Density control of highway traffic using approached dimension of spatial distribution controller

authors: V. Stoilova, E. Nikolov, N. Nikolova

vassilka_stoilova@hotmail.com; nikoloff@tu-sofia.bg; nina@tu-sofia.bg

Keywords: traffic control; distributed parameter system; approached dimension of spatial distribution controller

I INTRODUCTION

Transport flows to which relates and traffic highways are complex systems to which formalization are used different models. The exact field of application of the models has influence over the concrete dependences. The model chosen for the actual work is proposed first by ***Lighthill*** and ***Whitham*** while independently developed by ***Richards (LWR)***. It is based on the hydrodynamic analogy of traffic to river streams and describes the traffic flow as a one-dimensional compressible fluid. The object of the paper is transport flow, in particular traffic highway, which relates to distributed parameter systems characterized with inner complexity. The complexity of transportation systems demands looking for new methods for modeling and synthesis of control algorithms for these transport systems. The main goal of the paper is to apply non-integer control algorithm, which relates to distributed parameters system. For non-integer control algorithm is used approached dimension of spatial distribution controller.

II. TRAFFIC FLOW MODELING

The relation between speed v and density ρ delivers the correspondent flow-rate q (1). The LWR model is defined by the conservation equation (2), and the suggestion that the speed is known function of the density (3). This expression states that the number of vehicles is constant in each point of a considered highway section.

$$q\left(x,t\right)=\rho\left(x,t\right).v\left(x,t\right) \tag{1}$$

$$\frac{\partial q\left(x,t\right)}{\partial x}+\frac{\partial \rho\left(x,t\right)}{\partial t}=0 \tag{2}$$

$$v\left(x,t\right)=F\left(\rho\left(x,t\right)\right) \tag{3}$$

For solving conservation equation (2), is used ***Green***-function. After solving first order LWR model a transfer function is obtained (4).

$$G(x,\xi,p)\,\hat{=}\,\frac{\dfrac{e^{\frac{-L}{v_f\,(1-2\rho_0)}}}{v_f\,(1-2\rho_0)}}{\left(\dfrac{L}{2v_f\,(1-2\rho_0)}p+1\right)}\,e^{\frac{-L}{2v_f\,(1-2\rho_0)}p} \qquad (4)$$

III. DISTRIBUTED PATAMETERS CONTROL SYSTEM

The object of the paper is the density $\rho(x,t)$ of the traffic flow. It is dynamical distributed parameters system. In control system, a system is called distributed parameter system when being a function of both time and space. Fig.1 shows configuration of highway control system with equivalent dimensional distribution.

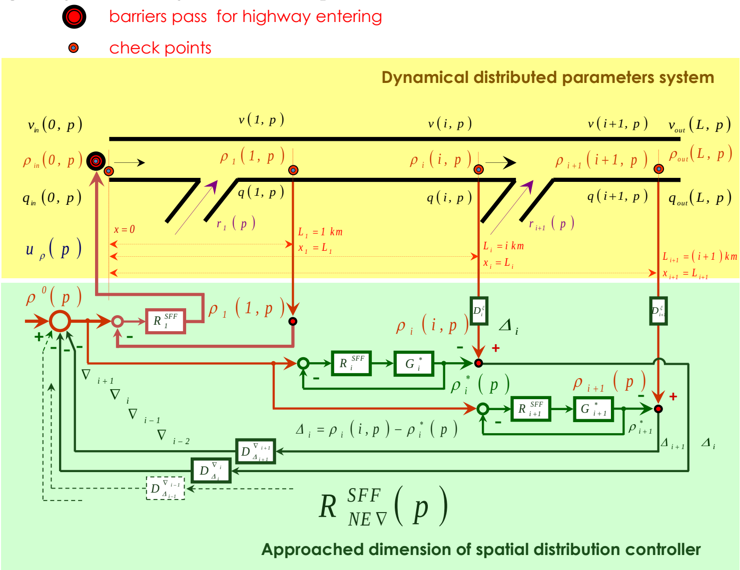

Fig.1. Highway density control system.

The distributed output of the system is the highway traffic density $\rho_i(i,p)$ in every check point $x_i = L_i$ of the highway. The input of the system is density $\rho_{in}(0,p)$ at the beginning section of the highway $(x=0)$. The input density $\rho_{in}(0,p)$ is a function of the number of passing entering barriers at the beginning of the highway, marked with „●”. The system controls the object at i points ●, which are defined in advance for each concrete object. Highway density control system uses $R^{SFF}_{NE\nabla}$ - approached dimension of spatial distribution controller.

Distributed parameters system (Fig.2) consists of ● main controller R^{SFF}_{NE} $(i=1)$, which includes the object at the first $(i=1)$ check point, from all check points i in x; ● Φ^{*}_{SFFi} - models, which are closed loop control systems (concentrated parameters). They use nominal plant $G^{*}_{i}(x,\xi,p),(i\geq 2)$ in previously defined $i-1$ check points in x and $R^{SFF}_{NE\ i}$, $(i\geq 2)$ controller with null dimensional distribution; ● ξ - filters D^{ξ}_{i}, making space transition projection of ξ-projection functions, realizing approached dimension of spatial distribution of $R^{SFF}_{NE\ i}$ controller to distributed parameter object $G\ (x,\xi,p)$; ● x-filters D^{x}_{i}, realizing space transition projection x-projection functions realizing approached dimension of spatial distribution of $R^{SFF}_{NE\ i}$ controller to distributed parameter object $G\ (x,\xi,p)$.

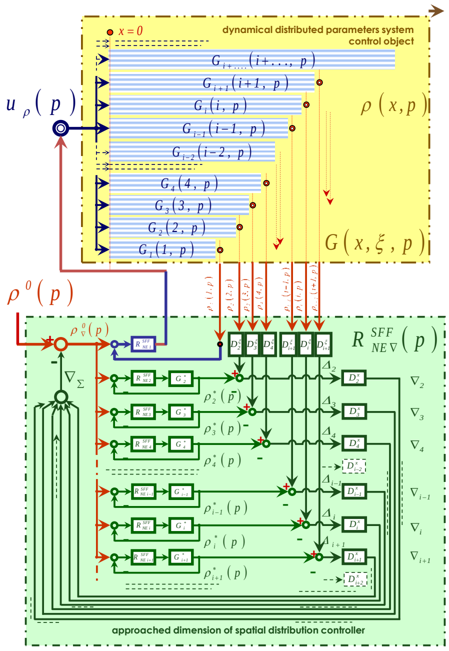

Fig.2. $R^{SFF}_{NE\nabla}$-approached dimension of spatial distribution from non-integer order highway density control system.

IV. RESULTS

For main controller $R^{SFF}_{NE\ i}$ in the structure of approached dimension of spatial distribution controller is used ID_{ne} non-integer controller. In the structure of closed loop

Φ^{*}_{SFFi} -models a PID controllers are used. For concrete conditions step response and Nyquist, Bode and Nichols frequency characteristics of system are shown (Fig.3.):

$$0\,km/h \le m_f \le 220\,km/h;\ \rho_{max}=160\,veh/km;\ 0.25 \le \rho_0 \le 0.4956;\ \rho_{cr}=0.5\rho_{max}=80\ veh/h;\ \rho_{cr_{ramp}}=0.1,$$
$$\rho_{max}=16\,veh/km;\ q_{max}=8800\ veh/km;\ 0\,km \le m \le 6.5\,km$$

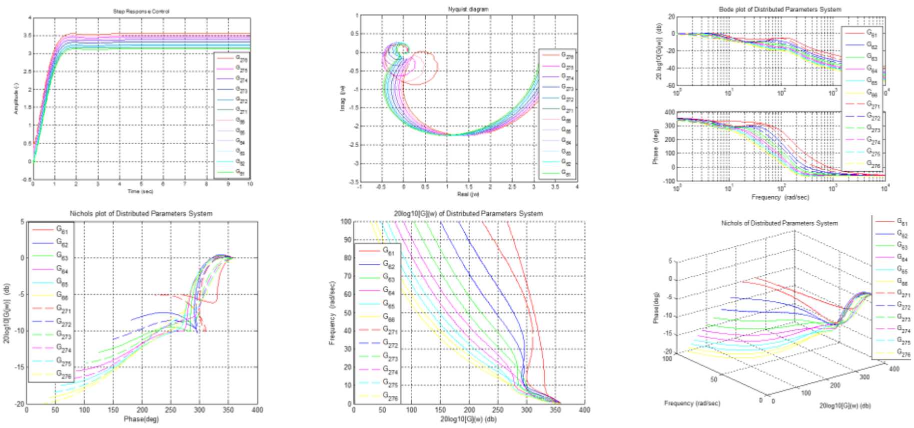

Fig.3. Time and Nyquist, Bode and Nichols frequency characteristics of system.

REFERENCES

[1] **Nikolov E.** (2010), ***Robust Fractional Control (Approaches Predictive and Algebraic, Distributed Control Systems)***, Sofia 2010, © 2010 Publishing House of Technical University of Sofia, 2010, ISBN 978-954-438-851-5, 375 p.

[2] **Nikolov E.** (2010), ***Method and Control System with a Approximated Dimension of the Spatial Distribution***, Automatics and Informatics Journal, 4/2010, © 2009 Union of Automatics and Informatics, ISSN 0861-7562, 32-37

[3] **Nikolov Emil, N. G. Nikolova, V. Stoilova, B. Grasiani** (2012), ***Fractional Control with Approximate Dimension of the Spatial Distribution,*** Journal Proceedings of the Technical University Sofia, © 2012 Publishing House of Technical University of Sofia, ISSN 0374-342X, ISSN 1311-0829, Vol 62, (4), pp. 169-178

[4] **Stoilova V., E. Nikolov, N. Nikolova** (2013), ***Analytical Derive of Second Order model of Payne from First order Lighthil-Whitham-Richards Model,*** Journal Cybernetics and Information Technologies, 2013, Volume 12, No X, ISSN 1311–9702, © 2013 BULGARIAN ACADEMY OF SCIENCES, pp. . 54-62

[5] **Stoilova Vassilka,E. Nikolov** (2014), ***Modeling of the Transport Traffic in the Class Distributed Parameters Plant,*** Journal Proceedings of the Technical University Sofia, © 2014 Publishing House of Technical University of Sofia, ISSN 0374-342X, ISSN 1311-0829, Vol 64, (1), pp. 111-120

[6] **Butkovskiy, A. G.** (1969). ***Distributed control systems.*** New York: © American Elsevier Publishing Company, p. 310

[7] **Butkovskiy A. G.** (1982), ***Green's Functions and Transfer Functions Handbook, first ed.,*** © Ellis Horwood, Chichester, NY, 1982

[8] **Curtain Ruth, Kirsten Morris** (2009), ***Transfer functions of distributed parameter systems: A tutorial,*** Automatica 45 (2009) © Elsevier Publishing Company, 1101-1116