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Kinematics of Mobile Robots with Three Active Wheels

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Abstract: *We give the solution of the direct and inverse kinematic problems for mobile robots with thee symmetric active wheels. The results can be implement in the control of such robots in the form of state feedback synthesized by using linear-quadratic optimization and/or robust control techniques.*

Keywords: *robotic arm, mobile platform, kinematic model, control, motion planing*

1. Statement of the problem

We study the kinematic scheme of a 3-wheel mobile robot (MR) proposed in an early paper [2] of the authors, see also [1, 4]. The solution of the direct and inverse kinematic problems for such robots is important in the framework of robust dynamic control [3].

Let XOY be a coordinate system, where each point A with coordinates x; y is identified by the vector

$$OA = [x; y]$$

We shall deal with symmetric MR with center A and three drive axes with length a. The centers of the wheels of radius r are projected at the points

$$O_i = [xi; yi]$$

(throughout the paper the index i takes values 1; 2; 3), which form an equilateral triangle, i.e.

$$|AO_i| = a$$

And

$$x_i = x + a\cos\varphi_i \qquad (1)$$
$$y_i = y + a\sin\varphi_i$$

Setting

$$\varphi_1 = \varphi$$

we have

$$\varphi_2 = \varphi + 2\pi/3, \varphi_3 = \varphi + 4\pi/3$$

The position and orientation of MR are de_ned by three parameters x, y and φ. The control variables are the angles ψ_i of rotation of the wheels, or the angular velocities $\omega = \psi_i$ Denote by

$$v = [\dot{x}, \dot{y}]$$

and

$$v_i = [\dot{x}_i, \dot{y}_i]$$

the (vector) velocities of the points A and Oi, respectively. Let $X_iO_iY_i$ be local coordinate systems associated with each wheel such that the ordinate axis O_iY_i is the line AO_i. Now we may decompose v_i as

$$v_i = u_i + w_i$$

where u_i and w_i are the projections of v_i on the axes O_iX_i and O_iY_i, respectively. Note that

$$|u_i| = r|w_i|$$

Furthermore it follows from (1) that

$$\dot{x}_i = \dot{x} - a\dot{\varphi}\sin\varphi_i \qquad (2)$$
$$\dot{y}_i = \dot{y} - a\dot{\varphi}\cos\varphi_i$$

Setting

$$v_i^2 = \dot{x}_i^2 + \dot{y}_i^2$$

we get

$$v_i^2 = v^2 + a^2\dot{\varphi}^2 - 2a\dot{\varphi}(\dot{x}\sin\varphi_i - \dot{y}\cos\varphi_i)$$

where

$$v_i^2 = \dot{x}^2 + \dot{y}^2$$

The vectors u_i and v_i may be represented as

$$u_i = \lambda_i(\sin\varphi_i - \cos\varphi_i)$$
$$w_i = \mu_i(\cos\varphi_i - \sin\varphi_i)$$

where

$$\lambda_i = \dot{x}\sin\varphi_i - \dot{y}\cos\varphi_i - a\dot{\varphi} \qquad (3)$$
$$\mu_i = \dot{x}\cos\varphi_i + \dot{y}\sin\varphi_i$$

The above relations yield

$$v_1 + v_2 + v_3 = [\dot{x}_1, \dot{y}_1] + [\dot{x}_2, \dot{y}_2] + [\dot{x}_3, \dot{y}_3] = 3v = 3[\dot{x}, \dot{y}]$$

And

$$u_1 + u_2 + u_3 = w_1 + w_2 + w_3 = \frac{3v}{2}$$

2. Inverse kinematic problem

Let the variables

$$x = x(t), y = y(t), \varphi = \varphi(t)$$

be given as functions of the time t on a certain interval $[t_0; t_1]$ and set $x_0 = x(t_0)$, etc. Then the solution of the inverse kinematic problem for the angular velocities of the wheels is given by

$$\omega_i = \dot{\psi}_i = \frac{\lambda_i}{r} = \frac{\dot{x}\sin\varphi_i - \dot{y}\cos\varphi_i - a\dot{\varphi}}{r} \qquad (4)$$

or, in a more detailed form

$$\omega_1 = \frac{\dot{x}\sin\varphi_i - \dot{y}\cos\varphi_i - a\dot{\varphi}}{r}$$

$$\omega_2 = \frac{\dot{x}(\sqrt{3}\cos\varphi - \sin\varphi) + \dot{y}(\cos\varphi + \sqrt{3}\sin\varphi) - 2a\dot{\varphi}}{2r}$$

$$\omega_3 = \frac{-\dot{x}(\sqrt{3}\cos\varphi + \sin\varphi) + \dot{y}(\cos\varphi - \sqrt{3}\sin\varphi) - 2a\dot{\varphi}}{2r}$$

3. Direct kinematic problem

If the functions

$$\omega_i = \omega_i(t)$$

are given then the expressions for $\dot{x}$, $\dot{y}$ and $\dot{\varphi}$ are obtained using (3) as

$$\dot{\varphi} = -\frac{r}{3a}(\omega_1 + \omega_2 + \omega_3)$$

and

$$\dot{x} = \frac{r}{\sqrt{3}}(\omega_2 - \omega_3)\cos\varphi + \frac{r}{3}(2\omega_1 - \omega_2 - \omega_3)\sin\varphi \qquad (5)$$

$$\dot{y} = \frac{r}{\sqrt{3}}(\omega_2 - \omega_3)\sin\varphi - \frac{r}{3}(2\omega_1 - \omega_2 - \omega_3)\cos\varphi$$

Hence

$$\varphi = \varphi_0 - \frac{r}{3a}(\psi_1 + \psi_2 + \psi_3) \qquad (6)$$

where the functions x, y are obtained by integration of the right-hand sides of (5) taking into account (6). The projections μ_i of the velocities v_i along the axes AO_i are given by

$$\mu_1 = \frac{r}{\sqrt{3}}(\omega_2 - \omega_3)$$

$$\mu_2 = \frac{r}{\sqrt{3}}(\omega_3 - \omega_1)$$

$$\mu_3 = \frac{r}{\sqrt{3}}(\omega_1 - \omega_2)$$

And

$$v^2 = \frac{2}{3}(\mu_1 + \mu_2 + \mu_3)$$

Case ω_i= const. In this particular case we have

$$\varphi = \varphi_0 - \omega_0 t$$

Where

$$\omega_0 = \frac{r\Omega}{3a}$$

And

$$\Omega = \omega_1 + \omega_2 + \omega_3$$

Case $\Omega = 0$. Here the MR is moving along a straight line without rotation. The coordinates of its center are

$$x = x_0 + mt$$

$$y = y_0 + nt$$

Where

$$m = r(\frac{\omega_1 + 2\omega_2}{\sqrt{3}} \cos\varphi_0 + \omega_1 \sin\varphi_0)$$

$$n = r(\frac{\omega_1 + 2\omega_2}{\sqrt{3}} \sin\varphi_0 - \omega_1 \cos\varphi_0)$$

and the velocity is determined from

$$|v| = \sqrt{m^2 + n^2} = 2r\sqrt{\frac{\omega_1^2 + \omega_1\omega_2 + \omega_2^2}{3}}$$

Case $\Omega \neq 0$. Here we have

$$\dot{x} = v\cos(\varphi_0 + \theta - \omega_0 t)$$

$$\dot{y} = v\sin(\varphi_0 + \theta - \omega_0 t)$$

Where

$$v = \frac{2r\Omega_1}{3}$$

$$\tan\theta = \frac{\omega_2 + \omega_3 - 2\omega_1}{\sqrt{3}(\omega_2 - \omega_3)}$$

And

$$\Omega_1 = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2 - \omega_1\omega_2 - \omega_2\omega_3 - \omega_3\omega_1}$$

Therefore

$$x_C = x_0 + \frac{v}{\omega_0}\sin(\varphi_0 + \theta)$$

$$y_C = y_0 - \frac{v}{\omega_0}\cos(\varphi_0 + \theta)$$

And

$$R = \frac{3av}{r|\Omega|} = \frac{a\Omega_1}{|\Omega|}$$

4. Conclusion

We have presented the complete solutions of the direct and inverse kinematic problems for 3-wheeled symmetric mobile robots. The results obtained can be used for the purpose of dynamic control of such robots.

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Кинематика мобильных роботов с трех активными колесами

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Аннотация: *Мы даем решение прямых и обратных кинематических задач для мобильных роботов с симметричными активными колесами. Результаты могут быть реализованы при управлении такими роботами в форме обратной связи состояния, синтезированной с использованием методов линейной квадратичной оптимизации и / или надежного управления.*

Ключевые слова: *роботизированная рука, мобильная платформа, кинематическая модель, управление, планирование движения*