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Linear Plastic Deformation of Solid Body

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Abstract: *This article looks at the types of deformations. Linear plastic deformation is considered. The ISO standard used to calculate the elongation of a solid body is discussed. The deformation of a rod under tension and formulas for calculation are considered. The physical properties of some common metals and alloys in mechanical engineering are shown. The elongation of a deformed test piece with tensile strength is calculated.*

Keywords: *plastic deformation, tensile, compression, extension, test piece.*

1. General concepts

The displacement of the building blocks of the bodies relative to each other and the change in the relative distance between them as a result of the action of forces is called deformation. The rigid bodies are characterized by two borderline cases of elastic and plastic deformations.

Deformations that disappear after the termination of forces are called elastic deformations. Elastic deformations are homogeneous and inhomogeneous. Homogeneous are, for example, the deformations of unilateral and all-round tensile (contraction) as well as slip. The bending and torsional deformations are inhomogeneous [1]. Deformations that remain in the bodies after the termination of forces are called plastic deformations.

Elasticity refers to the desire of bodies to restore their volume and shape, which have been altered by external forces. There are two types of elasticity: volume elasticity and shape elasticity. The volumetric elasticity is inherent in all bodies, while the elasticity of the form is held only by solid bodies. For some solid bodies, the elasticity of the shape is poorly expressed. Such bodies are called plastic.
Mechanical stress is a measure of the intensity of internal forces at a point in the solid body. Measurement unit for mechanical stress is the force per unit area or in basic units of the SI system is N/m^2. This unit is named after the French scientist Pascal and is called Pa, i.e. $1Pa \leqq 1N/m^2$.

The aim of the presented paper is to be analyzed linear plastic deformation of solid body.

2. Internal forces

With a normal shear force N_x (acting on all internal forces), Fig. 1, directed at the outer normal of the cross-section of the rod along the x-axis, i.e. out of the mentally cut part of the rod, it is positive ($N_x > 0$) and then we call the load tensile. In the opposite case, when N_x is directed inwards at the cut off part of the rod, it is negative ($N_x < 0$) and then we call the load pressure. The normal cutting force for the right part of the mentally cut rod is directed opposite to the left and equal in size to it. This also applies to internal forces in general, which is in line with Newton's third law [1, 2, 3, 4, 5, 6].

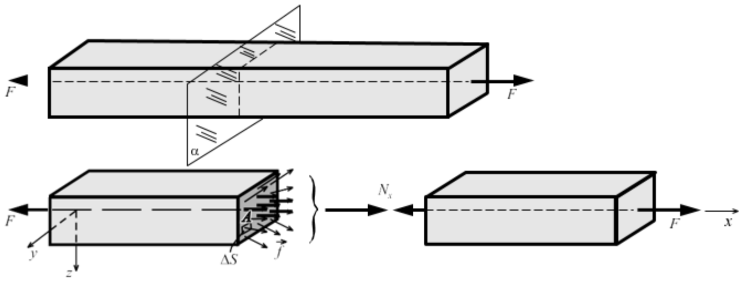

Fig. 1. Separate tensile forces (compression)

2.1. Deformation measure

The deformation of a rod subjected to tensile (pressure) can be best observed if it is of a more elastic material, such as rubber, and a rectangular paint net is applied on the outer surface of the rod. If we tighten the bar at one end and apply a tensile axial force at the other end, as shown in Fig.2, the rod will extend along its axis and bend in the transverse direction. The pressure is the opposite. The mesh retains its orthogonality in this deformation and the straight lines remain straight, with the distance between the transverse lines becoming larger and the distance between the longitudinal lines less. In addition, increasing or decreasing the distance between the respective adjacent lines is the same.

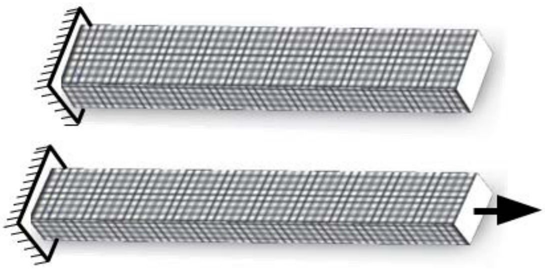

Fig. 2. Deformation of the rod under tension

To schematize the tensile rod of Fig. 2 and the variation of its dimensions as shown in Fig. 3. A rod of length ℓ and transverse size b after deformation acquires length ℓ ′ and transverse size b ′. The displacement of the free end of the bar is denoted by δ. The change in the dimensions of the rod is characterized by the absolute change in the length $\Delta\ell = \ell' - \ell$ and in the transverse dimension $\Delta b = b' - b$, as well as in the relative change in the length $\Delta\ell / \ell$ and the transverse dimensions $\Delta b / b$. An arbitrary segment of elementary length dx and oriented along the axis of the rod x, after deformation, is again in the direction of its axis, but there is already a new length dx ′ [1,2,4,5].

Linear deformation is a technical measure of deformation (dilation), which represents the limit of the relative change in the length of an elementary segment at a point and the direction it characterizes as the length of the segment decreases.

In our case, the linear deformation along the x-axis is:

$$\varepsilon_{\mathrm{x}} = \lim_{dx \to 0} \frac{dx^1}{dx} \qquad (1)$$

We denote the linear deformation by the Greek letter ε and the index defining the direction it characterizes. Linear deformation is a dimensionless dimension.

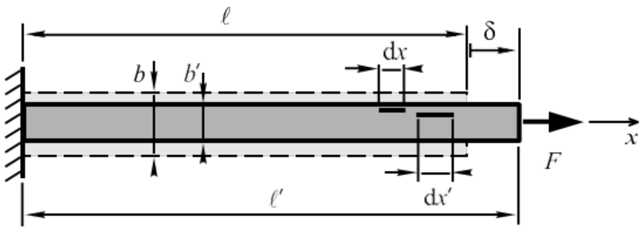

Fig. 3. Changing the dimensions of the tensile rod

The linear deformation along the x-axis for the tensile rod is the same at each point according to the deformation of the straight orthogonal mesh, and is positive, i.e. $\varepsilon x > 0$. Then the longitudinal linear deformation can be calculated from the relative elongation of the rod.

$$\varepsilon_x = \frac{\Delta l}{l} \qquad (2)$$

The transverse deformation in any transverse direction on the axis of the rod is also the same for any point on the rod and is negative in tension, ie. $\varepsilon < 0$. It can also be calculated from the relative change in transverse dimensions.

$$\varepsilon = \frac{\Delta b}{b} \qquad (3)$$

When pressed, the signs of linear deformation change - negative longitudinal linear deformation and positive transverse linear deformation.

2.2. Relationship between deformations, stresses and displacements

- Hooke's law - the linear dependence of displacements in a rigid body on external forces when deformed.

In our case of rod fixing and load application, we can write this law as a dependency:

$$\delta = kF, \qquad (4)$$

where: k is the coefficient of proportionality.

Depending on the linear dependence of movements on forces, the principle of independent action of forces immediately follows, or as it is called, the principle of superposition.

- The principle of superposition is the effect of the action of a system of forces, the sum of the effects of each of the forces acting individually.

The displacement of the free end of the rod δ as well as the linear deformations in the rod depend not only on its strength but also on its size. In order to ignore the influence of the dimensions and to study the behavior of the material of the rod under this type of loading - tensile or compression, it must be found the dependence between the stresses, as a measure of the intensity of the internal forces in the material, and the observed linear deformations, as a measure for deforming the material [4, 5, 6, 9, 10].

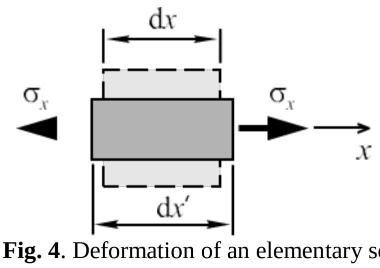

Fig. 4. Deformation of an elementary square

If the deformation of a simple square of a rectangular one grid applied to the rod (Fig. 4), then linear deformations along the axis of the rod x can only be caused by normal stresses σx acting in the cross section. Due to the uniform longitudinal linear deformations εx in the rod, it will have a uniform distribution of the normal stresses σx in the cross sections of the rod, as shown in Figs. 5 on the mentally separated right part. It follows from the uniform distribution of the normal stresses that they can be determined in the most elemental way by the normal shear force:

$$\sigma_x = \frac{N_x}{S}, \qquad (5)$$

where S is the face of the cross-section of the rod. In the testing of the tensile and compressive materials, which will be discussed below, a linear relationship between the normal stresses in the cross-sections and the linear longitudinal deformations at not very high loads has been established.

- Hook's law of pure tensile (compression)

$$\sigma_x = E\,\varepsilon_x\ , \qquad (6)$$

Where E is a material constant, called the modulus of elasticity of linear deformations (also a Jung module or a module of the first kind), with a basic unit of measure in the SI Pa system, but because of its usually very large value it is most often used unit of measure GPa (gigapascal 10^9 Pa).

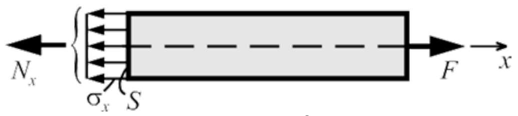

Fig. 5. Distribution of normal tensions

The transverse deformations ε are always opposite in sign of the longitudinal εx. It has been experimentally found that the ratio of the magnitudes of these deformations depends only on the properties of the material. The dependence of the transverse deformations on the longitudinal is expressed by Poisson's law of pure tensile (compression),

$$\bar{\varepsilon} = \mu\varepsilon_x, \qquad (7)$$

where μ is a dimensionless material constant, called the Poisson coefficient (also the coefficient of transverse linear deformation) and has a value of $-1 \leq \mu \leq 0.5$ (most known materials have μ> 0).

Apart from the action of external forces, solid bodies also change in size as a result of changes in temperature. They expand with increasing temperature and contract with decreasing temperature. This dependence has also been found to be linear. It can be defined this relative change in the size of solid bodies as a result of the change in temperature ΔT as a **temperature deformation**

$$\varepsilon_T = \alpha\Delta T \qquad (8)$$

where α is a material constant called the coefficient of linear temperature expansion and has a basic unit of measure in the SI - K − 1 system (Kelvin minus the first one equal to 1/°C) [1, 3, 4, 5].

The physical properties of some common metals and alloys in mechanical engineering are given in Table 1.

Table. 1. Material constants of some metals and alloys

Material	Modulus of elastisity E, GPa	Poisson's coefficient μ,	Coefficient of temperature expansion α, $\times 10^{-6}\,K^{-1}$
aluminum	70	0,33	23,0
bronze	96÷120	0,34	18,0÷21,0
copper	110÷120	0,33÷0,36	16,6÷17,6
brass	100	0,34	19,1÷21,2
steel	190÷210	0,27÷0,3	10,0÷18,0
cast iron	83÷170	0,20÷0,30	9,9÷12,0

The linear deformations, longitudinal and transverse, are the same at all points of the tensioned (compressed) rod. Therefore, the normal stresses at all points of the cross sections and in all cross sections are the same. The tensions at one point determine its stress state. When all points of the exerted load of a solid body are characterized by equal magnitudes of stress, we say that the stress state in the solid body is homogeneous.

3. Application of deformation in tensile strength of a solid body

Deformation testing (extension) of a test body is performed on tensile strength testing machines. When testing a low carbon steel material, a typical graph is obtained as shown in Fig. 6. When the initial parameters of the test piece are given, so called specimen Fig.7. The resulting graph, it can be calculated the extension at maximum force and the extension after breaking it [2].

In Fig. 7. a) test piece with a widening clip is shown in the jaws of the testing machine. In subparagraph (a), we see a view of a specimen before rupture, where Lo is the initial overall length, expressed in mm.

In subparagraph (b), the same test piece is shown after rupture by breaking the specimen in two, where L is the length after testing [5, 6].

Figure 7 shows a graph that is typical for tensile strength testing. The definitions of extension required for the analysis of a test piece are shown where:

- A is the percentage extension after rupture (calculated from the extensometer or directly from the sample) -)- $A_{\%} = \frac{Lu - Lo}{Lo}$x100;

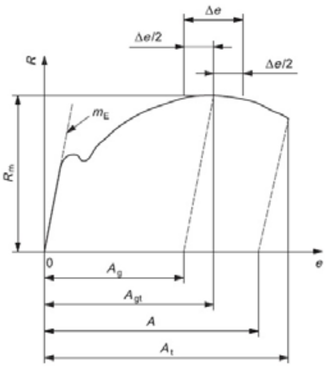

Fig. 6. Graph tension - percentage extension

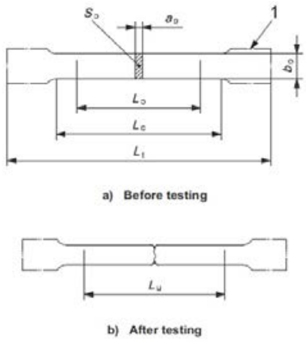

Fig. 7. Mechanically made test pieces of rectangular cross-section: a) before testing, b) after testing

- Ag is a plastic extension at maximum force
- $Ag_{\%} = (\frac{\triangle Lm}{Le} - \frac{Rm}{mE})x100$;
- Agt is the total percentage extension at maximum force- $Agt_{\%} = \frac{\triangle Lm}{Le}x100$;
- At is the total percentage extension after total rupture;
- "*e*" is a percentage extension; $At_{\%} = \frac{\triangle Lf}{Le}x100$;
- m_E is the slope of the elastic part of the graph tension-percentage extension;
- *R* is a tension;
- *Rm* is the tensile strength- $Rm_{\%} = \frac{Fmax}{So}x100$;
- $\triangle e$ is an extension zone;
- *Le* is the measuring range (extensometer).

Table 2 show an example for calculating a test piece extension for a tensile strength of a low carbon steel specimen. The tests were performed on a WDW-300 testing machine manufactured in China.

Table 2. Initial, final and calculated parameters for extension of the test piece

Initial parameters		Parameters after rupture		Calculated parameters	
a, mm	20	Fm, kN	8.28	Rm, MPa	360
b, mm	1.15			A, %	32
Lo, mm	80	Lu, mm	106	At,%	31.5
So, mm	23			Agt,%	22,5
Le,mm	120	m_E,MPa	240	Ag,%	21

4. Conclusion

This analysis deals with the deformation of a tension rod and the laws that apply - namely Newton's law, Hook's law, Jung's module, Poisson's law. All these laws apply to the calculation of the deformation parameters of a test specimen under tensile strength. The extension at maximum force and after the end of breakage of the test body is calculated. The analysis was performed on a WDW-300 testing machine. Future step is to be investigated the material after tension with computed tomography, and during the test it will be performed also temperature tests [7, 8].

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Линейная пластическая деформация твердого тела

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Аннотация: *В данной статье рассматриваются виды деформаций. Рассматривается линейная пластическая деформация. Обсуждается стандарт ISO, используемый для расчета удлинения твердого тела. Рассмотрены деформации стержня при растяжении и формулы для расчета. Показаны физические свойства некоторых распространенных металлов и сплавов в машиностроении. Рассчитано удлинение деформированного образца для испытаний с пределом прочности на разрыв.*

Ключевые слова: *пластическая деформация, растяжение, сжатие, растяжение, образец для испытаний.*