Deblurring Poissonian Images via Multiple-Constraint Optimization

Extended Abstract

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This research deals with the restoration of images corrupted by a non-invertible or ill-conditioned linear transform and Poisson noise. Poisson data typically occur in imaging processes where the images are obtained by counting particles, e.g., photons, that hit the image support. The Poisson distribution exhibits a mean/variance relationship. This mean/variance dependence can be reduced by using variance-stabilizing transformations (VST), one of which is the *Anscombe transform* [1] defined as

$$T\colon [0,+\infty)^n \to (0,+\infty)^n : v = (v_i)_{1\le i\le n} \mapsto 2\left(\sqrt{v_i + \frac{3}{8}}\right)_{1\le i\le n}.$$

It transforms Poisson noise to approximately Gaussian noise with zero-mean and unit variance (if the variance of the Poisson noise is large enough). The Anscombe transform has been employed in order to solve inverse problems where one wants to recover an original signal $\overline{u} \in [0,+\infty)^m$ from observations

$$f = \mathcal{P}(H\overline{u}),$$

where $\mathcal{P}$ denotes an independent Poisson noise corruption process and $H \in [0,+\infty)^{n\times m}$ is a linear degradation operator, e.g. a blur. Note that we consider images of size $M \times N$ columnwise reshaped as vectors of length $m = MN$.

In [2], we solved the Anscombe Total Variation (TV) [3] constraint optimization problem

$$\underset{u\in[0,+\infty)^m}{\text{minimize}} \quad \|\nabla u\|_{2,1} \quad \text{subject to} \quad \|T(Hu) - T(f)\|_2^2 \le \tau_A, \tag{1}$$

where $\nabla \in \mathbb{R}^{2m\times m}$ is the discrete gradient operator (forward differences and Neumann (mirror) boundary conditions were used), and $\|\cdot\|_{2,1}$ denotes the $\ell_{2,1}$ norm. For the purpose, we applied a primal-dual algorithm together with a projection onto the epigraph of a convex function related to the Anscombe transform. We showed that this epigraphical projection can be efficiently computed by Newton's methods with an appropriate initialization.

Based on the statistical properties of the Anscombe transform and the law of large numbers, a consistent choice for the above bound is $\tau_A = n$, provided the size n of the original image $\bar{u}$ is large enough. We denote

$$C_A := \left\{u \in C \;:\; \|T(Hu) - T(f)\|_2^2 \le \tau_A\right\}.$$

The choice of the constraint parameter τ_A is based only on the fidelity term $\|T(Hu) - T(f)\|_2^2$. It places the true image $\bar{u}$ with high probability very close to the boundary of the constraint set C_A, thus guaranteeing the set is non-empty and a minimizer u exists. Moreover, since the TV functional is a semi-norm and every semi-norm is positively-homogeneous, for $\tau_A = n$ our optimization problem admits a solution $u_A \in \partial C_A$, which is unique whenever C_A contains no constant images.

Being an admissible candidate for the solution of the optimization problem is not enough for the initial image $\bar{u}$ to be always "close" to that solution! If the constraint set is too large, then the two images might still differ a lot. This is indeed the case for the numerical examples, considered in [2]. The constraint set C_A is too large and we "oversmooth" the image a lot. Hence, we need to restrict the former.

Even though decreasing τ_A may improve the output of (1), it doesn't seem like a good strategy. We need to adapt the value of τ_A, which is computationally expensive and we lose the nice properties of $\bar{u}$ being close to the boundary of C_A, thus the existence and the uniqueness of the minimizer as well as the possibility of $\bar{u}$ to be that minimizer. Therefore, here we follow a different approach for restricting C_A that is based on subdividing the image domain into smaller regions and using different constraints for them. In such a setup, equation (1) is reformulated into

$$\underset{u\in[0,+\infty)^m}{\text{minimize}} \ \|\nabla u\|_{2,1} \quad \text{subject to} \quad \|T(Hu) - T(f)\|_{A_i} \leq \tau_i, \qquad i = 1, \dots, K$$

where $\{A_i\}_{i=1}^K$ is a tessellation of the image domain ($A_i \cap A_j = \emptyset$), and $\|\cdot\|_{A_i}$ is a short notation for the squared 2-norm, restricted to the region A_i. We consider different options for $\{A_i\}_{i=1}^K$, namely block subdivision, intensity tessellation, and 2-step combined tessellation.

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References

1. F. J. Anscombe. The transformation of Poisson, binomial and negative-binomial data. *Biometrika*, 35:246–254, 1948.
2. S. Harizanov, J.-C. Pesquet, and G. Steidl. Epigraphical Projection for Solving Least Squares Anscombe Transformed Constrained Optimization Problems. In *Scale-Space and Variational Methods in Computer Vision (SSVM 2013)*, A. Kuijper et al., (eds.), LNCS 7893, 125–136, Springer-Verlag: Berlin, 2013.
3. L. I. Rudin, S. Osher, and E. Fatemi. Nonlinear total variation based noise removal algorithms. *Physica D*, 60:259–268, 1992.