БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ • BULGARIAN ACADEMY OF SCIENCES

ПРОБЛЕМИ НА ТЕХНИЧЕСКАТА КИБЕРНЕТИКА И РОБОТИКАТА, **58**
PROBLEMS OF ENGINEERING CYBERNETICS AND ROBOTICS, **58**

София • 2007 • Sofia

Two Approaches for Solving Multiple Criteria Decision Making (MCDM) Problems with an Illustrative Example

Nikolay Tontchev[1], *Leoneed Kirilov*[2]

[1] *Higher School of Transport, 1574 Sofia*
[2] *Institute of Information Technologies, 1113 Sofia*

1. Introduction

One of the basic problems in the process of design, control and planning of functioning engineering systems is the determination of the system structure, the material and power balances, the dimensionality of the equipment, the controlling variables in normal and emergency modes of system operation and also of production planning for short-term / long-term exploitation – [1, 2]. The solution of this problem must be found under the restrictions of different requirements postulated by economic (minimization of material and power consumption, maximization the profit from the production), technical (minimization of the equipment size), ecological (minimal pollution of the environment by a specific waste product) and other specifications. It is evident that the requirements cannot be satisfied as a whole but a reasonable compromise is a solution of the problem. In fact this is a multicriteria model for solving such problems.

The above formulation can be denoted as follows:

(1) $$\max_X f(x) = (f_1(x), \ldots, f_k(x)),$$

where x is an n-dimensional vector of the variables (alternatives) of the solution and $f(x)$ is a k-dimensional vector of the criteria (objective functions). We admit that the variables x_i are continuous for the set of admissible solutions.

The set of the admissible solutions is defined as follows:

(2) $$X = \{x \mid g_j(x) \geq 0,\ j = 1,\ 2, \ldots, m;\ x \in R^n\},$$

where $g_j(x)$ are the functional constraints imposed by the multicriteria problem.

From an engineering and technological standpoint the criteria forming the problem (1) are very often loaded by a different physical meaning. For this reason it is necessary to normalize the physical criteria in dimensionless criteria. The normalization is realized introducing and using special dimensionless functions known as transfoms (transforming coefficients). It is typical for these transformations that they include the option of interactivity to the solution of the MCDM problems. This is possible in accordance with the way of their generation. The next stage is finding a solution of the problem (1). Most often this takes place by a single-criteria (scaling) problem. The optimal solution of the scaling problem is a non-dominating problem solution (1) satisfying the DM most of all consistently with some additional conditions (criteria). There are several basic types of transforms in bibliography. Below we present an example with some of them.

One approach is to form the scaling function in such a way that it will simultaneously maximize the excess of this value and to maximize its shortage. The algorithms of G r a u e r [3], M a s u d and H w a n g [4], N a k a y a m a and F u r u k a w a [5], W i e r z b i c k i [6], realize the different varieties of this type of transformation.

In the transformation for an optimistic decision the DM initially defines optimistic (his best) values y_j^{opt} for the object functions that cannot be obtained simultaneously: the process continues with a gradual decrease of the optimistic values.

An opposite approach is the so called “pessimistic” approach. In this case the DM defines values for the object functions known as “necessary” – y_j^{pes} that can be reached simultaneously. The decision is obtained by maximization of the excess for every object function in accordance with the necessary values. Once he/she has obtained a concrete decision, the DM can define other necessary values.

2. The proposed approaches

The aim of this paper is to propose an approach for solving the above MCDM model that comes from solving a number of technological problems from the area of building-up by welding [2, 9, 10].

The main idea of method is to use a simple and effective tool for solving SC problem to present non-dominated solutions to the DM. For this purpose we use graphical method on the base of limits, given in advance by the DM. The DM can change them in the time until acceptable solution is located for the Scalarizing (SC) problem.

The general scheme of approach can be presented in four steps.

1. Generating a MCDM model.

For this purpose for example regression models can be used. The result of first step is nonlinear multiple objective programming problem. It could include some or all discrete or integer variables.

2. Discretization of a feasible set.

For example if we use the interval [–1, 1] it is recommended the step to be 0.25 for the technological problems.

3. Implementing an MCDM strategy (method) – scalarization function that combine heuristic rule and transformation property and interaction with the Decision Maker (DM).

In Points 2.1 and 2.2 we describe two MCDM methods.

4. Implementing a graphical approach (two-way joining result) for solving the chosen scalarising problem.

This approach is presented in Point 2.1.

2.1. Apriori method

In this method the DM gives his/her preferences in terms of reservation levels in the objective space – f^r. In other words every solution with values greater than or equal to these levels (thresholds) is acceptable and vise versa. For each objective function a number of levels are defined. For example we use the interval [0, 100] where the values of transformed objectives get. It can be divided into 6 portions by five levels – say 10, 35, 50, 65, 80. The sense of each level is acceptability of the solution by the DM with 10%, 35%, etc. These levels can be changed during the process of solving and their number also can be decreased up to one or vise versa. In our computer realization the maximal number of levels is limited up to 5. The reason is that the maximal number of values for easy and comfortable evaluation does not exceed 7 or 8.

The following SC problems are used:

a) a linear filter – arithmetic mean:

(3) $$l(x) = \sum_{i=1}^{k} \frac{f_1(x) + \ldots + f_k(x)}{k};$$

b) geometrical mean

(4) $$g(x) = \sqrt[k]{f_1(x) \ldots f_k(x)};$$

c) a max-min filter

(5) $$m(x) = \min_{i=1,\ldots,k} \{ f_i(x) \}.$$

Each of these filters is maximized over the feasible discrete set in the following way.

On the basis of a given set of levels for each objective a two-way joining results are plotted with the corresponding colour, defined in advance.

The DM moves these limits along the scale and in this way he filters the visualized multi-dimensional spaces and focuses precisely on the decisions, which he/she is interested in.

In other words a graphical solution of one of the above filters is performed on the base of two-way joining results approach.

It is known that the solutions of linear, geometrical and max-min filter are efficient points.

For an example we shall prove the result for a max-min filter.

P r o o f. The SC problem is: $\max_X m(x)$.

Let us assume that x' is an optimal solution and that it is not an efficient solution. Then another point x'' exists such that $f(x)'' \geq f(x')$ and for at least one index strong inequality holds. Then $m(x'') \geq m(x')$ and this is in contradiction with the optimality of x'. ■

Note that the different filters produce different efficient points depending on the properties of the problem solved.

The algorithm of an apriori method can be summarized as follows:

Step 1. Perform discretization of the feasible set.

Step 2. Select number and values of limits for each objective

Step 3. Select type of filter.

Step 4. Solve the SC problem with a chosen filter as objective function and by using two-way joining results.

Step 5. Evaluate the received efficient solution. If it is satisfactory – stop. If not, start the procedure again.

Remarks:

1. In step 4 the DM locates the efficient point that is optimal solution of a filter by moving the limits and possibly changing their number.

2. In our realization the maximal number of levels for each objective is up to 5 and can be reduced up to 1.

3. As it is seen this approach is non-interactive. Its advantage is the simplicity and the new way of geometrical solving of scalarizing problem.

4. When the number of decision variables is greater than 2, two-way joining result approach includes the next variables.

5. Current limitations of the proposed approach are that the number of decision variables has not to be too large. Also the number of levels is recommendable to be approximately no more than 5.

2.2. An interactive method

We again consider the above MCDM problem.

Again we suppose that the DM sets levels in the objective space to express his/her preferences for acceptable solution.

We know that the ideal point f^* is un-achievable. Then the question is how to modify it into a point $f^{*\mathrm{M}}$ so the new point to be achievable in some sense and also to keep the preferences of the DM as high as possible.

Further, an efficient point is generated in accordance to the modified ideal point.

If it is acceptable, the procedure stops. If not, the ideal point is modified according to new preferences of the DM and process continues until acceptable solution is found.

This is in general the idea of here presented interactive method.

For this purpose we use the following SC problem:

(5) $$\min_X \varphi(x) = \min_X \sum_{i=1}^{k} (p_i f_i^* - f_i(x))^+ ,$$

where the set X is the set of feasible alternatives and $\sum_{i=1}^{k} p_i = 1$.

This scalarazing function $\varphi(x)$ realizes our idea about the thresholds as follows.

The term $p_i f_i^*$ is the threshold value. It is formed by the ideal value f^* multiplied by p_i, a number from the interval [0, 1]. The term $(p_i f_i^* - f_i(x))^+$ is the underachievement (from below) to the threshold and the SC function minimizes the sum of underachievements. Of course another L_p norm can be used also where p is a number within the interval [1, +ω].

Let us consider some properties of the proposed SC problem.

Theorem 1. If at least one or a group or all reservation levels are feasible (dominated) then there exists a nondominated solution that dominates the corresponding level(s).

P r o o f. Let us assume for example that the dominated thresholds have the indices 1, 2, …, k_1; $k_1 \le k$. Then at least one feasible solution x' exists such that $f_i(x') \ge p_i f_i^*$ for $i = 1, 2, \ldots, k_1$.

Then following this sequence at least one nondominated solution exists that dominates the set of dominated thresholds. ■

Theorem 2. If the set of thresholds are all non-feasible, then the solution is non-dominated point that minimizes the sum of under-achievements to thresholds.

P r o o f. Our SC problem is correctly defined and it has at least one feasible solution. Let us assume that its optimal solution x' is not non-dominated point. Then another point x'' exists such that $f(x'') \ge f(x')$ and for at least one index strong inequality holds.

From this and from the definition of $\varphi(x)$ it follows that $\varphi(x'') \le \varphi(x')$ and this is contradiction with the optimality of x'. ■

Some comments about uniqueness of the solutions of SC problem. Because we use sum term the solution is not necessarily unique. But at least one non-dominated point exists that is a solution of problem (6). This fact follows by using some non-dominance test.

Theorem 3. If the point x' is an efficient point, then a set of thresholds exists such that the point x' belongs to the set of solutions of SC problem.

P r o o f. We can expect more than one set of thresholds to be a solution.

Consider $f' = f(x')$ and $F' = \sum_{i=1}^{k} f_i(x')$.

Then the coefficients $\{p_i\}$ have to satisfy the following conditions:

$$\sum p_i f_i^* = F',$$

(6) $$\sum_{i=1}^{k} p_i = 1,$$

$$p_i \geq 0,\ i = 1, 2, \ldots, k.$$

This system has an infinite number of solutions. ■

If the DM has difficulties in setting the levels p_i, we propose to compute them by reservation levels f_i^r. Namely

$p_i = \dfrac{f_i^r - f_i^-}{f_i^* - f_i^-}$, $i = 1, \ldots, k$, where f^* is the ideal point, f^- is the point formed by the minimal values of the objectives

The proposed SC can be compared with the approaches of interactive goal programming of W e i s t r o f f e r [7] and the reference point approach of W i e r - z b i c k i [6]. In these methods the possible under-achievements and possibly over-achievements are compared directly with the desired levels or reference points.

3. An illustrative example

We shall make (comparative) analysis of the geometric parameters and the quality of welded-up seams obtained with surfaceelectric arch welding up in a protective environment of a gas mixture using the tubular electrode wire Fluxofil 56 and the compact electrode wire LNM 420FM.

To achieve the aim preset, a number of planned experiments have been carried out. During the laboratory experiments surface samples of steel trade Ст20 (St20) have been welded up with two types of electrode wire, Fluxofil 56 and LNM420FM. The protective gas mixture consists of 83% argon and 17% CO_2.

The range of variation of the controlling factors is given in Table 1.

Table 1. Values of controlling factors used to carry out the experiment

Levels	Main factors			
	X_1 Current, A	X_2 Voltage, V	X_3 Speed of welding up, m/min	X_4 Electrode output, mm
Main level (0)	200	22	0.82	13
Step of changing	50	4	0.54	3
Low limit (–1)	150	18	0.28	10
Upper limit (1)	250	26	1.36	16

To solve the main problem in the process of restoring details, namely to achieve big stiffness of the layer welded up with a minimal area of thermal influence, and having analyzed the data of tubular electrode wires for welding up, the following wires have been chosen:

– Tubular wire Fluxofil 56 – 1.4 mm (OERLIKON company) DIN 8555 with the following chemical composition: C-0.35, Mn-1.5, Si-0.53, S-0.007, P-0.012, Cr-5.2, Mo-0.64.

– Compact wire LNM 420FM (LINKOLN company) with the following chemical composition: C-0.45, Mn-0.4, Si-3, Cr-9.5.

Each geometric parameter of the weld (strengthening and width) as well as the quality of welding up is described using the regulation of functioning, in complience with which this parameter reacts to the influence of the factors shown in Table 1.

It is approximation that plays an extremely important part in developing the models as it allows using convenient polinomials binding the complexity of the models with the requiments put.

Each parameter of quality is contrallable by measuring as it is evaluated in quality in ist limited interval of changing.

The main aim of each multi-factor examination is reduced to a study on the problem described with identification and mathematical description. From the initial information, the parameters of quality under examination have a clearly expressed non-linear character related to the factors examined. Due to that reason, the determination of the functions $f_i(z)$ (equations (1)-(6)) and of their number is a complex and non-formalized procedure.

– **FLUXOFIL 56**

$$K = 2.536 + 0.148X_1 + 0.3X_2 - 0.648X_3 + 0.0865X_4 - 0.876X_1^2 +$$
$$+0.255X_1X_2 + 0.215X_1X_3 + 0.0908X_1X_4 + 0.249X_2^2 -$$
$$-0.00594X_2X_3 + 0.0733X_2X_4 - 0.257X_3^2 + 0.0972X_3X_4 + 0.362X_4^2;$$

$$H = 1.643 + 0.242X_1 - 0.212X_2 - 0.69X_3 - 0.00313X_4 - 0.0174X_1^2 - 0.0965X_1X_3 +$$
$$+0.0342X_1X_4 + 0.232X_2^2 + 0.252X_2X_3 + 0.583X_3^2 - 0.0537X_3X_4 - 0.0789X_4^2;$$

$$B = 6.405 + 1.003X_1 + 1.196X_2 - 1.757X_3 - 0.076X_4 + 0.197X_1^2 +$$
$$+0.24X_1X_2 - 0.493X_1X_3 + 0.0416X_1X_4 + 0.0588X_2^2 - 0.268X_2X_3.$$

– **LNM 420 FM**

$$K = 3.0792 - 0.41X_1 + 0.3795X_2 - 0.713X_3 + 0.1016X_4 - 0.3785X_1^2 + 0.1675X_1X_2 +$$
$$+0.1924X_1X_3 - 0.1328X_1X_4 - 0.03909X_2^2 - 0.1415X_2X_3 + 0.1195X_2X_4 -$$
$$-0.1155X_3X_4 - 0.3536X_4^2;$$

$$H = 2.389 + 0.7845X_1 - 0.247X_2 - 0.908X_3 + 0.2429X_4 + 0.06129X_1X_2 -$$
$$-0.2114X_1X_3 + 0.2278X_1X_4 + 0.2506X_2^2 + 0.02067X_2X_3 -$$
$$-0.1036X_2X_4 + 0.489X_3^2;$$

$$B = 5.6486 + 07628X_1 + 1.297X_2 - 1.416X_3 - 0.1061X_4 - 0.05479X_1^2 + 0.052X_1X_2 -$$
$$-0.3108X_1X_3 - 0.08426X_2^2 - 0.36X_2X_3 + 0.9506X_3^2 + 0.09435X_3X_4 - 0.3834X_4^2.$$

The adequacy of the models is determined by the coefficient of the set correlation and has been proved by the computated value of Fisher's criterion F, which has to be bigger than the one in the table.

Using the standard methods [8], the following regression dependencies for the quality parameters examined have been obtained:

The graphic interpretation of the models developed is presented in Figs. 1-12. From them, besides the quality influence of the parameters known in practice, it is possible to determine the exact value of any parameter of the corresponding electrode wire that is under examination.

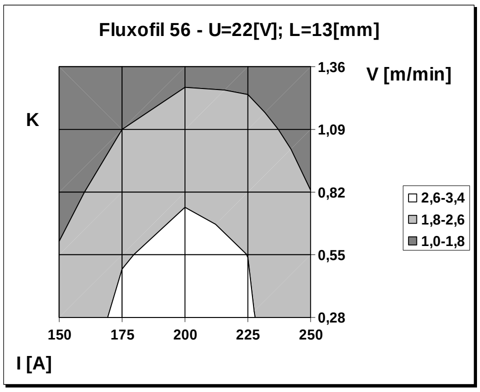

Fig. 1

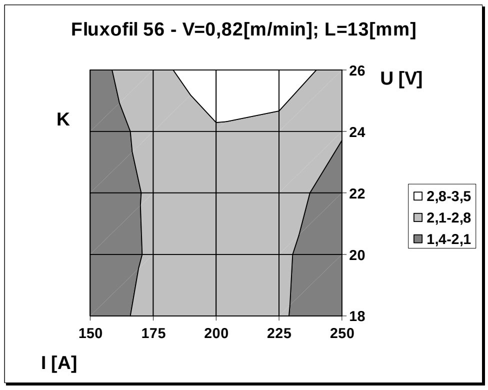

Fig. 2

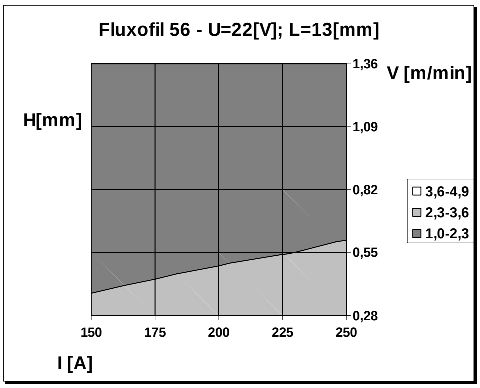

Fig. 3

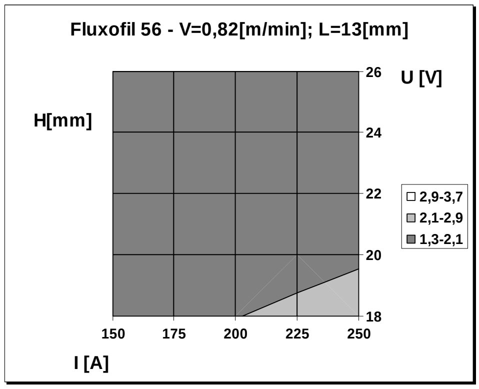

Fig. 4

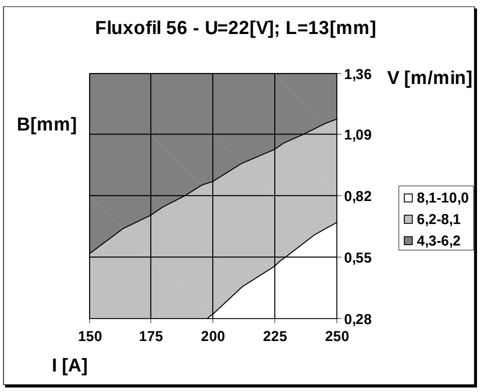

Fig. 5

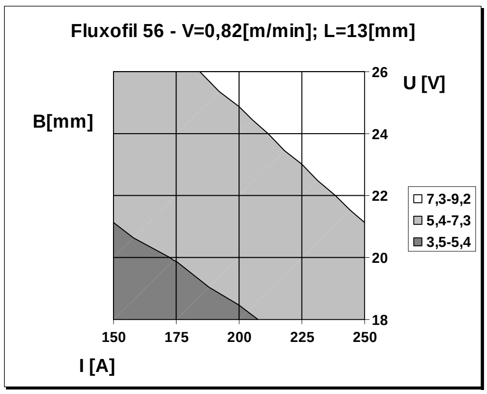

Fig. 6

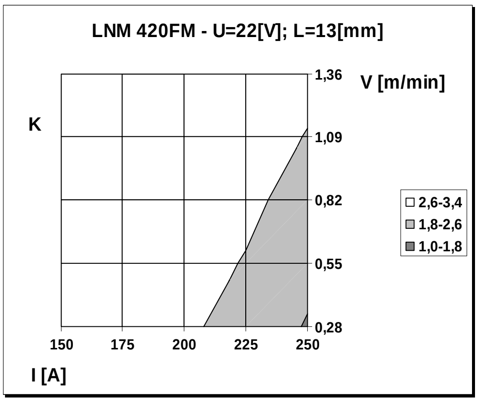

Fig. 7

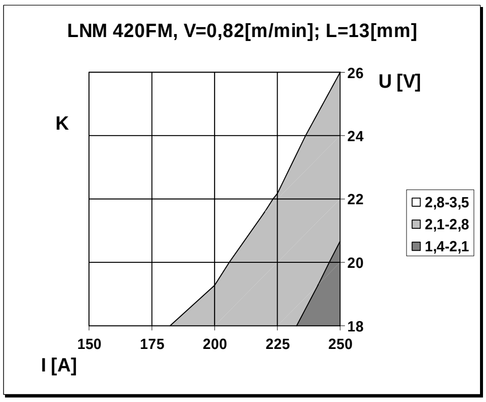

Fig. 8

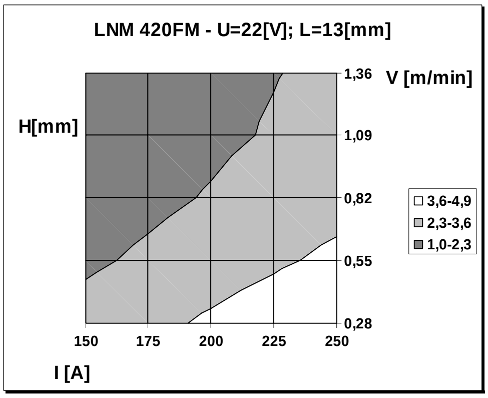

Fig. 9

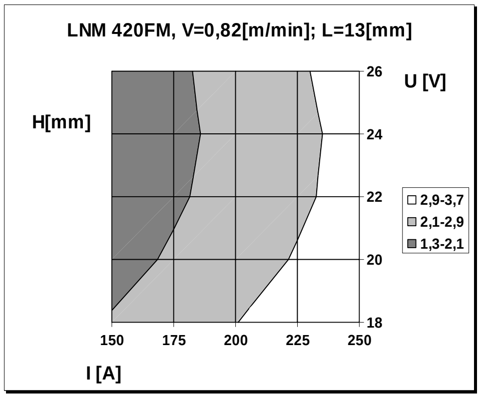

Fig. 10

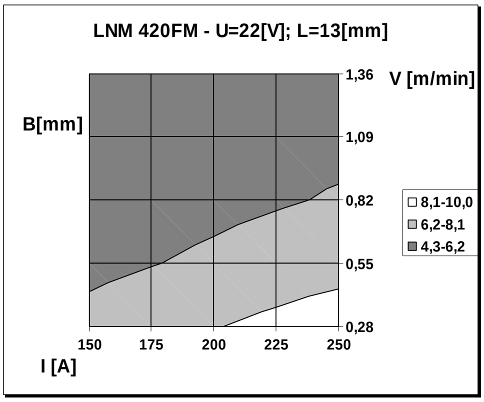

Fig. 11

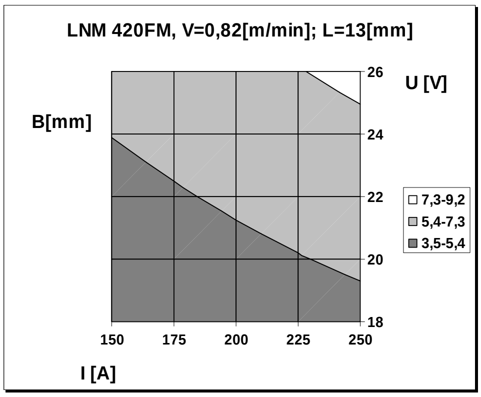

Fig. 12

In a considerably wider interval of varying the values of the voltage bigger than 22 V, the values of the welding current over 200 A and speeds up to 0.55 m/min using electrode wire Fluxofil 56, it is possible to weld up seams with a width of about 9 mm. However, the height of welding for this electrode wire under these modes is not more than 2.5 mm.

Covers of a bigger thickness up to 4 mm with the range mentioned can be implemented using LNM 420 FM.

It has been found that there are considerable differences between the electrode wires under examination while analyzing the visual assessment of the welding quality with different technological parameters. After the end of the experiment an expertise assessment of the external appearance of welding has been made using a three-rate scale and on this basis the regression dependencies 3 and 6 have been drawn.

Using the electrode wire LNM 420 FM, it is possible to achieve a considerably better quality of the seams welded up within the range of technologic modes used.

The recomended mode for a particular electrode wire according to the requirements of users can be determined by the graphic study carried out.

4. Conclusion

Two methods for solving nonlinear MCDM problems are presented. Their basic characteristics can be summarized as:

1. Dialog in terms of reservation levels.
2. Solving of current SC problem in a graphical way.
3. Comparing the possible under-achievements to the modified ideal point.
4. The methods can be used for solving non-convex MCDM problems and/or with discrete decision variables.

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Два подхода к решении задач принятия многокритериального решения (МЗПР) с ильюстративным примером

Николай Тончев[1], *Леонид Кирилов*[2]

[1] *Высшая школа транспорта, 1574 София*
[2] *Институт информационных технологий, 1113 София*

(Р е з ю м е)

В работе представлены два подхода к решении нелинейных задач принятия многокритериального решения – априорный и интерактивный. Их философия взаимствана из внутренной природы задачи применения, которая решается при помощи модели МЗПР. Рассматривается задача решения технологических проблем. Первый метод неинтерактивный, а второй – интерактивный метод с обучением.