<0)$; $D'(x,y,z,\alpha)=0$ if $(M(f/x,y,z,\alpha)=0)$. We approximate $D'(x,y,z,\alpha)$ by a function of the type :

$G(x,y,z,\alpha)=(\alpha g(x)+(1-\alpha)g(y)-g(z))$, where $g(x)=\sum_i c_i\Phi_i(x)$.

The coefficients c_i^n take part in the polynomial approximation of G(x,y,z,α): $G^n(x,y,z,\alpha)=(c^n,\Psi(t))=\alpha g^n(x)+(1-\alpha)g^n(y)-g^n(z)$, $g^n(x)=\sum_{i=1}^{N} c_i^n \Phi_i(x)$. The function $G^n(x,y,z,\alpha)$ is positive over $\boldsymbol{A_{u^*}}$ and negative over $\boldsymbol{B_{u^*}}$ depending on the degree of approximation of D'(x,y,z,α). **The function $g^n(x)$ is the approximation of the utility function u(.).**

The stochastic convergence of the evaluation given by Equation (1) is analyzed in ((M. Aizerman, E. Braverman, L. Rozonoer, 1970) and is described by the following stochastic procedure:

$$c_i^{n+1}=c_i^n+\gamma_n\left[D'(t^{n+1})+\xi^{n+1}-(c^n,\Psi(t^{n+1}))\right]\Psi_i(t^{n+1}),$$

$$\sum_n \gamma_n=+\infty,\ \sum_n \gamma_n^2<+\infty,\ \forall n,\gamma_n>0.$$

The following decomposition is used:

$$f(t^{n+1})=\left[D'(t^{n+1})+\xi^{n+1}\right].$$

The following theorem determines the convergence of the stochastic procedure (1) ((M. Aizerman, E. Braverman, L. Rozonoer, 1970; Y. Pavlov, R. Andreev, 2013):

THEOREM: We denote by $(t^1,...,t^n,..)$ a sequence of independent random vectors $t=(x,y,z,\alpha)$ with one and the same distribution F. We suppose that the sequence of random values $(\xi^1,\xi^2,..,\xi^n,..)$ (Procedure 3) satisfies the conditions: $M(\xi^n/(x,y,z,\alpha),c^{n-1})=0$, $M((\xi^n)^2/(x,y,z,\alpha),c^{n-1})<d$, $d\in R$. It is supposed that the Euclidian norm of $\Psi(t)$ is limited by a constant, $\|\Psi(t)\|<\theta,\theta\in\boldsymbol{R},\theta>0,\ for\ \forall\ t, t=(x,y,z,\alpha)$. The convergence follows from Procedure (3):

$$J_{D'}(G^n(x,y,z,\alpha))=M\left(\int_{D'(t)}^{G^n(t)}(\bar{v}-D'(t))dv\right)=\int\left(\int_{D'(t)}^{G^n(t)}(\bar{v}-D'(t))dv\right)dF\ \xrightarrow[n]{p.p.}$$

$$\xrightarrow[n]{p.p.}\ \inf_{S(t)}\int\left(\int_{D'(t)}^{S(t)}(\bar{v}-D'(t))dv\right)dF\quad.$$

In the theorem above p.p. denotes “almost sure” and M denotes mathematical expectation. The functions $S(t)$ in the limits of the integral belong to L_2 (defined by the probability measure F) and have the presentation described in Equation (2). The integral $J_{D'}(G^n(x,y,z,\alpha))$ fulfills:

$$\int\left(\int_{D'(t)}^{G^n(t)}(\bar{v}-D'(t))dv\right)dF\ \geq\ \frac{1}{2}\int(\overline{G^n(t)}-D'(t))^2dF\ .$$

The proof is based on the extremal approach of the potential function method ((M. Aizerman-E. Braverman-L. Rozonoer 1970).

We know that the utility function is defined in the interval scale (in the proposed conditions) [Fishburn, 1970, Pavlov-Andreev, 2013]. A decision support system for subjective utility evaluation is developed (figure 1). The utility U(.) is approximated by a polynomial:

$$U(x)=\sum_{i=0}^{6} c_i x^i\ .$$

The polynomial representation permits easy mathematical implementation of the utility function and utilization of computers and information systems for mathematical description, optimization and control of complex process with human participation

The proposed procedure and its modifications are machine learning. The computer is taught to have the same preferences as the DM. The DM is comparatively fast in learning to operate with the procedure: session with 128 questions (learning points) takes approximately 45 minutes and requires only qualitative answers “yes”, “no” or “equivalent ”.The learning points $((x,y,z,\alpha),f(x,y,z,\alpha))$ are set with a Sobol’s pseudo random sequence ((M. Sobol 1970).

LEARNER’S MODEL CONSTRUCTION: USE CASES OF MEASUREMENT OF SUBJECTIVE PREFERENCES

In this section we present our assertion that through measurement of human preferences it is possible to determine various learner attributes and in this way to build learner model. In that way it is possible to ensure the realization on value based management of e-learning personalization. Using learner's preferences we can construct objective learner model, since the most objective assessment of an attributes of a learner can be given by himself/herself. The paper presents the determination of an aspect of cognitive ability of a learner – comprehension of knowledge. This ability is determined in the following limits: theoretical presentation of knowledge and example-based presentation. We have made this attempt with two learners using the instrument for measurement of their specific preferences. The results of this measurement concern the choice of proportion of theoretical presentation to the example-based presentation of knowledge.

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