БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ • BULGARIAN ACADEMY OF SCIENCES

ПРОБЛЕМИ НА ТЕХНИЧЕСКАТА КИБЕРНЕТИКА И РОБОТИКАТА, **59**
PROBLEMS OF ENGINEERING CYBERNETICS AND ROBOTICS, **59**

София • 2008 • Sofia

Synthesis of Power-Polynomial Motion Laws

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1. Introduction

Opportunities for using power polynomials at synthesis motion laws of the working devices of the mechanisms, machines, and robots are showed. Motions, one of necessary conditions is velocities and accelerations at the beginning and end of the end-effector moving to be equal zero, hold a basic place.

At the mechanisms synthesis as well as control of different mechatronic devices in particular industrial robots, one of the main problems for their correctly working is the synthesis of suitable motion laws for their working devices. A choice of mathematical function, describing such a motion law, which satisfies wanted kinematic characteristics of the purpose motion, is the base of the synthesis.

Almost all elementary functions, including Bessel functions, Chebishev polynomials, Legendre polynomials [7], using by engineers, chemists and mathematicians, are particular cases of the hypergeometric function [1], which represented in power series has the form

(1) $$F(a,\, b,\, c;\, \xi) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!}\, \xi^n, \quad c \neq 0, -1, -2, \ldots,$$

(2) $$\begin{cases} (a)_0 = 1, \\ (a)_n = \dfrac{\Gamma(a+n)}{\Gamma(a)} = a(a+1)\ldots(a+n+1), \quad n = 1, 2, 3, \ldots \end{cases}$$

Characteristic elementary functions are utilized for synthesis of motion laws of the output links of the cam mechanisms [2, 3, 5, 9], centroid type mechanisms

[6, 8], as in planning of the robot end-effectors motions. The power polynomials "hide" little "surprises" from the elementary functions at planning of the motions.

A purpose of this work is to show using of power polynomials at synthesis of motion laws of the working devices of the mechanisms, machines and robots. Motions, on which one of necessary conditions is the velocities and accelerations at the beginning and the end of the end-effector moving to be equal zero, hold a basic place.

2. Normalized power polynomials

Let a family of power polynomials is written as follows:

$$u(\xi) = \sum_j a_j \xi^j, \quad j = k,\ m,\ p,\ \dots, \tag{3}$$

and when $\xi \in [0;\ 1]$ function u changes at the same interval, $u \in [0;\ 1]$. A determination of the constant coefficients of the polynomial (3) can be done by solving the following algebraic system

$$\begin{cases} \sum_{j=b}^{n} a_j = 1 \\ \sum_{j=b}^{n} a_j j = 0 \\ \sum_{j=b}^{n} a_j j(j-1) = 0 \\ \dots \\ \sum_{j=b}^{n} \prod_{k=j-m}^{j} a_j k = 0 \end{cases} \tag{4}$$

obtained under substitutions $u = 1,\ u' = u'' = \dots = u^{(m+1)} = 0$ for the end of the range $\xi \in [0;\ 1]$. A solution of the system (4), rewritten in the matrix form $J.A = B$, where

$$J = \begin{bmatrix} 1 & 1 & 1 & \dots\ 1 \\ b & c & d & \dots\ n \\ b(b-1) & c(c-1) & d(d-1) & \dots\ n(n-1) \\ \dots & & & \\ \prod_{k=b-m}^{b} k & \prod_{k=c-m}^{c} k & \prod_{k=d-m}^{d} k & \dots \prod_{k=n-m}^{n} k \end{bmatrix}, \quad A = \begin{bmatrix} a_b \\ a_c \\ a_d \\ . \\ . \\ a_n \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

has the form $A = J^{-1}.B$. The system solving is reduced actually to finding an inverse matrix of the matrix J. When the number of equations in the system (4) is low, then its solution obtains convenient by the *rule of Sarrus*.

If the coefficients $a_j \neq 0$, when $j = k, m, p$, then the coefficients a_k, a_m, a_p of the polynomial

(5) $$u(\xi) = a_k \xi^k + a_m \xi^m + a_p \xi^p$$

are determined by the next expressions:

(6) $$a_k = \frac{D_k}{D}; \qquad a_m = \frac{D_m}{D}; \qquad a_p = 1 - a_k - a_m,$$

where

$$D = \begin{bmatrix} 1 & 1 & 1 \\ k & m & p \\ k(k-1) & m(m-1) & p(p-1) \end{bmatrix};$$

$$D_k = \begin{bmatrix} 1 & 1 & 1 \\ 0 & m & p \\ 0 & m(m-1) & p(p-1) \end{bmatrix};$$

$$D_m = \begin{bmatrix} 1 & 1 & 1 \\ k & 0 & p \\ k(k-1) & 0 & p(p-1) \end{bmatrix}.$$

The values of the coefficients a_k, a_m, a_p, when $k, m, p,$ are integer numbers in the range from 2 to 6, written in Table 1.

Table 1

No	a_1	a_2	a_3	a_4	a_5	a_6
1	0	6	–8	3	0	0
2	0	5	–5	0	1	0
3	0	4.5	–4	0	0	0.5
4	0	10/3	0	–5	8/3	0
5	0	3	0	–3	0	1
6	0	2.5	0	0	–4	2.5
7	0	0	10	–15	6	0
8	0	0	8	–9	0	2
9	0	0	5	0	–9	5
10	0	0	0	15	–24	10

When the low polynomial power is equal to two, then the second derivative $u'' = u''(\xi)$ of the polynomial (3) is possible to be equal to zero only for the right end of the ξ range. For the conditions $u' = u'' = 0$ are fulfilled at the ends of the range $\xi \in [0;\ 1]$ it is needed $j \geq 3$, but this leads to higher extreme values of u' and

u'', which increase in the polynomials with coefficients up from a line No 7 in Table 1.

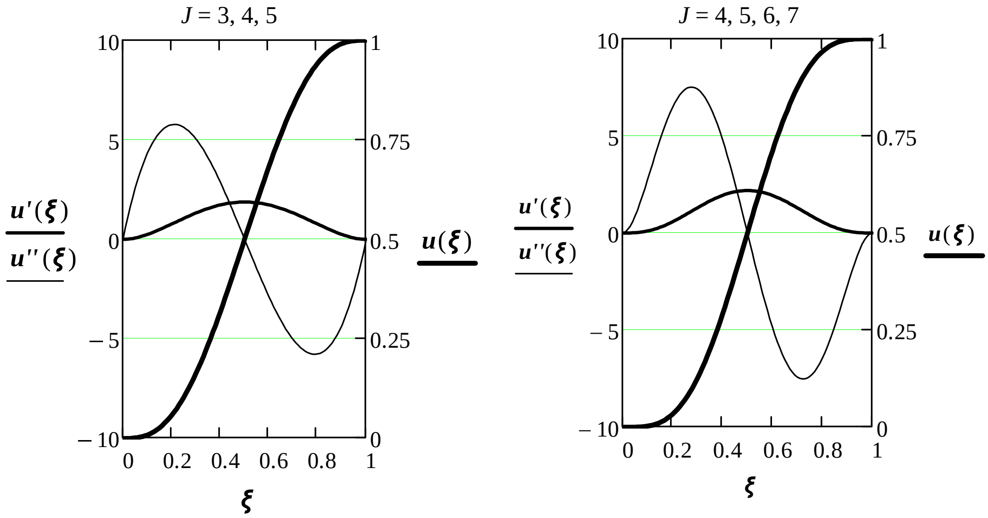

Fig.1. Power polynomials $u(\xi)$and their derivatives u' and u'' when J = 3, 4, 5 and J = 4, 5, 6, 7

The functions u, u', u'' are represented on the Fig. 1, when j takes values 3, 4, 5. The same conditions can be kept as well as j takes values greater than 6, but higher extreme values of the functions u', u'' are obtained. For example, from (6) the coefficients a_j are a_3 = 4.375, a_5 = –5.25, a_7 = 1.875, when $j = 3, 5, 7$, and $a_3 = 3$ $a_5 = -3$, $a_7 = 1$ when $j = 3, 6, 9$.

For the conditions $u = 1, u' = u'' = u''' = 0$ to be fulfilled, it is necessary function $u = u(\xi)$ to be a polynomial, which consists of at least four terms and $j \geq 4$. Then the coefficients of the polynomial

(7) $$u(\xi) = a_k \xi^k + a_m \xi^m + a_p \xi^p + a_s \xi^s$$

are defined from the following expressions:

(8) $$a_k = \frac{D_k}{D}; \qquad a_m = \frac{D_m}{D}; \qquad a_p = \frac{D_p}{D}; \qquad a_q = 1 - a_k - a_m - a_p,,$$

where

$$D = \begin{bmatrix} 1 & 1 & 1 & 1 \\ k & m & p & q \\ k-1 & m(m-1) & p(p-1) & q(q-1) \\ k(k-1)(k-2) & m(m-1)(m-2) & p(p-1)(p-2) & q(q-1)(q-2) \end{bmatrix};$$

$$D_k = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & m & p & q \\ 0 & m(m-1) & p(p-1) & q(q-1) \\ 0 & m(m-1)(m-2) & p(p-1)(p-2) & q(q-1)(q-2) \end{bmatrix};$$

$$D_m = \begin{bmatrix} 1 & 1 & 1 & 1 \\ k & 0 & p & q \\ k-1 & 0 & p(p-1) & q(q-1) \\ k(k-1)(k-2) & 0 & p(p-1)(p-2) & q(q-1)(q-2) \end{bmatrix};$$

$$D_p = \begin{bmatrix} 1 & 1 & 1 & 1 \\ k & m & 0 & q \\ k-1 & m(m-1) & 0 & q(q-1) \\ k(k-1)(k-2) & m(m-1)(m-2) & 0 & q(q-1)(q-2) \end{bmatrix}.$$

According to (8), coefficients a_j are obtained a_4 = 35, a_5 = –84, a_6 = 70, a_4 = –20, j = 4, 5, 6, 7. The functions u, u', u'' are represented on Fig.1. The obtained results, compared with the results of a polynomial when $j = 3, 4, 5$, show that when u''' equals zero in the ends of the range $\xi \in [0;\ 1]$, the extreme values of u' and u'' increase.

3. Power-polynomial motion laws

Each component η of the input or output coordinate can be written as a sum of initial value η_0 of η and normalized power polynomial $u(\xi)$, multiplied by a factor equal to maximum change $\Delta\eta_{\max}$ of η:

(9) $$\eta = \eta_0 + \Delta\eta_{\max} u(\xi),$$

where ξ is substituted by ration $\xi = t / T$ current time t to time T for realizing of $\Delta\eta_{\max}$.

The choice of normalized polynomial, describing the motion on defined coordinate, is determined by conditions for zero derivatives of the polynomial $u(\xi)$ in the ends of the range $\xi \in [0;\ 1]$.

The derivatives u', u'', u''' correspond respectively to the velocity, acceleration and second acceleration (pulse) on the appropriate coordinate.

If it is necessary that the velocity and acceleration to be equal to zero, respectively u' and u'' in the ends of the ξ range, at the synthesis of the motion laws, then the power polynomials with constant coefficients, written in Table 1, can be used. These polynomials guarantee zero inertial load, caused by the mass of the end-effector, in the ends of the ξ range.

If it is necessary apart of the velocity and acceleration to be equal to zero, the second acceleration to be equal to zero to, respectively u', u'' and u''', in the ends of the ξ range, then the power polynomial (7) when $j = 4, 5, 6, 7$ can be used or other polynomials, consisting of at least four terms and $j \geq 4$. These polynomials guarantee zero change of the inertial load (without jump), caused by the mass of the end-effector, in the ends of the ξ range. This “suppress” in major degree the unfavorable influence of the oscillations of the mechanical system, but leads to higher extreme values of the derivatives u' and u''.

The parabolic laws, based on the normalized polynomial of the form (5), are synthesized under conditions for straight path and zero velocities and accelerations in the path ends of the SCARA robots [4].

Example. A motion law is to be synthesized at the coordinate η under the following data: $\eta_0 = 0$, $\Delta\eta = 0 \div 640$ mm, $T = 2$ s and condition for zero $\dot{\eta} = \ddot{\eta} = 0$ in the range of $\Delta\eta$.

The polynomials $u = u(\xi)$ with constant coefficients, written in Table 1 from line No 7 to line No 10, are appropriate under this condition. The polynomial (No 7), leading to low extreme values of the velocity and acceleration, is $u(\xi) = 10\xi^3 - 15\xi^4 + 6\xi^5$, which substituted in (9), along with the above data, leads to the positional function $\eta(t) = 40(20t^3 - 15t^4 + 3t^5)$. Its derivatives $\dot{\eta}(t) = 600(4t^2 - 4t^3 + t^4)$, $\ddot{\eta}(t) = 2400(2t - 3t^2 + t^3)$ with respect to t represent corresponding components of the velocity and acceleration at the coordinate η. The derivatives $\dot{\eta}$ and $\ddot{\eta}$ are obviously equal to zero in the ends of the interval $t = 0 \div 2\ s$. On this way all conditions on the put task are satisfied.

4. Conclusion

Opportunities for using power polynomials at the synthesis of motion laws of the working devices of the mechanisms, machines and robots are showed:

1. Families of power-polynomial laws of movement, giving the different opportunities for zero the velocities and the accelerations on the ends of the intervals of the mechanical systems end - effectors movement, are identified.

2. A comparative analysis are done for different power-polynomial laws of motion at relation on extreme values of their derivatives with respect to the time (velocities and accelerations).

3. The possibilities for synthesis of power-polynomial motion laws, in which the derivatives and accelerations are zero, are mentioned. On this way the harmful influence of the oscillations of the mechanical system is reduced.

An illustrative example shows approach for synthesis of motion law at one input or output coordinate of the mechanical system.

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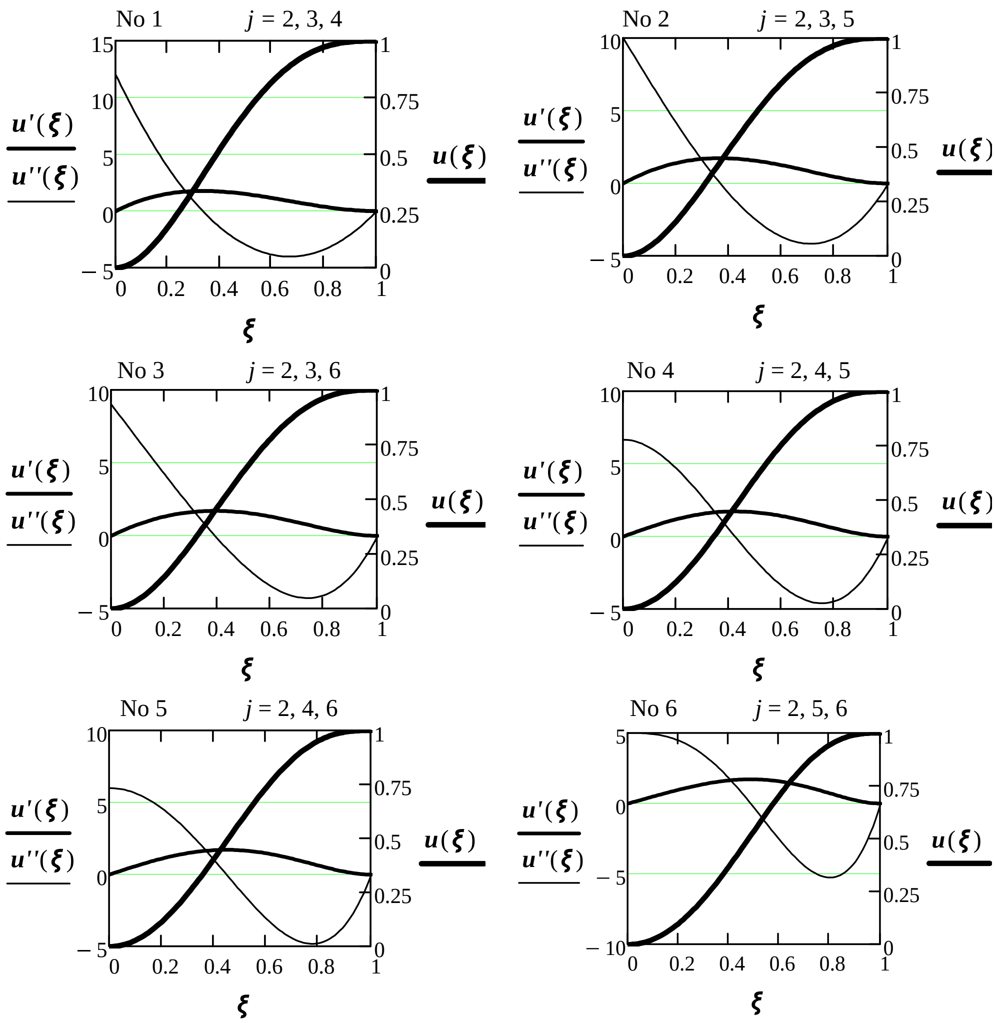
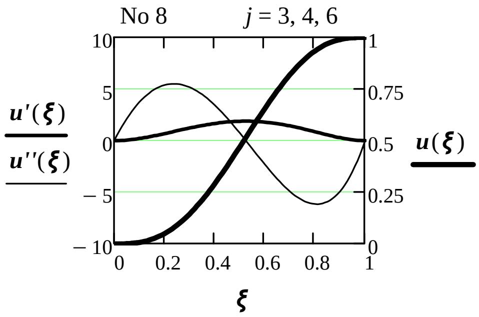
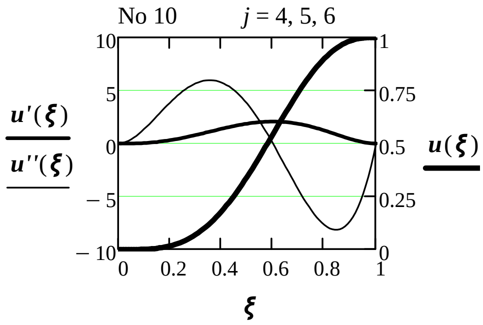
Appendix

The functions u, u', u'' are represented from Table 1 without line No 7 and line No 9 (Fig. 1).







Синтез степенно-полиномных законов движения

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(Р е з ю м е)

Показаны возможности использования степенных полиномов для синтеза законов движения исполнительных устройств механизмов машин и роботов. Одно из необходимых условий в начале и в конце движения исполнительного механизма (захвата) является установление нулевой скорости и ускорения.