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Comparison of Various Space-Filling Curves for Lossless Data Hiding Using a Modified Histogram

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Abstract*: In the area of data-hiding and digital watermarking there is a special group of approaches to embed data that are based on the specific property of the cover image – the histogram. In this group of methods a very promising subset comprises the methods, using histogram modification, especially those that exploit the high correlation between adjacent pixels. In order to take full advantage of this statistical phenomenon, the 2D representation of the image data should be transformed to 1D array of values, applying a space-filling curve over the discrete pixel plane. This paper focuses on comparing various curves for better understanding of their use and applicability. First the space-filling curves are introduced with their specific characteristics. In the second part the data-hiding method using a histogram modification is described. The third part compares the space-filling curves analytically. The last part presents the experimental results of applying every curve through data-hiding methods and concludes the results.*

1. Introduction

In the image data-hiding area numerous methods have been developed for embedding message data within visual information with the idea to make it undetectable and an integral part of the cover image. The general separation of these methods is according to the domain they are operating in – frequency ([4]) or spatial ([1]-[3]). Usually the latter group is considered more simplistic as it does not require any transformation from one domain to the other and back. In the methods utilizing the spatial domain there is a distinct class that relies on the histogram of the image and its distinct properties (usually the extreme values). Such a method is the method of Ni [1], a lossless data-hiding algorithm using the histogram minimum and maximum of the image. As an improvement in the transferred data capacity of this method there is another basic approach that uses a modified histogram based on the difference between neighboring pixels. Any method based on this approach tries to take the maximum out of the fact that in an ordinary photograph the adjacent pixels are highly correlated, i.e., the difference between them is stacked around the beginning of the histogram coordinate system. This approach has been described in details in [12].

In order to take full advantage of the neighboring pixels it is of great importance how the image is iterated through. This determines the adjacent values by transforming and ordering the values in an one-dimensional vector. For this reason it is important which space-filling curve will be used to go through the image pixel values. Preference should be given to those curves that go through directly adjacent pixels.

2. Space-filling curves

The space-filling curves that should be used to iterate through the image should have some obligatory features in order to be deemed appropriately.

First of all they should not go through one and the same pixel more than once. Otherwise this would result in wrong encoding of the message.

Another important feature, which is not obligatory, is that Hausdorff dimension [8] must be equal to 2. This means that the space-filling curve covers the whole pixel plane, which in return may result in a better embedding capacity.

In this paper 6 different space filling curves will be investigated:

1. Left-to-right-top-to-bottom (zig-zag) (Fig. 1)
2. Reverse-S space-filling curve (Fig. 2)
3. Spiral-like centric space-filling curve (Fig. 3)
4. Z-order space-filling curve (Fig. 4)
5. Hilbert space-filling curve (Fig. 5)
6. Peano space-filling curve (Fig. 6)

For simplicity of description we will assume that the curves fill a square area with size equal to n. Furthermore, the length of a path $\mathbf{P}$ is the sum of the Euclidean distance between the neighboring pixels, i.e., the total distance a cursor would travel, if they were following the iteration through the image pixels.

2.1. Left-to-right-top-to-bottom space-filling curve

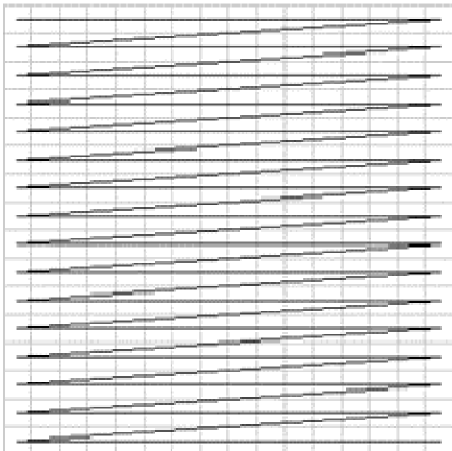

Fig. 1. Left-to-right-top-to-bottom space-filling curve

This iteration method is the simplest to implement as it goes in a very natural way through the image pixels.

Nevertheless, it is very impractical in the current case since the transition between the end of the row up to the beginning of the next row is not optimal when tracking adjacent pixels. The only way this space-filling curve to be efficient is if the left and right edges of the image have similar pixel values.

The length of the path is measured by the following formula – $P = \left(n + \sqrt{n^2 + 1} - 1\right)n$.

2.2. Reverse-S space-filling curve

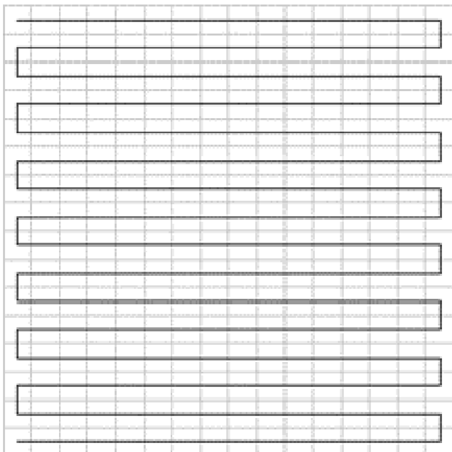

Fig. 2. Reverse-S space-filling curve

The reverse S space-filling curve is the improved version of the left-to-right-top-to-bottom space-filling curve. The most important characteristic is that the path goes through neighboring pixels only. Going from a previous to the next row is done by the corresponding pixel in the next row and reversing the iteration direction.

This path offers the ideal conditions for the histogram modification data-hiding algorithm based on the adjacent pixel difference. Apart from this it is simple and easy to implement.

The path length in pixels is measured through the following formula – $P = (n-1)(n+1)$.

2.3. Spiral-like centric space-filling curve

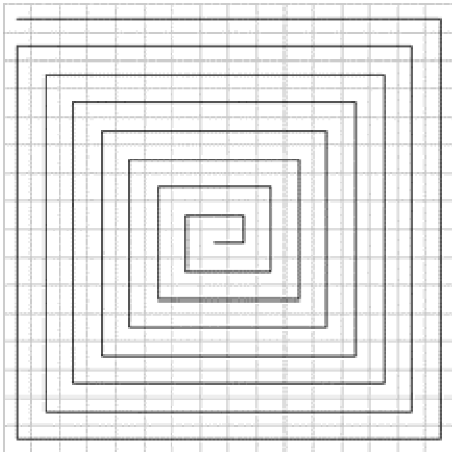

Fig. 3. Spiral-like centric space-filling curve

This space-filling curve iterates through the pixels from the outer border inwards towards the center of the image.

This method has improved locality in the center of the image. If there is a region of similar pixel values, then the result would be optimal, as the correlation between these pixels will be very exploited.

The behavior around the borders could be compared with that of the reverse-S space-filling curve.

The path length in pixels is measured through the following formula – $P = (n-1)(n+1)$.

2.4. Z-order space-filling curve

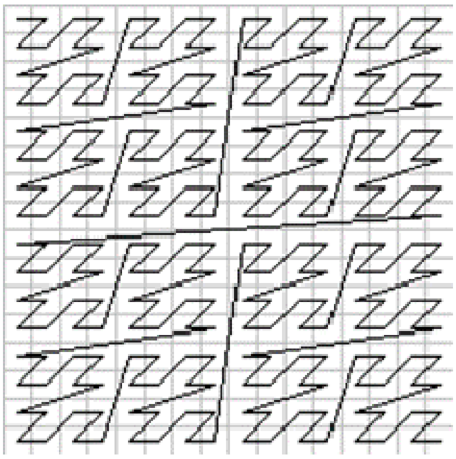

Fig. 4. Z-order space-filling curve

Z-order, Morton order, or Morton code is a function which maps multidimensional data to one dimension while preserving locality of the data points. It was introduced in 1966 by G. M. Morton [8].

It is extremely easy to implement this space-filling curve since the position is calculated simply by interleaving the binary representations of its coordinate values. Let us assume that x is represented in a binary form by the binary string – $x_0x_1x_2x_3$. The same goes for y – $y_0y_1y_2y_3$. The position in the z-order path will be $y_0x_0y_1x_1y_2x_2y_3x_3$. For example the image position (5, 2), will be at position 25 of the z-order path, since 5 (binary 101) and 2 (binary 010) will become 25 (binary 011001). Please, note that in order these calculations to be correct the dimension should start from 0, not 1.

Z-order is mainly used in storing data for fast searches due to the good locality it offers. Still, for the purpose of maximizing the correlation between neighboring pixels, it does not offer the greatest results as leaving one cluster of Zs, a jump occurs which breaks the adjacent iteration among the pixel values.

2.5. Hilbert space-filling curve

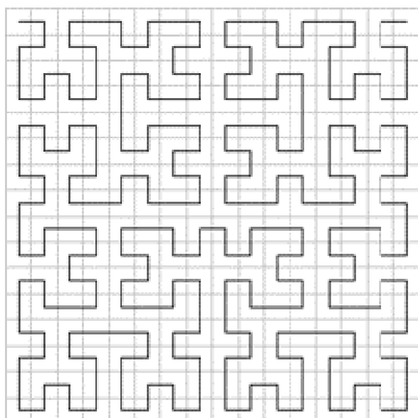

Fig. 5. Hilbert space-filling curve

A Hilbert curve (also known as Hilbert space-filling curve) is a continuous fractal space-filling curve first described by the German mathematician David Hilbert in 1891, as a variant of the space-filling curves discovered by Giuseppe Peano in 1890.

It consists of a sequence of curves defined iteratively. This space-filling curve offers great locality as it exploits the correlation between pixels within small regions and thus it can be more robust in certain scenarios.

The length of the path in pixels can be expressed through the following formula – $P = n^2 - 1$

2.6. Peano space-filling curve

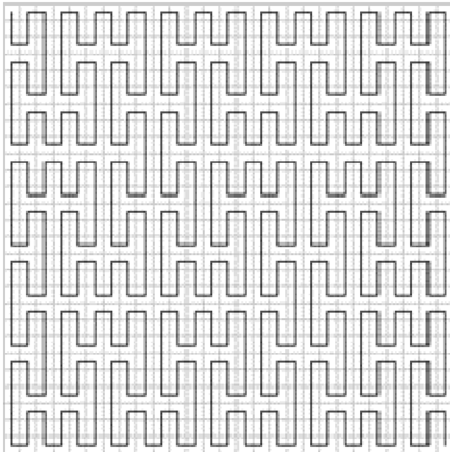

Fig. 6. Peano space-filling curve

Although all space-filling curves that map a plane are known under the name Peano, there is one distinct construction that was the original work of the Italian mathematician Giuseppe Peano.

It result is a curve with a high degree of self-similarity. The original Peano space-filling curve is again based on iterative constructs.

The major difference between this curve and the others is that this one has 3 as a base, opposed to the Hilbert and *Z*-order, which are based on dimensions that are powers of 2.

The length of the path in pixels is represented in the same way as Hilbert space-filling curve – $P = n^2 - 1$

2.7. The embedding algorithm using a modified histogram

One needs to use a modified approach in order to construct a histogram so that the correlation of neighboring pixels can be utilized; a much higher peak point can occur, which is exactly the property used to embed the message by. The difference between the adjacent pixels is the property that is registered in the histogram. As this difference is more frequently around 0, then the peak point in the histogram should be expected to be at the beginning.

Once the modified histogram has been constructed and there is a prominent peak point, the embedding of the message can be accomplished [12]. The basic idea is pretty straight-forward – clear the pixel value position after the peak point by extending any difference larger than that by 1. Since this value is cleared, it will be the perfect candidate for storing the 1s in the data bit stream. The 0s will remain in the peak point difference. The embedding is done by traversing the image in the same way as the histogram has been constructed. If a peak point difference is met, an embedding should occur. If a 0 should be embedded, then do nothing, otherwise extend the difference by 1, thus occupying the peak point at +1 position.

The decoding part is the same as the embedding. Knowing the peak point, one should traverse the image and if an embedding position is found, extract the respective bit and add it to the message bit stream. Every difference greater than the peak point difference should be shrunk by 1 in order to restore the original cover work.

2.8. Comparison of space-filling curves

Since the algorithm is defined and the importance of having an optimal space-filling curve is clear, it is very important to investigate the advantages and disadvantages of each of the space-filling curves presented in this paper.

The basic way of categorizing the space-filling curves is the way they handle locality. The first group of such curves are those that do not handle locality at all, i.e., they do not intentionally strive to exploit smaller regions while iterating. Such are the left-to-right-top-to-bottom, reverse S and the centric space-filling curves. All of them go through the image in a straight-forward manner. Only the centric one has a form of locality in the center of the image, where the spiral iteration converges. Z-order, Hilbert and Peano space-filling curves are in the other group of curves striving to use locality as much as possible. They intentionally combine neighboring pixels in more complex regions so that more close-related positions can be used, thus improving the correlation between the resulting one-dimensional vector.

There is another property of the space-filling curves that is of great importance for the case of embedding messages. This is the ratio between the Euclidean distance (length of the path) and scalar distance (total count of elements in the path) of the space-filling curve, which results in better utilization of the neighboring pixels if this ratio is low. A simple observation on the curves is enough to see that there are two of them that do not follow the neighboring pixels very closely, i.e., there are jumps to more distant positions in the image matrix. These are the left-to-right and z-order space-filling curves. There is a very good analytic and experimental comparison carried out by Aubrey Jaffer [11] that demonstrates the benefits of the Hilbert's curve and Peano's curve in various space dimensions.

Despite of being more complex as structures, the Hilbert, Peano and z-order curves have a certain limitation in their usage. All of them require that the area, image, matrix they should iterate through, must be a power of 2, 3 and 2. This adds an additional level of limitations, what must be done with images that do not satisfy this requirement. Well, in the implementation provided, the area that is not present to complete the needed side dimension is just ignored, which will result in a line following the edges of the image. This will not break the neighboring properties of the respective space-filling curve, sinces they still follow the next pixel on the edge. This will even improve the performance of the z-order space-filling curve in that respect.

3. Experimental results

In order to get better understanding of the different aspects of the performance of the embedding algorithm, using various space-filling curves, tests were run on six images altogether (Fig. 7).

Fig. 7. Test images used for evaluating the experimental results

The experiments must compare the pure payload capabilities of the used embedding algorithm using each of the space-filling curves.

The results are represented in charts, one for each test image, where the pure payload of each of the space-filling curves is denoted. The *x*-axis shows the name of the space-filling curve and the *y*-axis – the value of the pure payload for the corresponding image.

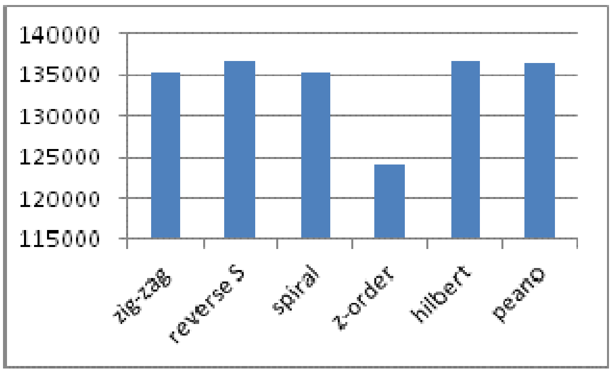

Fig. 8. Bee pure payload

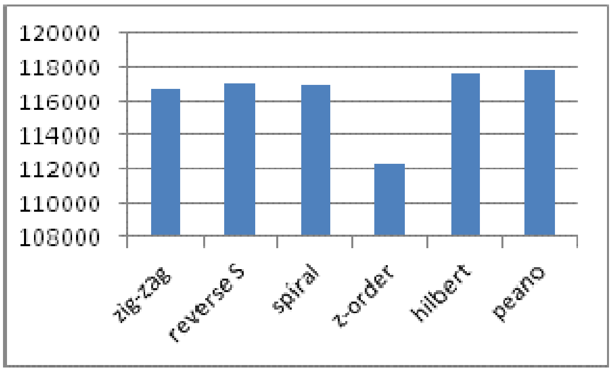

Fig. 9. Cat pure payload

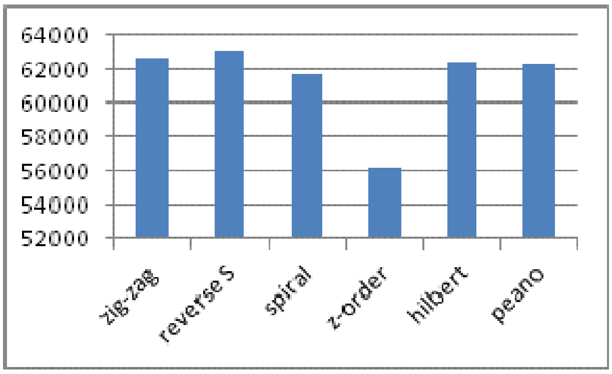

Fig. 10. Jet pure payload

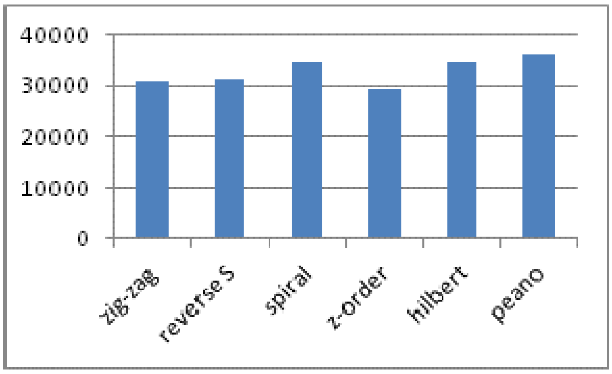

Fig. 11. Barbara pure payload

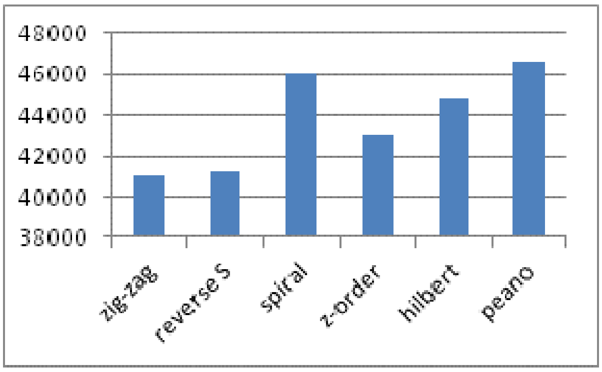

Fig. 12. Lena pure payload

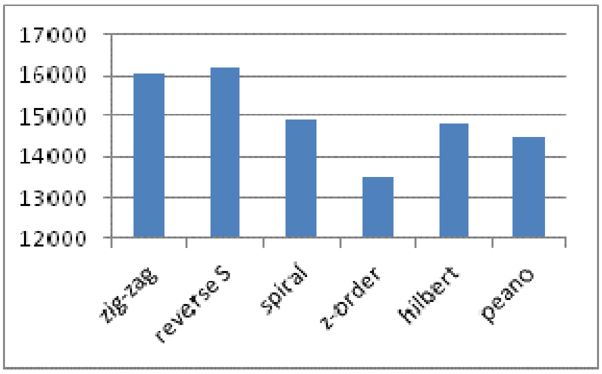

Fig. 13. Mandrill pure payload

Analyzing the results one cannot distinguish a space-filling curve, which functions much better than the rest. It turns out that the performance of the space-filling curve is greatly dependent on the image spatial structure. Therefore, for every image there is an optimal space-filling curve. In this aspect a data-hiding system based on this approach could have enumeration of space-filling curves that is passed along with the peak point as necessary information before decoding the message.

As far as the left-to-right space-filling curve is concerned, it performs well, despite its lack of continuity when changing rows. Nevertheless, the Reverse-S space-filling curve still outperforms it. These good results could be due to the fact that the left and right edges of the test images are fairly similar. Furthermore, these transitions are merely 512, which is a small number compared to the total number of differences denoted in the modified histogram.

The reverse S space-filling curve shows very good and consistent results. It is among the top performers in all of the test images except for the Lena image. Due to its simplicity and continuous path, these results might be expected. This space-filling curve is great with landscapes, where the top part is usually a uniform sky background.

The spiral space-filling curve shows consistent results as well, but overall shows lower results, especially in the mandrill image. Nevertheless in the Lena image it performs very strongly, as it takes full advantage of the convergence in the center region. This space-filling curve is very suitable for images that display objects of uniform pixel values in the center of the image.

The z-order space-filling curve proves to be not that good when it comes to exploiting the high correlation between neighboring pixels. It performs very poorly compared to the others. Its use should be avoided for this kind of purposes.

The Hilbert and Peano curves prove to be very good form embedding messages, using the modified histogram approach. Exploiting locality seems to be beneficial especially in the Barbara and Lena images.

4. Conclusion

Using various space-filling curves for building a modified histogram, based on pixel difference, and subsequent message embedding in images shows that this is an area that needs further investigation. Different space-filling curves do not perform uniformly for different images. Therefore, there could be two directions for improvement. One is to come up with an algorithm of choosing the most appropriate space-filling curve from the set of predefined such. The other option could be to implement an adaptive space-filling curve that is able to take full advantage of the spatial relations between adjacent pixels. The issue with the second variant is how these new space-filling curves will communicate to the decoding side in an optimal and secure manner.

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Сравнение между кривыми заполняющими пространствами при встраивании данных без потери в изображениях

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В области встраивания данных и цифровых водяных знаков без потери, в изображениях широко используется подход модификации гистограммы несущего изображения. Чтобы использовалась корелляция между соседными пикселями в изображении, необходимо исследовать разные способы трансформирования изображения от 2D в 1D представлении посредством заполняющими пространства кривыми. Сначала кривые, заполняющие пространства вводятся посредством своих специфических характеристик. Во второй части статьи описывается метод встраивания данных на основе гистограмной модификации. Представлены экспериментальные результаты с применением разных кривых. В конце статьи сделаны некоторые выводы насчет применения разных кривых.