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Kinematics of SCARA Robots

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1. Introduction

Manipulation mechanisms are the main part of technological robots (K o z a r e v, 1988). These mechanisms direct a point in a given path (path-generating mechanisms), or guide a solid body along a given trajectory of its characteristic point and orientation of this solid body (transpose mechanisms) [S u h et al. 1978], (E r d m a n et al., 1991; G a l a b o v, 1992). The geometry of the purpose motion can be achieved by manipulation mechanism or by active control of the motors as in the case of SCARA robots.

The definition of the variable input geometrical and kinematic mechanisms parameters as a result from a given motion of the end-effector, usually is referred to the inverse kinematics (E r d m a n, 1993). Explicit solutions can be obtained only for particular cases of kinematics chains, which most of the utilized in practice mechanisms possess (G a l a b o v, 1998).

In this publication, the inverse kinematic problem is brought to determination of the functions, over which the input parameters (positions, angle velocities, accelerations) are changed. These functions are necessary for control of the purpose motions of the robot. The solution of the direct kinematic problem serves as a test for the solution of the inverse kinematic problem.

The kinematics of the SCARA robots can be researched by different methods (L e b e d e v, 1966; M i n k o v, 1985), however the most effective method in this case is the method of the vector loop, developed in detail by Zinoviev, since the essence of the structure of SCARA robots is planar.

2. Essence of the method

The Zinoviev method for solving the direct and inverse kinematic problem of planar and spatial linkages is based on the theory of closed vector loops, substituting the mechanisms kinematic scheme. Thus, a vector with a defined direction corresponds to every mechanisms link. The vector direction is positive along the direction of the loop circuit and backward. The vector sum of these vectors represents an equation of the closed vector loop, equivalent to the mechanism loop.

The universal robots have mainly open kinematic chain and an open vector loop corresponds to it. This open vector loop is conditionally closed by a vector describing the purpose robot path. The vector equation is presented by projection equations along the axes of a properly selected coordinate system. From the obtained system of equations, the positional direct or inverse problem is solved. This problem is nonlinear by definition.

The projection equations are differentiated with respect to time *t*. The aim is to solve two problems in relation to the velocities and accelerations. The obtained system of derivative equations is linear in relation to unknown velocities and accelerations.

3. Direct kinematic problem

The parameters of the robot kinematic scheme are given. From these parameters $l_1 = \overline{OC}$, $l_2 = \overline{CH}$, z_1, z_2 are constants. The generalized coordinates $\varphi_{1,0}$, $\varphi_{2,1}$, z_3 and their derivatives with respect to time *t*, so called kinematic input parameters, are variables (Fig. 1). The problem is to find the law of motion of the characteristic point *H* of the robot end-effector: a trajectory τ with vector equation $\vec{r} = \vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$ and projection equations $x = x(t)$, $y = y(t)$, $z = z(t)$; velocity $\dot{\vec{r}} = \dot{\vec{r}}(t) = \dot{x}(t)\vec{i} + \dot{y}(t)\vec{j} + \dot{z}(t)\vec{k}$ and projection equations $\dot{x} = \dot{x}(t)$, $\dot{y} = \dot{y}(t)$, $\dot{z} = \dot{z}(t)$; acceleration $\ddot{\vec{r}} = \ddot{\vec{r}}(t)\vec{r} = \ddot{x}(t)\vec{i} + \ddot{y}(t)\vec{j} + \ddot{z}(t)\vec{k}$ and projection equations $\ddot{x} = \ddot{x}(t)$, $\ddot{y} = \ddot{y}(t)$, $\ddot{z} = \ddot{z}(t)$. The units vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ are constants.

From the equations of the closed vector loop

$$\vec{z}_1 + \vec{l}_1 + \vec{z}_2 + \vec{l}_2 + \vec{z}_3 - \vec{r} = 0 , \qquad (1)$$

the coordinates of the characteristic point *H* are obtained

$$\begin{aligned} x &= l_1 \cos\varphi_{1,0} + l_2 \cos(\varphi_{1,0} + \varphi_{2,1}), \\ y &= l_1 \sin\varphi_{1,0} + l_2 \sin(\varphi_{1,0} + \varphi_{2,1}), \\ z &= z_1 - z_2 - z_3 \end{aligned} \qquad (2)$$

and the distance $l_{OA} = |\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$, together with the direction cosines

$$\text{(3)} \qquad \cos(\widehat{x,\vec{r}}) = \frac{x}{r}; \ \cos(\widehat{y,\vec{r}}) = \frac{y}{r}; \ \cos(\widehat{z,\vec{r}}) = \frac{z}{r}.$$

After differentiation of the equations (2), the velocity components of point *H* are obtained

$$\text{(4)} \qquad \begin{aligned} \dot{x} &= -l_1 \sin\varphi_{1,0}.\dot{\varphi}_{1,0} - l_2 \sin(\varphi_{1,0} + \varphi_{2,1})(\dot{\varphi}_{1,0} + \dot{\varphi}_{2,1}), \\ \dot{y} &= l_1 \cos\varphi_{1,0}.\dot{\varphi}_{1,0} + l_2 \cos(\varphi_{1,0} + \varphi_{2,1})(\dot{\varphi}_{1,0} + \dot{\varphi}_{2,1}), \\ \dot{z} &= -\dot{z}_3 \end{aligned}$$

and the magnitude of the velocity $|\dot{\vec{r}}| = \dot{r} = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$ with directions, defined through direction cosines

$$\text{(5)} \qquad \cos(\widehat{\dot{x},\dot{\vec{r}}}) = \frac{\dot{x}}{\dot{r}}; \ \cos(\widehat{\dot{y},\dot{\vec{r}}}) = \frac{\dot{y}}{\dot{r}}; \ \cos(\widehat{\dot{z},\dot{\vec{r}}}) = \frac{\dot{z}}{\dot{r}}.$$

After differentiation of the equations (4), the acceleration components of point *H* are obtained,

$$\text{(6)} \qquad \begin{aligned} \ddot{x} &= -l_1\left(\cos\varphi_{1,0}(\dot{\varphi}_{1,0})^2 + \sin\varphi_{1,0}.\ddot{\varphi}_{1,0}\right) - l_2\left(\cos(\varphi_{1,0} + \varphi_{2,1})(\dot{\varphi}_{1,0} + \dot{\varphi}_{2,1})^2 + \right. \\ &\qquad \left. + \sin(\varphi_{1,0} + \varphi_{2,1})(\ddot{\varphi}_{1,0} + \ddot{\varphi}_{2,1})\right), \\ \ddot{y} &= -l_1\left(\sin\varphi_{1,0}(\dot{\varphi}_{1,0})^2 - \cos\varphi_{1,0}.\ddot{\varphi}_{1,0}\right) - l_2\left(\sin(\varphi_{1,0} + \varphi_{2,1})(\dot{\varphi}_{1,0} + \dot{\varphi}_{2,1})^2 - \right. \\ &\qquad \left. - \cos(\varphi_{1,0} + \varphi_{2,1})(\ddot{\varphi}_{1,0} + \ddot{\varphi}_{2,1})\right), \\ \ddot{z} &= -\ddot{z}_3 \end{aligned}$$

and the magnitude of the acceleration $|\ddot{\vec{r}}| = \ddot{r} = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$ with directions, defined through direction cosines

$$\text{(7)} \qquad \cos(\widehat{\ddot{x},\ddot{\vec{r}}}) = \frac{\ddot{x}}{\ddot{r}}; \ \cos(\widehat{\ddot{y},\ddot{\vec{r}}}) = \frac{\ddot{y}}{\ddot{r}}; \ \cos(\widehat{\ddot{z},\ddot{\vec{r}}}) = \frac{\ddot{z}}{\ddot{r}}.$$

After differentiation of equations (6), the second acceleration of point *H* can be obtained, but they are rarely used in practice.

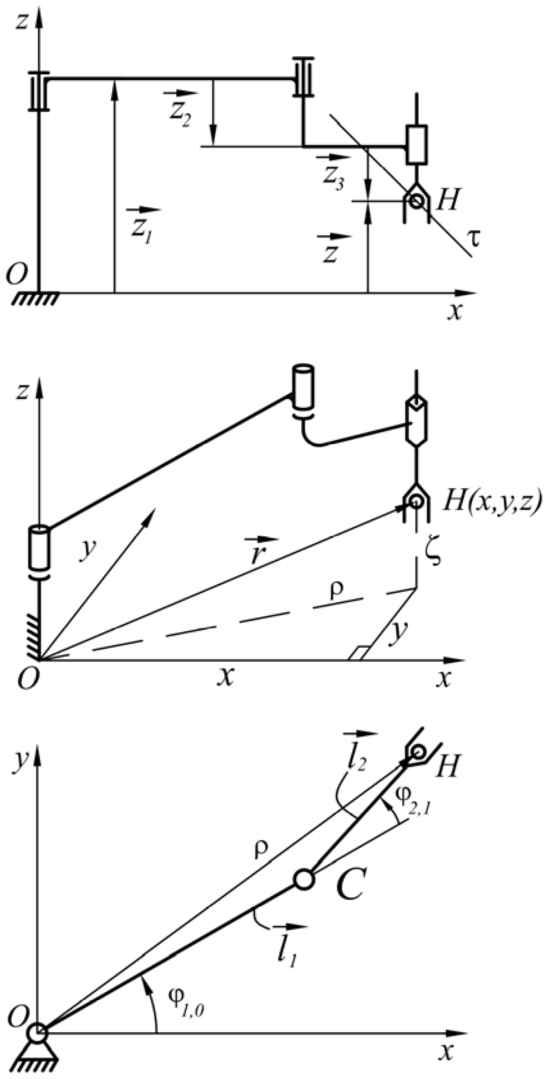

Fig. 1. Kinematics scheme of the SCARA Robot

4. Inverse kinematics problem

On given: trajectory τ of point H with the corresponding equations $\vec{r} = \vec{r}(t)$, respectively $x = x(t)$, $y = y(t)$, $z = z(t)$; velocity $\dot{\vec{r}} = \dot{\vec{r}}(t)$, respectively $\dot{x} = \dot{x}(t)$, $\dot{y} = \dot{y}(t)$, $\dot{z} = \dot{z}(t)$; acceleration $\ddot{\vec{r}} = \ddot{\vec{r}}(t)$, respectively. $\ddot{x} = \ddot{x}(t)$, $\ddot{y} = \ddot{y}(t)$, $\ddot{z} = \ddot{z}(t)$ and known constant parameters $l_1 = \overline{OC}$, $l_2 = \overline{CH}$, z_1, z_2 of the robot kinematic scheme, the generalized coordinates $\varphi_{1,0}$, $\varphi_{2,1}$, z_3 and their derivatives with respect to time t – generalized (input) velocities and accelerations are sought.

The projection equations system (2) is utilized for solving the positional problem. This system is nonlinear according the unknowns $\varphi_{1,0}$ and $\varphi_{2,1}$ and has a solution

(8) $$\varphi_{1,0} = \arcsin \frac{l.y \mp \sqrt{(l.y)^2 - \rho^2\left(l^2 - x^2\right)}}{\rho^2},$$

(9) $$\varphi_{2,1} = \arcsin \frac{y - l_1 \sin\varphi_{1,0}}{l_2} - \varphi_{1,0},$$

where the symbols are introduced

(10) $$l = \frac{\rho^2 + l_1^2 - l_2^2}{2l_1}, \quad \rho^2 = x^2 + y^2.$$

After successive k-times differentiation of the equations (2) with respect to t, systems of three linear equations according the input velocities $\dot{\varphi}_{1,0}, \dot{\varphi}_{2,1}, \dot{z}_3$ (where k=1), input equations $\ddot{\varphi}_{1,0}, \ddot{\varphi}_{2,1}, \ddot{z}_3$ (where k=2) and etc., are obtained. These systems have the form

$$a\,\varphi_{1,0}^{(k)} + b(\varphi_{1,0}^{(k)} + \varphi_{2,1}^{(k)}) = c_k, \quad a = -l_1 \sin\varphi_{1,0}, \quad b = -l_2 \sin(\varphi_{1,0} + \varphi_{2,1}),$$

$$d\,\varphi_{1,0}^{(k)} + e(\varphi_{1,0}^{(k)} + \varphi_{2,1}^{(k)}) = f_k, \quad d = l_1 \cos\varphi_{1,0}, \quad e = l_2 \cos(\varphi_{1,0} + \varphi_{2,1}),$$

$$z^{(k)} = -z_3^{(k)}$$

and solutions

(11) $$\varphi_{1,0}^{(k)} = \frac{(D_a)_k}{D}, \quad \varphi_{2,1}^{(k)} = \frac{(D_b)_k}{D} - \varphi_{1,0}^{(k)}, \quad z_3^{(k)} = -z^{(k)}, k = 1, 2, 3, ...,$$

where

(12) $$D = \begin{vmatrix} a & b \\ d & e \end{vmatrix} = l_1 l_2 \sin\varphi_{2,1}.$$

If k=1 then $c_1 = \dot{x}, \ f_1 = \dot{y}$ and determinants:

(13) $$(D_a)_1 = \begin{vmatrix} c_1 & b \\ f_1 & e \end{vmatrix} = l_2\left(\dot{x}\cos(\varphi_{1,0} + \varphi_{2,1}) + \dot{y}\sin(\varphi_{1,0} + \varphi_{2,1})\right),$$

(14) $$(D_b)_1 = \begin{vmatrix} a & c_1 \\ d & f_1 \end{vmatrix} = -l_1(\dot{x}\cos\varphi_{1,0} + \dot{y}\sin\varphi_{1,0}).$$

The input velocities are respectively

(15) $$\dot{\varphi}_{1,0} = \frac{(D_a)_1}{D} = \frac{\dot{x}\cos(\varphi_{1,0} + \varphi_{2,1}) + \dot{y}\sin(\varphi_{1,0} + \varphi_{2,1})}{l_1 \sin\varphi_{2,1}},$$

(16) $$\dot{\varphi}_{2,1} = \frac{(D_b)_1}{D} = \frac{\dot{x}\cos\varphi_{1,0} + \dot{y}\sin\varphi_{1,0}}{-l_2 \sin\varphi_{2,1}} - \dot{\varphi}_{1,0}, \quad \dot{z}_3 = -\dot{z}.$$

Similar at k=2 is it obtained:

(17) $c_2 = \ddot{x} + l_1 \cos\varphi_{1,0} . \dot{\varphi}_{1,0}{}^2 + l_2 \cos(\varphi_{1,0} + \varphi_{2,1})(\dot{\varphi}_{1,0} + \dot{\varphi}_{2,1})^2,$

(18) $f_2 = \ddot{y} - l_1 \sin\varphi_{1,0} . \dot{\varphi}_{1,0}{}^2 + l_2 \sin(\varphi_{1,0} + \varphi_{2,1})(\dot{\varphi}_{1,0} + \dot{\varphi}_{2,1})^2,$

(19) $$(D_a)_2 = \begin{vmatrix} c_2 & b \\ f_2 & e \end{vmatrix}, \; (D_b)_2 = \begin{vmatrix} a & c_2 \\ d & f_2 \end{vmatrix}$$

and input accelerations

(20) $$\ddot{\varphi}_{1,0} = \frac{(D_a)_2}{D}, \; \ddot{\varphi}_{2,1} = \frac{(D_b)_2}{D} - \ddot{\varphi}_{1,0}, \; \ddot{z}_3 = -\ddot{z}, \; k = 1, 2, 3, ...$$

From the equations (2), (4) and (6) follows: $z_3 = z - z_1 - z_2$; $\dot{z}_3 = z$; $\ddot{z}_3 = z$.

In a similar way, the higher accelerations can be defined, but they are not used for a robot control.

5. Motion laws and verification of the results

Let the problem is to synthesize the laws of motion $x(t), y(t), z(t)$ on the corresponding axis under the following conditions: motion of characteristic point *H* from point *A*(500, 0, 50) to point *B* (0,500, 260) on a straight line for time *T*=2 s, at nullification of the first and second derivatives of $x(t), y(t), z(t)$ for the boundary point *A* and *B*. Like this in these points, the manipulated object is gripped or left, the inertial load originated from the mass of the end-effector is nullified.

Determination of laws of motion. The trajectory straightness condition will be fulfilled, if the laws of motion on the according axes are homogenous from the type:

(21) $$\begin{aligned} x &= x_A + \Delta x(t) = x_A + C_x u(t), \\ y &= y_A + \Delta y(t) = y_A + C_y u(t), \\ z &= z_A + \Delta z(t) = z_A + C_z u(t). \end{aligned}$$

Then the angle coefficients, which the projections of the trajectory in the planes *xy*, *yz*, *xz* enclose, are constant:

$$k_{xy} = \frac{\Delta y}{\Delta x} = \frac{C_y}{C_x}, \; k_{yz} = \frac{\Delta z}{\Delta y} = \frac{C_z}{C_y}, \; k_{xz} = \frac{\Delta z}{\Delta x} = \frac{C_z}{C_x} = k_{xy} \; k_{yz},$$

which proves the statement for the straightness of the trajectory mentioned above.

Let for the definition of the motion laws (21) a normalized power polynomial $u(\xi) = \sum_j a_j \xi^j$ is utilized, where the argument $\xi = t / T$ (*t* is the flowing time, *T* is the time for realization of the transposition) and the function *u* are changed in the same interval [0; 1]. To nullify the components of the velocity and acceleration of point *H*, respectively $u' = u'' = 0$ in the range $[0; 1]$, is necessary $j \geq 3$. At this condition the polynomial leading to minimal values of the velocity and acceleration, is $u(\xi) = a_3\xi^3 - a_4\xi^4 + a_5\xi^5$ with coefficients $a_3 = 10$; $a_4 = -15$; $a_5 = 1 - a_3 - a_4 = 6$, defined from the system of algebraic equations

(22) $$\sum_{j=3}^{5} a_j = 1, \quad \sum_{j=3}^{5} a_j j = 0, \quad \sum_{j=3}^{5} a_j j(j-1) = 0.$$

The last system is obtained by substitutions $u = 1, u' = u'' = 0$ in the end of the interval $[0;\ 1]$. When the normalized polynomial $u(\xi) = 10\xi^3 - 15\xi^4 + 6\xi^5$ is utilized, the positional functions

(23)
$$x = x_A + C_x u(\xi) = 500 - 31.25(20t^3 - 15t^4 + 3t^5)\ ,$$
$$y = y_A + C_y u(\xi) = 0 + 54.125(20t^3 - 15t^4 + 3t^5),$$
$$z = z_A + C_z u(\xi) = 500 - 15(20t^3 - 15t^4 + 3t^5)\ ,$$

are formed, where the constants C_x, C_y, C_{xz} are defined from the conditions t=T=2 s: $x = x_B = 500$ mm; $y = y_B = 0$ mm; $z = z_B = 500$ mm, and ξ is substituted with the relation $\xi = t / T$.

The second differentiation of (23) leads to determination of the velocity and acceleration components of point *H:*

(24)
$$\dot{x} = -468.75(4t^2 - 4t^3 + t^4)\ ,$$
$$\dot{y} = 811.875(4t^2 - 4t^3 + t^4),$$
$$\dot{z} = -225(4t^2 - 4t^3 + t^4)\ ,$$

(25)
$$\ddot{x} = -1875(2t - 3t^2 + t^3)\ ,$$
$$\ddot{y} = 3247.5(2t - 3t^2 + t^3),$$
$$\ddot{z} = -900(2t - 3t^2 + t^3).$$

Solution of the inverse problem at parameters $l_1 = l_2 = 500$ mm. The input coordinates $\varphi_{1,0}$, $\varphi_{2,1}$ and their derivatives (the input angle velocities and accelerations) $\dot{\varphi}_{1,0}$, $\ddot{\varphi}_{1,0}$, $\dot{\varphi}_{2,1}$, $\ddot{\varphi}_{2,1}$ are determined by the equations (8), (9), (15)-(21), respectively. From equation (8), two solutions of the given problem are found. At the first solution the initial values of the generalized coordinates are $\varphi_{1,0} = -60°$ and $\varphi_{2,1} = 120°$, and at the second $\varphi_{1,0} = 60°$ and $\varphi_{2,1} = -120°$. The first solution is preferred due to the closer to the translation motion of the second link in the second stage of motion from $t = 1$ s to $t = 2$ s. The functions $\varphi_{1,0}$, $\dot{\varphi}_{1,0}$, $\ddot{\varphi}_{1,0}$ and $\varphi_{2,1}$, $\dot{\varphi}_{2,1}$, $\ddot{\varphi}_{2,1}$ are represented on the Figs. 2 and 3.

The direct problem is utilized for verification of the inverse problem solution, where the obtained functions $\varphi_{1,0}$, $\dot{\varphi}_{1,0}$, $\ddot{\varphi}_{1,0}$ and $\varphi_{2,1}$, $\dot{\varphi}_{2,1}$, $\ddot{\varphi}_{2,1}$ are substituted by the equations (2), (4) and (6). The determined functions $x = x(t)$, $y = y(t)$, $z = z(t)$, $\dot{x} = \dot{x}(t)$, $\dot{y} = \dot{y}(t)$, $\dot{z} = \dot{z}(t)$, $\ddot{x} = \ddot{x}(t)$, $\ddot{y} = \ddot{y}(t)$, $\ddot{z} = \ddot{z}(t)$ coincide with the given (23), (24), (25), which means that the inverse problem solution is correct.

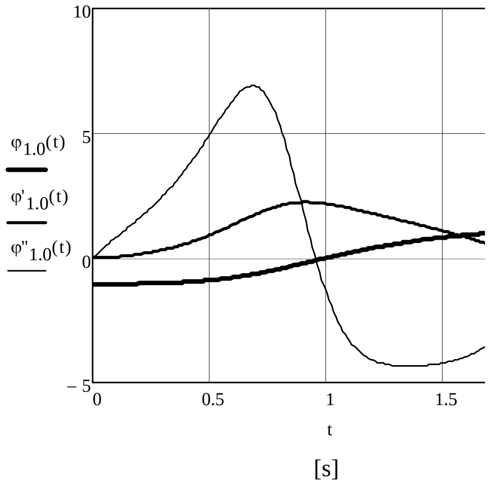

Fig. 2. Functions $\varphi_{1,0}$, $\dot{\varphi}_{1,0}$, $\ddot{\varphi}_{1,0}$

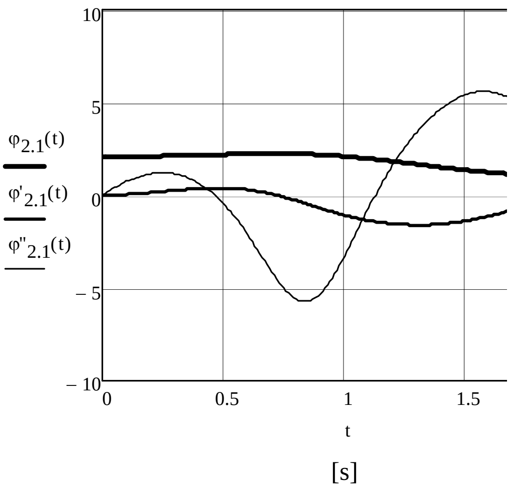

Fig. 3. Functions $\varphi_{2,1}$, $\dot{\varphi}_{2,1}$, $\ddot{\varphi}_{2,1}$

6. Conclusion

For the control of the SCARA robot, the inverse kinematic problem is solved for determination of the angle parameters and their derivatives (angle velocities and accelerations) in the function of coordinates (trajectory) and their derivatives (velocities and accelerations) of the characteristic point of the end-effector. So defined input parameters, verified by solving of the direct kinematic problem are achieved through control of the gear-motors.

The statement of straightness is proved of the generated trajectory, which is obtained when the laws of motion $x(t)$, $y(t)$, $z(t)$ of the corresponding axes are homogenous from the type (21). There are synthesized parabolic laws of the purpose motion on the coordinate axes, over the basis of normalized power polynomial, derived from the condition for nullifying of the inertial load, originated from the end-effector's mass.

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Кинематика SCARA роботов

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(Р е з ю м е)

Поставлены и решены прямая и инверсная задача кинематики. Эти задачи касают определения функций, в соответствии с которыми меняются входящие параметры для управления движением SCARA роботов по заданному пути и их производных.

Синтезируются параболические законы движения на основе полученного нормализированного полинома заданных траекторий.