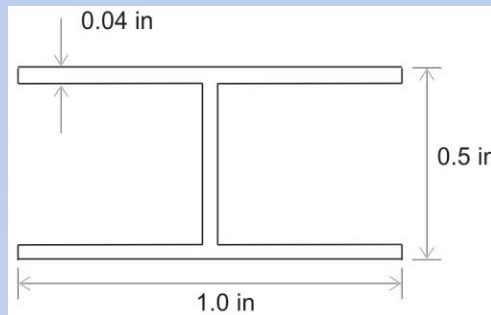
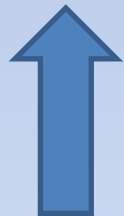
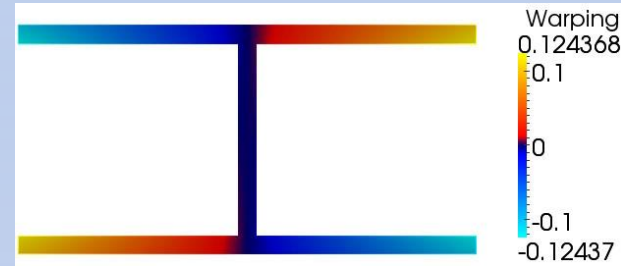


1. Derivation and validation of the equation of motion of complex beams	Planned	Done
Arbitrary cross section	Yes	Yes (TOBECS)
Tapered beams	Yes	Yes
Initially twisted beams	Yes	Yes
Composite materials	Yes	Yes
Rotating beams (including setting angle and hub radii)	No	Yes
Validation of all these models with three-dimensional FEM	Yes	Yes (with Elmer)
Nonlinear normal modes	Yes	Yes (composite beams)
2. Investigation of nonlinear frequency-response diagrams of elastic three-dimensional structures		
Implementation of shooting method within Elmer software	Yes	Yes
Validation of the shooting method	Yes	Yes
Investigation of efficiency and acceleration of Elmer for nonlinear elastic problems	Yes	Yes
Determination of secondary branches	No	Yes
Continuation of secondary branches	No	50 %
Parallel implementation of shooting method	No	50 %

TOBECS (TOol for BEam Cross Sectional analysis) was developed.



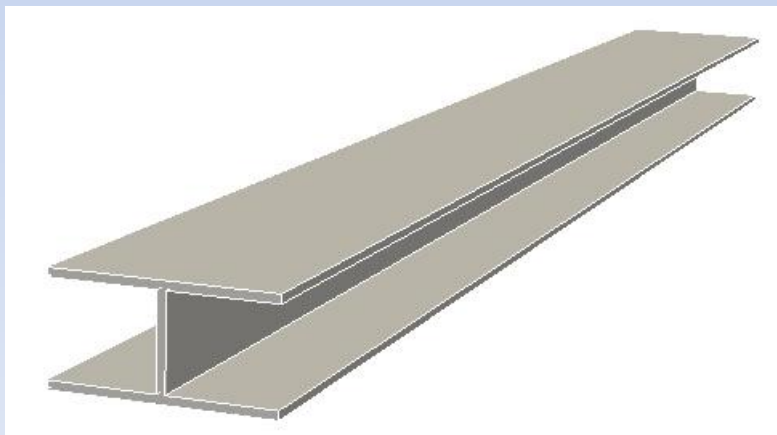
TOBECS



2D cross section
(input - geometry of
the cross section)

output – all cross sectional properties

$$I_{y_c}, I_{z_c}, I_{y_c z_c}, C_{\omega}, I_t, \dots$$



p-FEM

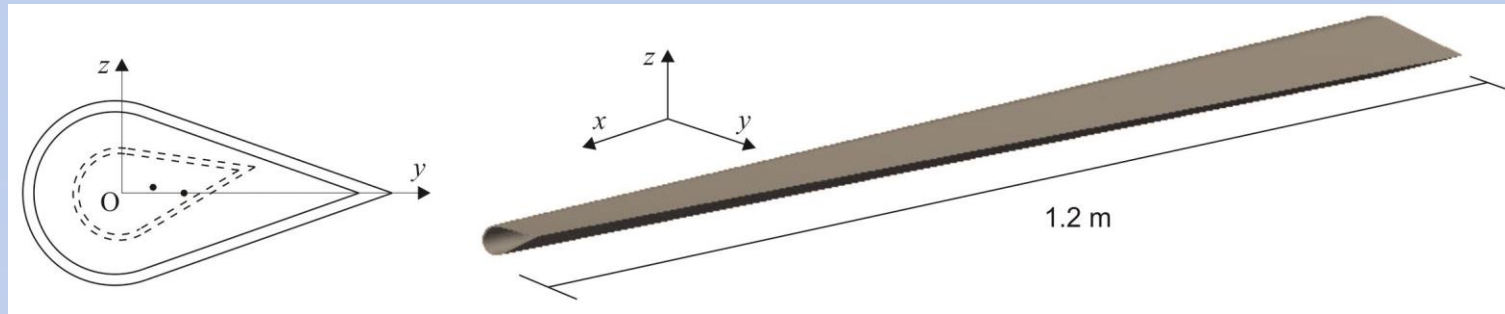


$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}(\mathbf{q})\mathbf{q} = \mathbf{F}$$

Equation of motion

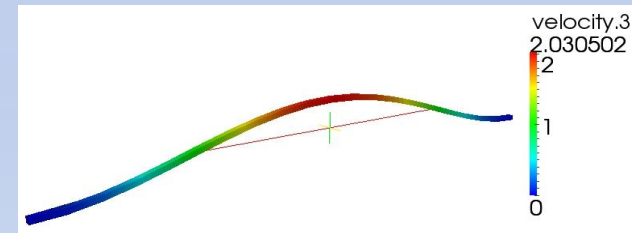
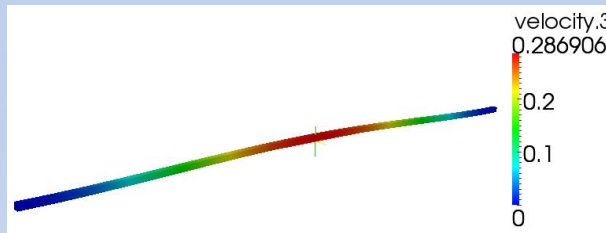
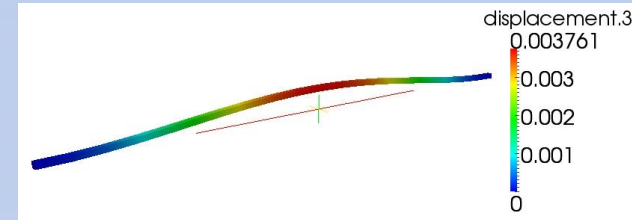
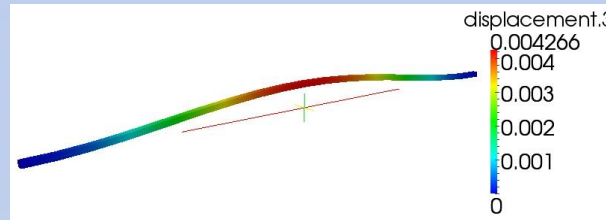
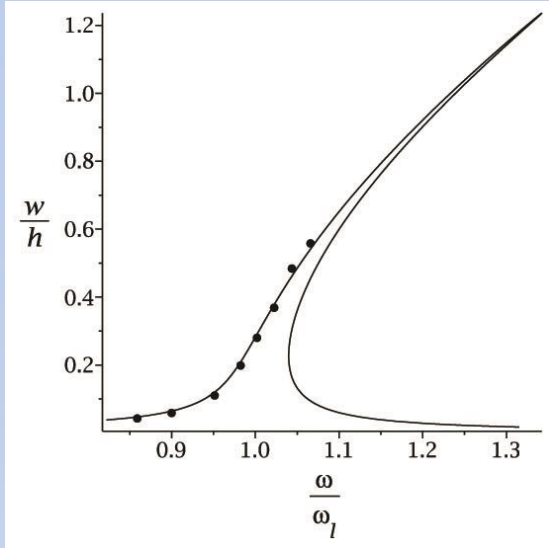


Validation of tapered and initially twisted beam with arbitrary cross section



Mode	Beam model 60 DOF	Elmer 1.2 million DOF	Difference %
1	123.733	123.624	0.09
2	239.118	239.231	0.05
3	523.143	519.622	0.68
4	1003.18	1000.79	0.24
5	1299.46	1284.24	1.18
6	2389.96	2359.37	1.30

Natural frequencies (rad/s)



Validation

— beam model using harmonic balance method

••• shooting method with three-dimensional finite elements by Elmer

$$F = A \cos(168.9 t)$$

$$F = A \cos(183.2 t)$$

Initial conditions of periodic solutions obtained by shooting method, for different excitation frequencies

Published:

- S. Stoykov, S. Margenov, Nonlinear free vibrations of 3D composite beams, In: Z. Dimitrovova, J. Almeida, R. Goncalves (Eds.), Proceedings of the 11th International Conference on Vibration Problems, Lisbon, Portugal, 2013, ISBN: 978-989-96264-4-7, Paper id: 164, 10 pages.

Accepted:

- S. Stoykov, S. Margenov, Nonlinear forced vibration analysis of elastic structures by using parallel solvers for Large-Scale Systems, In: I. Lirkov, S. Margenov, J. Wasniewski (Eds.), Large-Scale Scientific Computing, Lecture notes in computer sciences, Springer, Sozopol, Bulgaria, 2013.
- S. Margenov, S. Stoykov, Y. Vutov, Numerical homogenization of heterogeneous anisotropic linear elastic materials, In: I. Lirkov, S. Margenov, J. Wasniewski (Eds.), Large-Scale Scientific Computing, Lecture notes in computer sciences, Springer, Sozopol, Bulgaria, 2013.
- S. Stoykov, S. Margenov, Nonlinear vibrations of rotating 3D tapered beams with arbitrary cross sections, In: M. Papadrakakis, V. Papadopoulos, V. Plevris (Eds.), Proceedings of the 4th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, Kos, Greece, 2013, Paper id: 1479, 15 pages.
(<http://eccomasproceedings.org/cs2013/pdf/1479.pdf>)

Under review:

- S. Stoykov, E. Manoach, S. Margenov, An efficient beam model based on the p -version finite element method and preliminary cross sectional design, Finite Element in Analysis and Design.