Bi-level Formalization of Urban Area Traffic Lights Control

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I INTRODUCTION

The formal approaches for the control of transportation systems are categorized mainly in two general formal descriptions:
- Store and forward modeling;
- Free way traffic control on high ways.

These two families of models have their domain applications as noted above for urban transportation networks and for high way system of transport network. But the formal background of these types of models is totally different from mathematical point of view.

For the free traffic model the basics of the control address models, applying partly differential equations. The control theory applies first and second order of such equations by means to make relations between the traffic density, traffic flows and the velocity of the vehicles. These models are mainly used for the definition of ramp metering control policy, where the control influence is the new incoming traffic flows from corresponding ramp metering points [4, 5]. Despite the complex formalism and the needs for many additional parameters, which have to be identified both in real time as in off-line environments, the control algorithms successfully implements feedback control [2], model predictive control [6], optimal control, flatness based control [1] and others.

The control of the transportation in urban areas mainly applies relatively simple formalism. It is based on the discontinuity of flows, which refers that the input flows are equal in quantities like the output flows of a transport node. The formal description of these relations presents the store and forward model, which is very easy to describe in the discrete time models. The control low for that case can be easily defined by solving appropriate optimization problems. In more simple case the solution of the optimization problem can give a feedback control loop, which is easily implemented by the technical systems. The optimization problem is defined mainly assuming the cars or waiting queues in the transport network as arguments of the problem. The goal function is a relation from the number of cars or queues and can express physical events like waiting time, time for traveling, and others. The constraints of the problem express the store and forward relations for each node of the urban network. The control influences are the green lights on the traffic crossroad points. Additional control influences could be the duration of the traffic lights cycle and the offset between the cycles on different cross sections. By increase of the control space, the control problem becomes very complicated, which constraints its solution in real time and respectively the practical implementation of such a control policy.

This paper makes an attempt to introduce an innovation approach for the formal description of the urban traffic control. A hierarchical, bi-level model is applied for the definition of the control problem. Such an approach gives potential for the increase of the control space for the control problem. The paper presents a real control case of the town of Enna, Sicily, Italy. Having a particular transportation case of a complex cross-road section, a bi-level formalism is applied for the definition of optimal control problem. The comparisons with the classical optimization problem illustrate partly the potential of the bi-level optimization. The general drawback for the implementation of this innovative formalization could be the lack of fast algorithms for solving these types of problems. This can be an obstacle for the real time control of urban transportation networks.

II Traffic control in urban area by bi-level approach

The hierarchical systems theory is a base for solving global optimization problem which solution is obtained by solving interconnected problems solved at the two levels of the hierarchical system. This manner of solving the global optimization problem is known as bi-level approach [3]. The advantage of its usage is the possibility to find a solution, satisfying the restrictions of the both problems, solved by the two-level hierarchical system. In that manner the obtained solution represents an integration of the both criteria, solved at each hierarchical level. The benefit of the usage of the hierarchical approach is the possibility of unifying both solutions, satisfying both problems' limitations. The formalization of the bi-level approach is given below and it is illustrated in Fig.1.

At the upper level is solved problem (1)

$$\min_x f_x(x, y^*),$$

(1)

$$x \in S_x(y^*) \;,$$

with solution $x^*(y)$, where it is supposed that $y=y^*$ are known parameters. The solution $x^*(y)$ is a function of y. At the same time y is a solution of the optimization problem, solved at the first hierarchical level. Its problem is

$$\min_y f_y(y, x^*)$$

(2)

$$y \in S_y(x^*)$$

The lower level problem has a solution $y^*(x)$ as a function of x assuming $x=x^*$ as known parameters.

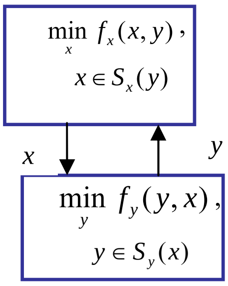

Fig.1. Bi-level hierarchical formalization

The benefit of the hierarchical approach is the fact that these two interconnected optimization problems (1) and (2) give the solution of the global problem

$$\min_x f_x(x,y) \quad (3) \quad x \in S_x(y) \quad y \in \arg\left\{ \begin{array}{c} \min_y f_y(y,x) \\ y \in S_y(x) \end{array} \right\},$$

which solution is optimal both to x and y because according to the hierarchical optimization both optimization functions $f_x(x, y)$ and $f_y(x, y)$ are found. In the classical optimization there is only one optimization function with argument x or y.

The bi-level approach is applied for the traffic light control in order to increase the optimization values. The crossroad, considered in the paper has a topology, which is a critical point in the center of the New Enna town in Sicily, fig.2. Due to the difficulties on the exploitation and management of this urban section, the paper makes appropriate modeling and assessments, applying the bi-level formalism as innovative approach for the traffic control.

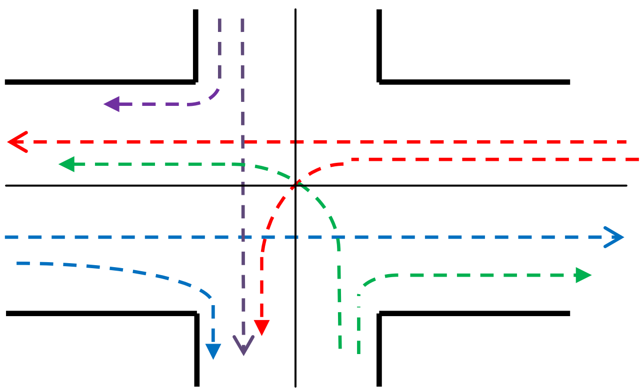

Fig2. Crossroad topology in Enna

The low level problem is defined as classical optimization problem: minimization of the queue lengths in front of the traffic lights. The optimal green durations targets the minimization of the total waiting cars in the system. The control influences are the relative duration of the lights towards the duration of the traffic light cycle. This low level optimization problem is well known and widely used but the duration of the cycle is assumed constant and predefined parameter.

The bi-level formalism allows being defined new optimization problem, which evaluates the optimal duration of the traffic lights cycle. Thus it can vary, if the cars in the corresponding direction increase. In that manner the traffic light cycle becomes an argument for the new bi-level optimization problem. The increase of the control space both with the green light relative duration and the duration of the traffic lights cycle gives additional opportunity to satisfy goals, related with the transport network, which are not performed in the well known optimization problems.

The upper optimization problem targets the maximization of the important part of the outgoing flows. Both problems are interconnected by their decisions but the final solution gives both the optimal durations of the traffic cycle and the green light duration. Comparisons with the case of fixed time cycle and classical optimization case gives proves for the potential of this new formal model for traffic light control.

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