БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ • BULGARIAN ACADEMY OF SCIENCES

ПРОБЛЕМИ НА ТЕХНИЧЕСКАТА КИБЕРНЕТИКА И РОБОТИКАТА, **61**
PROBLEMS OF ENGINEERING CYBERNETICS AND ROBOTICS, **61**

София • 2009 • Sofia

The Interval Shortest-Route Problem on Sofia Transportation Network

Akter Hossain[1], *Stavri Dimitrov* [2], *Emil Madjarski*[3]

[1] *French Language Faculty of Electrical Engineering, Technical University of Sofia, 1700 Sofia*
E-mail: akter_h@yahoo.com

[2] *Department of Technology, Organization and Management of Transport Higher School of Transport "Todor Kableshkov" , 1113 Sofia*
E-mail: stavri_dimitrov@hotmail.com

[3] *Faculty of Transport, Technical University of Sofia, 1700 Sofia*
E-mail: majarski@tu-sofia.bg

1. Introduction

There are many design and managment problems, for example, the design of telecommunication networks, design of transportation networks, design of large-scale irrigation systems, etc., which can solved with the aid of network models/algorithms.

The Shortest Route Problem (SRP) is a classical network problem, and it is the most popular model/algorithm among all network models/algorithms. The applications of shortest route computations are too numerous to cite in detail. Areas of application include capacity planning, equipment replacement, vehicle routing and scheduling problems, capacity planning, design and/or expansion of transportation and communication network, critical route scheduling, generalized assignment, traveling salesman, knapsack, traffic equilibrium, etc. [7, 8, 10-12]. The shortest route algorithms are better suited for implementation on micro computers than any of the mathematical programming algorithms [7].

The Interval Shortest Route Problem is concerned with determining the *interval shortest route* from an *origin* to a *destination* through a connecting network, given non-negative distances associated with the respective arcs of the network [10].

In literature, Dijkstra algorithm [1] is considered as a classical algorithm for SRP, and the last five decades hundreds of different variants of Dijkstra algorithm have been developed. It is the most implemented shortest route algorithm. It is simple, easy to understand and implement, and highly efficient algorithm [3]. The basic idea of Dijkstra algorithm is as follows: it assigns to all nodes labels, which are either *temporary* or *permanent*. At the beginning, each node receives a temporary label, which is further converted into a permanent label. This conversion is performed when it becomes evident that the last obtained distance in the label is the shortest distance to the node. In Dijkstra algorithm, in each iteration one temporary label is converted into a permanent label [3, 8, 11, 12].

An insightful interval algorithm (interval acyclic algorithm) is proposed for solving a network problem under parametric uncertainty in [4]. The exact values of the parameters of a given network are unknown, but upper and lower limits within which the values are expected to fall are considered. The interval algorithm is developed on the base of midpoint and half-width representation of intervals. Considerable unification and simplification are obtained by using the mean-value lemma.

An initial version of interval cyclic algorithm is presented for solving the well-known shortest route problem for cyclic network in [5, 6, 9]. The final version of interval cyclic algorithm is presented for solving the shortest route problem for cyclic network in [10]. The general approach in [4] is applied by the author for solving the interval shortest route problem. The formulation of the interval shortest route algorithm is an interval extension of classical shortest route algorithm given in [1]. The author considered the interval generalized distance between nodes *i* and *j* is a non-negative, interval number represented by D_{ij}, $D_{ij} = [\underline{d}_{ij}, \overline{d}_{ij}]$. The interval algorithm is developed on the base of midpoint and half-width representation of intervals. The interval algorithm is applicable when the parameters are interval and real. The author analyzed the complexity of the interval cyclic algorithm in [10], it is a polynomial algorithm.

The paper is organized as follows. The application of the interval cyclic algorithm is discussed in the second section. The conclusion is given in section three.

2. Application of the interval cyclic algorithm

In this section, a real application of the interval cyclic algorithm [5, 6, 9, 10] will be considered. We will determine interval shortest route and the interval shortest distance from a source node (node 1/center) to any destination node in Sofia transportation network for special transportation services, for example, **Police cars**,

Fire service, **Ambulance**, etc. on the basis of length arcs (routes) obtained by Google Earth.

Google Earth is used to find the distance between two nodes. Google Earth shows the transportation network of all the countries, for example, Fig. 1 shows Sofia transportation network [13]. Sofia transportation network has 86 regions [2] and all the regions have a center.

In Sofia transportation network, if one wants to travel from node *i* to node *j*, there are more than one route with different distances. For example, from center (node 1) to node 3 (see, in Fig. 1), there are several routes with different distances: 0.65 km, 0.72 km, 0.89 km, etc. We took the minimum distance, 0.65 km, between the center and node 3.

To find the distance between two nodes, we considered a main road (*bul.*), a main street (*ul.*). Most of the times, we first considered the main road, then the main street. But sometimes, we considered the main street first instead of the main road because the aim is to find the shortest distance between the two nodes. For example, one needs to travel from node 6 to node 17. There are at least two ways to travel between these two nodes: the first way is by bul. Tsar Osvoboditel, then bul. Tsarigradsko shosse, after that bul. Mihay Eminesku $\rightarrow$ the distance is 1.96 km, and the second way is by bul. Tsar Osvoboditel, then ul. Tsar Ivan Assen 2 $\rightarrow$ the distance is 1.66 km. The difference (distances) between these two routes is 0.30 km, and it is relatively big. So, we chose the second option, refer to [10].

The real distance between node *i* and node *j* of Sofia transportation network is obtained by Google Earth. Because of measurement and add placemark inaccuracies, 5% minus and plus is taken, and it gives the interval distance D_{ij} between node *i* and node *j* ($D_{ij} = [\underline{d}_{ij}, \overline{d}_{ij}] = [(d_{ij} - d_{ij} \times 0.05), (d_{ij} + d_{ij} \times 0.05)]$). For example, the interval distance between node 1 and node 3 is D_{13}, $D_{13} = [(0.65 - 0.65 \times 0.05), (0.65 + 0.65 \times 0.05)] = [0.6175, 0.6825]$. The real distances, traditional interval representation of real distances, and midpoint and half-width of real distances of node *i* and node *j* of Sofia transportation network are given in [10]. Some real distances, traditional interval representation of real distances, and midpoint and half-width of real distances of node *i* and node *j* of Sofia transportation network are given in Table 1.

To find the shortest route between the source node (node 1 or center of the city) to any destination, 243 arcs are considered (connections between two nodes) for Sofia transportation network. Sometimes, we did not consider the distance between two consecutive nodes *i* and *j*. For example, the distance between node 82 and node 83 is not considered, because there is no direct connection between these two nodes. If one needs to travel from node 82 to node 83, he has to travel via node 44. Another situation, the distance between node 84 to node 85 is 6.14 km, and node 84 to node 63, then node 63 to node 85 is 6.13 km (3.29 km + 2.84 km), the distance is almost the same, this type of connection or arc is not considered [10].

Let us assume that the density of the traffic jam between two nodes *i* and *j* is such, that the special service transportation car can move, the distance between node *i* and node *j* equals the distance between node *j* and node *i*, etc.

Table 1. Distance between two regions (nodes) in Sofia transportation regions

Number of Arc	From node or region	To node or region	Distance between two nodes, *l*, km	Traditional interval representation, *L*, km	Midpoint and half-width of interval
1	1	6	1.25	[1.1875, 1.3125]	[1.25, 0.0625]
2	6	17	1.66	[1.577, 1.743]	[1.66, 0.083]
3	17	18	1.55	[1.4725, 1.6275]	[1.55, 0.0775]
4	18	47	2.05	[1.948, 2.153]	[2.05, 0.1025]
5	47	64	1.41	[1.3395, 1.4805]	[1.41, 0.0705]
6	64	86	3.14	[2.983, 3.297]	[3.14, 0.157]

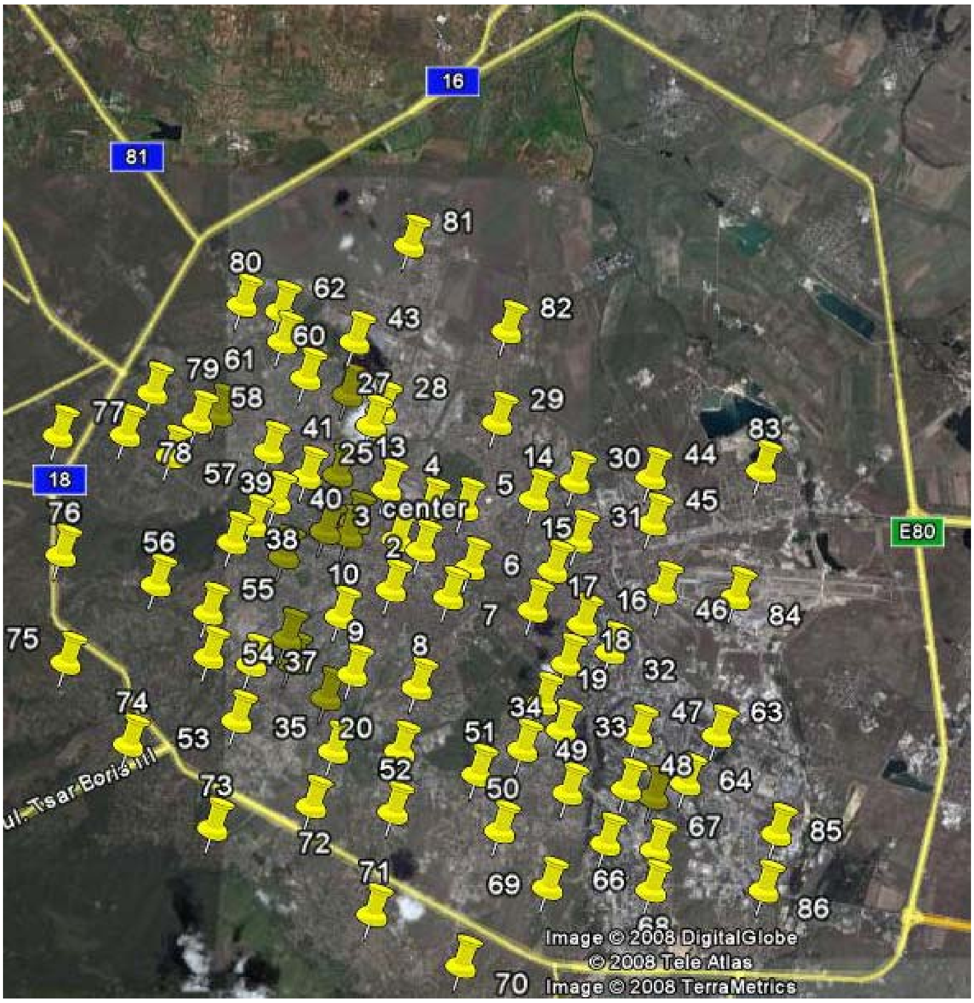

Fig. 1. Sofia transportation network

Using the interval cyclic algorithm the interval shortest route between the center (source node or node 1) to any destination node n, n = 2, 3, ..., 86, is obtained. The calculations of the temporary labels, permanent labels, labels of the nodes, etc. are given in [10].

The interval shortest route between center or source node 1 to any destination node in Sofia transportation network is determined by starting from the desired destination node and backing through the nodes using label's information. Using the label's information we obtain the following interval shortest route between the source node (node 1) to the destination node (node 86):

86 → (11.06, 64, 0.157) → 64 → (7.92, 47, 0.0705) → 47 → (6.51, 18, 0.1025) → 18 → (4.46, 17, 0.0775) → 17 → (2.91, 6, 0.083) → 6 → (1.25, 1, 0.0625) → 1

The interval shortest route is: **1 → 6 → 17 → 18 → 47 → 64 → 86**.

The half-width of the optimal solution is obtained by summing the third elements in the labels:

$$\Delta_{86} = \Delta_{64-86} + \Delta_{47-64} + \Delta_{18-47} + \Delta_{17-18} + \Delta_{6-17} + \Delta_{1-6} =$$
$$= 0.157 + 0.0705 + 0.1025 + 0.0775 + 0.083 + 0.0625 = 0.553.$$

Hence,

$$U_{86} = [\underline{u}_{86}, \bar{u}_{86}] = [11.06 - 0.553, 11.06 + 0.553] = \mathbf{[10.507, 11.613]}.$$

The interval shortest distance between center (node 1) to destination node 86 is [**10.506**, **11.616**]. The algorithm provides the interval shortest route and the interval shortest interval distance between node 1 (center) and any destination node, see, e.g. [10].

There are some other ways to find the route in a transportation network, for example, the Global Positioning System (GPS). GPS is used to find the route in the transportation network. GPS is a **Global Navigation Satellite System** (GNSS) developed by the **United States Department of Defense**. It uses a **constellation** of between 24 and 32 **Medium Earth Orbit satellites**, that transmit precise **microwave** signals, which enable GPS **receivers** to determine their current **location**, the time, and their velocity. It is the only fully functional GNSS in the world [14].

The GPS satellite constellation is managed by the **United States Air Force 50th Space Wing**. GPS is often used by civilians as a navigation system. A GPS receiver calculates its position by carefully timing the signals sent by the GPS **satellites** high above the Earth. The accuracy of a typical GPS receiver is about 10 - 15 meters. This may not be practical for locating a small object such as an automobile, which is about three meters long. The Differential GPS (DGPS) is a system that improves the accuracy of the GPS receiver to about one to two meters [14].

The GPS technology is becoming popular in cars, but some limitations still exist. The biggest problem is the blockage of signal transmission by obstacles such as mountains, high buildings, tunnels, urban canyons, etc.

Practical application aspects. It is assumed that any special car has a Decision Support System. A part of its knowledge base is the set of the shortest routes between any starting node *i* and any destination node *j*, when the driver (operator) specifies nodes *i* and *j*, he gets on the scan $(i, j) \Rightarrow$ the shortest route between node *i* and node *j*. The actualization of the set roads is performed in frequency, as needed, in the central office of the service.

Early morning, later evening, and during the night the set of the shortest routes can be used without any difficulties. The applicability during the day is based on two assumptions:

1. The density of the transportation jam is such that after receiving the signal from a special car, priority is given to the special car.

2. Any car has a special device to receive the signal from a special car and to inform the driver that he or she must change the position of his car.

3. Conclusion

In this paper, we considered a real application of interval cyclic algorithm. Using the new interval cyclic algorithm, we obtained the interval shortest route and the interval shortest distance from a source node to any destination node for special transportation services in Sofia transportation network, for example, Police car, Fire service, Ambulance, etc. It would the possible to determine the interval shortest route and the shortest distance in Sofia transportation network from a source node to any destination node for ordinary (public and private) transportation services. For ordinary transportation services, we have to consider some additional parameters, for instance, the traffic jam, the direction of the route segments, the allowed speed of the route segments, etc.

References

1. D i j k s t r a, E. W. A Note on Two Problems in Connection with Graphs. – Numerische Mathematik **1**, 1959, 269-271.
2. D i m i t r o v, S. D. Study of the Parameters of the Individual and the Public Passenger Transport in the Cities. PhD Thesis, Technical University of Sofia, 2006.
3. D r e y f u s, S. E. An Appraisal of Some Shortest-Path Algorithms. – Operations Research, Vol. **17**, 1969, No 3, 395-412.
4. G a t e v, G. Interval Analysis Approach and Algorithms for Solving Network and Dynamic Programming Problems under Parametric Uncertainty. – In: Proc. of the Technical University of Sofia, Vol. **48**, 1995, 345-350.
5. G a t e v, G., A. H o s s a i n. Interval Shortest Route Algorithms. – In: Proc. of the Int. Conf. on Automation & Information '2000, Vol. **2**, Sofia, Bulgaria, October 24-26, 2000, 68-71.
6. G a t e v, G., A. H o s s a i n. Interval Algorithms for Solving Minimal Spanning Tree and Shortest-Route Models. – In: Information Technologies and Control, Ed. K. Boyanov, Bulgarian Union of Automatics and Informatics, 2007, No 4, 37-46.
7. G l o v e r, F., D. K l i n g m a n, N. V. P h i l l i p s, R. F. S c h n e i d e r. New Polynomial Shortest Path Algorithms and their Computational Attributes. – Management Science, Vol. **31**, 1985, No 9, 1106-1128.
8. H i l l i e r, S. F., G. J. L i e b e r m a n. Introduction to Operations Research. Fifth Edition. N. J., McGraw-Hill Publishing Company, 1990.

9. H o s s a i n, A. Interval Algorithms for Solving Network and Dynamic Programming under Parametric Uncertainty. Master's Thesis, Technical University of Sofia, 1999.
10. H o s s a i n, A. Network Models under Parametric Uncertainty. PhD Thesis, Technical University of Sofia, 2009.
11. P h i l l i p s, D o n T., A. G a r c i a-D i a z. Fundamentals of Network Analysis. N. J., Prentice-Hall, Inc., Englewood Cliffs, 1981.
12. T a h a, H. A. Operations Research, An Introduction. Eighth Edition. Delhi, Pearson Education Inc., 2007.
13. **http://www.google earth.com**
14. **http://en.wikipedia.org/wiki/Global_Positioning_System**

Интервальная задача о кратчайшем пути в транспортной сети Софии

Актер Хосейн[1], *Ставри Димитров* [2], *Емил Маджарски*[3]

[1] *Факультет электротехники на французком языке, Технический университет – София, 1700 София*
E-mail: akter_h@yahoo.com

[2] *Кафедра технологии, организации и управления транспорта, Транспортный университет „Тодор Каблешков", 1113 София*
E-mail: stavri_dimitrov@hotmail.com

[3] *Факультет транспорта, Технический университет – София, 1700 София*
E-mail: majarski@tu-sofia.bg

(Р е з ю м е)

Определение кратчайшего пути между начальным узлем и конечной цели, или между двумя локациями в большой реальной сети, является важной задачей в многих применениях. В статье обсуждается реальное применение интервального варианта хорошо известного алгоритма кратчайшего пути в транспортной сети Софии. Это очень необходимо для движения некоторых специальных транспортных средств – например автомобили полиции, пожарной команды, скорей помощи. Маршрут определяется на основе длин дуг, полученных при помощи Google Earth.