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Development of the Evolutionary Approaches in Multiobjective Optimization

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1. Introduction

At the end of the 50s scientists from different countries [25, 18, 35] explored in detail evolutionary systems and independently came to the conclusion that they could use the theory of evolution as an instrument for optimization in the process of solution for problems of different nature with the main goal creating a population of eventual solutions using some of the most characteristic peculiarities of nature – heredity, changeability, selection and so on.

Genetic algorithms are a method for search based on the selection of the best species in the population in analogy to the theory of evolution of Ch. Darwin [18].

Their origin is based on the model of biological evolution and the methods of random search. From the bibliographical sources [25, 18, 35, 14] it is evident that the random search appeared as a realization of the simplest evolutionary model when the random mutations are modeled during random phases of searching the optimal solution and the selection is modeled as "removal" of the unfeasible versions.

From the point of view of the information change, the evolutionary search is a sequential transformation of a single fuzzy (imprecise) set of some solutions into another one. The transform itself can be named a searching algorithm or a Genetic Algorithm (GA).The GA is not simply a random search, but an efficient usage of information in the evolutionary process [14, 19].

The main goal of GA-s is twofold:

– abstract and formal explanation of the adaptation processes in evolutionary systems;

– modeling natural evolutionary processes for efficient solution of determined class of optimization and other problems.

During the last years a new paradigm is applied to solve optimization problems GA-based and modifications of GA. GA realize searching a balance between efficiency and quality of solutions at the expense of selecting the strongest alternative solution [19].

Usually in the multi-objective optimization problems several criteria (objective functions) are optimized simultaneously in a set of feasible alternatives. In the general case there does not exist an alternative (solution), which is optimal for all the solutions. But there exists a set of alternatives (solutions), characterized by the following property: each improvement of the value of one of the criteria leads to the deterioration of the value of at least one of the other criteria. A set of alternative solutions is obtained, each of the alternatives in this set could be a solution of the multiobjective problem.

The notion of optimality was originally introduced by F. Edgeworth in 1881 and later generalized by V. Pareto in 1896. It is called Edgeworth-Pareto optimum or Pareto optimum, use of this concept almost always gives not a single solution but a set of them, which is called the Pareto optimal set. The vectors of the decision variables corresponding to the solutions included in the Pareto optimal set are called nondominated. The plot of the objective functions whose nondominated vectors are in the Pareto optimal set is called the Pareto front [6].

The Operations Research community has developed approaches to solve MOPs since the 1950s. Currently, a wide variety of mathematical programming techniques to solve MOPs are available in the specialized literature. However, mathematical programming techniques have certain limitations when tackling MOPs. Many of them are susceptible to the shape of the Pareto front and may not work when the Pareto front is concave or disconnected. Others require differentiability of the objective functions and the constraints. Also, most of them only generate a single solution from each run. Thus, several runs (using different starting points) are required in order to generate several elements of the Pareto optimal set. In contrast, evolutionary algorithms deal simultaneously with a set of possible solutions (the so-called population) which allows us to find several members of the Pareto optimal set in a single run of the algorithm. Additionally, evolutionary algorithms are less susceptible to the shape or continuity of the Pareto front (they can easily deal with discontinuous and concave Pareto fronts) [6].

2. The first steps in the application of the evolutionary approaches in solving Multiobjective Optimization Problems (MOP)

The initial research of the possible application of the evolutionary approaches in solving MOP has appeared in the middle of the 20th century in which, however, no actual multi-objective evolutionary algorithm (MOEA) was developed (the multi-objective problem was restated as a single-objective problem and solved with a genetic algorithm). David Schaffer is normally considered to be the first to have designed an MOEA during the mid-1980s [40]. Schaffer's approach, called Vector Evaluated Genetic Algorithm (VEGA) consists of a simple genetic algorithm with a modified selection mechanism. At each generation, a number of sub-populations were generated by performing proportional selection according to each objective function in turn. These sub-populations would then be shuffled together to obtain a new population, on which the GA would apply the crossover and mutation operators in the usual way. VEGA had a number of problems, from which the main one had to do with its inability to retain solutions with acceptable performance, perhaps above average, but not outstanding for any of the objective functions. These solutions were perhaps good candidates for becoming nondominated solutions, but could not survive under the selection scheme of this approach.

After VEGA, researchers adopted for several years other naive approaches. The most popular were the linear aggregating functions, which consist in adding all the objective functions into a single value which is directly adopted as the fitness of an evolutionary algorithm [13]. Lexicographic ordering was another interesting choice. In this case, a single objective (which is considered the most important) is chosen and optimized without considering any of the others. Then, the second objective is optimized but without decreasing the quality of the solution obtained for the first objective. This process is repeated for all the remaining objectives [17].

Despite all these early efforts, the direct incorporation of the concept of Pareto optimality into an evolutionary algorithm was first hinted by David E. Goldberg in his seminal book on genetic algorithms [18]. While criticizing Schaffer's VEGA, Goldberg suggested the use of nondominated ranking and selection to move a population toward the Pareto front in a multi-objective optimization problem. The basic idea is to find the set of solutions in the population that are Pareto nondominated by the rest of the population. These solutions are then assigned the highest rank and eliminated from further contention. Another set of Pareto nondominated solutions is determined from the remaining population and are assigned the next highest rank. This process continues until all the population is suitably ranked. Goldberg also suggested the use of some kind of niching technique to keep the GA from converging to a single point on the front.

3. The enhancement of the research in the last decade of the 20th century

Goldberg does not provide a real execution of his procedures in multi-objective optimization (MOP), but in fact all the variants of this algorithm, later developed, are on the basis of his theory and are influenced by them.

3.1. Genetic algorithm of Srinivas and Deb

This algorithm is suggested by S r i n i v a s and D e b [42] and it is known as Nondominated Sorting Genetic Algorithm (NSGA). The NSGA is based on several layers of classifications of the individuals as suggested by G o l d b e r g [18]. Before selection is performed, the population is ranked on the basis of nondomination: all nondominated individuals are classified into one category (with a dummy fitness value, which is proportional to the population size, to provide an equal reproductive potential for these individuals). To maintain the diversity of the population, these classified individuals are shared with their dummy fitness values. Then this group of classified individuals is ignored and another layer of nondominated individuals is considered. The process continues until all individuals in the population are classified. Since individuals in the first front have the maximum fitness value, they always get more copies than the rest of the population. The algorithm of the NSGA is not very efficient, because Pareto ranking has to be repeated over an over again.

3.2. Genetic algorithm of Horn, Natpliotis and Goldberg

This algorithm is suggested in [26] and it is known as Niched-Pareto Genetic Algorithm (NPGA). The NPGA uses a tournament selection scheme based on Pareto dominance. The basic idea of the algorithm is: two individuals are randomly chosen and compared against a subset from the entire population (typically, around 10% of the population). If one of them is dominated (by the individuals randomly chosen from the population) and the other is not, then the nondominated individual wins. All the other situations are considered a tie (i.e., both competitors are either dominated or nondominated). When there is a tie, the result of the tournament is decided through fitness sharing.

3.3. Genetic algorithm of Fonseca and Fleming

This algorithm is suggested in [15] and it is known as Multi-Objective Genetic Algorithm (MOGA). In MOGA, the rank of a certain individual corresponds to the number of chromosomes in the current population by which it is dominated. All nondominated individuals are assigned the highest possible fitness value (all of them get the same fitness, such that they can be sampled at the same rate), while dominated ones are penalized according to the population density of the corresponding region to which they belong (i.e., fitness sharing is used to verify how crowded is the region surrounding each individual).

3.4. Investigations of Tanaka

Tanaka has developed the first scheme to incorporate user’s preferences into an MOEA [43]. In real-world applications it is normally the case that the user does not need the entire Pareto optimal set, but only a small portion of it. Normally the user can define certain preferences that can narrow the search and that can magnify certain portions of the Pareto front.

3.5. Comparative analysis

Making comparative analysis of the algorithms, above pointed, it is established with no doubt, that MOGA is excelling, followed by NPGA and NSGA.

The main conclusion about the implementations of this generation of GA is, that in order to be successful, one MOEA, a good mechanism has to be combined for the selection of the nondominated species, with a good mechanism for variety support, which will enable the generation of MOEA.

4. The new studies for MOEA – variety of measures for realization quality, enabling quantitotive comparison

The wide development of MOEA in the recent years has begun after the works of E. Zitzler and L. Thiele [46], due to it the elitism has become a standard mechanism in the development in this direction. In the context of multi-objective optimization, elitism usually (although not necessarily) refers to the use of an external population (also called secondary population) to retain the nondominated individuals found along the evolutionary process. The main motivation for this mechanism is the fact that a solution that is nondominated with respect to its current population is not necessarily nondominated with respect to all the populations that are produced by an evolutionary algorithm. Thus, what we need is a way of guaranteeing that the solutions that we will report to the user are nondominated with respect to every other solution that our algorithm has produced. Therefore, the most intuitive way of doing this is by storing in an external memory (or archive) all the nondominated solutions found. If a solution that wishes to enter the archive is dominated by its contents, then it is not allowed to enter. Conversely, if a solution dominates anyone stored in the file, the dominated solution must be deleted.

After the offered by Zitzler theory, most of researchers began to started to incorporate external populations in their MOEAs and the use of this mechanism (or an alternative form of elitism) became a common practice. In fact, the use of elitism is a theoretical requirement in order to guarantee convergence of an MOEA and therefore its importance.

Further on the exposition we shall make an attempt to discuss some of the new MOEA suggested, which have been implemented in the last years.

4.1. Algorithm of Zitzler and Thiele

This algorithm is known as Strength Pareto Evolutionary Algorithm (SPEA) and was introduced in [46]. This approach was conceived as a way of integrating different MOEAs. SPEA uses an archive containing nondominated solutions previously found (the so-called external nondominated set). At each generation, nondominated individuals are copied to the external nondominated set. For each individual in this external set, a strength value is computed. This strength is similar to the ranking value of MOGA [15], since it is proportional to the number of solutions to which a certain individual dominates. In SPEA, the fitness of each member of the current population is computed according to the strengths of all external nondominated solutions that dominate it. The fitness assignment process of SPEA considers both closeness to the true Pareto front and even distribution of solutions at the same time. Thus, instead of using niches based on distance, Pareto dominance is used to ensure that the solutions are properly distributed along the Pareto front. Although this approach does not require a niche radius, its effectiveness relies on the size of the external nondominated set. In fact, since the external nondominated set participates in the selection process of SPEA, if its size grows too large, it might reduce the selection pressure, thus slowing down the search. Because of this, the authors decided to adopt a technique that primes the contents of the external nondominated set so that its size remains below a certain threshold.

4.2. Second algorithm of Zitzler and Thiele

This algorithm is known as Strength Pareto Evolutionary Algorithm 2 (SPEA2) [33, 45] has three main differences with respect to its predecessor:

(1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals by which it is dominated;

(2) it uses a nearest neighbor density estimation technique which guides the search more efficiently;

(3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions.

4.3. Algorithm of Knowles and Corne

This algorithm is known as Pareto Archived Evolution Strategy (PAES) is introduced in [31]. PAES consists of a 1+1 evolution strategy (i.e., a single parent that generates a single offspring) in combination with a historical archive that records the nondominated solutions previously found. This archive is used as a reference set against which each mutated individual is being compared. Such a historical archive is the elitist mechanism adopted in PAES. Special feature of this algorithm is the procedure used to maintain diversity which consists of a crowding procedure that divides objective space in a recursive manner. Each solution is placed in a certain grid location based on the values of its objectives (which are used as its “coordinates” or “geographical location”). A map of such grid is maintained, indicating the number of solutions that reside in each grid location. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective space). This adaptive grid (or variations of it) has been adopted by several modern MOEAs [3].

4.4. Algorithm of Deb and Agarwal

This algorithm is knon as Nondominated Sorting Genetic Algorithm II (NSGA-II) is introduced in [10] as an improved version of the NSGA [42]. In the NSGA-II. for each solution one has to determine how many solutions dominate it and the set of solutions to which it dominates.The NSGA-II estimates the density of solutions surrounding a particular solution in the population by computing the average distance of two points on either side of this point along each of the objectives of the problem. This value is the so-called crowding distance. During selection, the NSGA-II uses a crowded-comparison operator which takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred). The NSGA-II does not use an external memory. Instead, the elitist mechanism of the NSGA-II consists of combining the best parents with the best offspring obtained (i.e., a ($\mu + \lambda$)-selection). Its mechanism is better.

4.5. Some other algorithms

Due to the enhancement of the research in this area, the algorithms, pointed in Section 4 are just a small part of those, offered in different references. We shall mention some other interesting developments:

– algorithm of C o e l l o and P u l i d o [3]– a micro-genetic algorithm is proposed for multi-objective optimization (MOMGA). The basis of this algorithm is a variant of adaptive grid, described in PAES, using one parent in combination with historical archive, which records the earlier found nondominated solutions.

– algorithm of C o r n e and al. [8] – PESA2 – an improved variant of PAES.

– The use of alternative bio-inspired heuristics for multi-objective optimization [7]. The most remarkable examples are particle swarm optimization and differential evolution [36], whose use has become increasingly popular in multi-objective optimization [1, 4]. However, other bio-inspired algorithms such as artificial immune systems and ant colony optimization have also been used to solve multi-objective optimization problems [2, 23]

5. Application domains

The advance in the research of MOEA ensures them their widening application. In order to give a general fancy for the type of applications, they could be classified in four main directions [5]: science, engineering, industry and various other directions (miscellaneous applications). Some specific areas inside any of these directions are discussed below.

– Engineering applications [5, 14, 20, 22] – electrical, hydraulic, structural, aeronautical, robotics and control and all.

– Industrial applications [5, 29, 36, 37] – design, manufacture, scheduling, management and all.

– Scientific applications [5, 21, 27, 32, 34, 39]– of chemical, analysis of spectroscopy, medical image reconstruction [32], computer aided diagnosis, machine-learning in high-dimensional data, the analysis of promoters in biological sequences in the problem to deal with [37] and all.

– Miscellaneous applications [5, 43, 41, 24] – problem of attribute selection in data mining, decisions support system, finance, optimization a forecast model, forest management and all.

The strong interest for MOEA in so many different disciplines reinforces the idea that there will be new possibilities for solving still more real-life problems.

6. Conclusions

After the attempt for a short survey it could be noted, that the scientific research in the area considered are directed towards different aspects, but one of the major aspects is the efficiency, which is regarded at algorithmic level and at data structure level [28, 30]. A variety of measures for implementation quality are suggested, it allow a quantitative (rather than only qualitative), comparison of results [44, 16, 46]. Zitzler et al. [44] stated that, when assessing performance of an MOEA, one was interested in measuring three things:

– Maximize the number of elements of the Pareto optimal set found.

– Minimize the distance of the Pareto front produced by algorithm with respect to the global Pareto front (assuming we know its location).

– Maximize the spread of solutions found, so that we can have a distribution of vectors as smooth and uniform as possible.

Concurrently with the research on performance measures, other researchers were designing test functions. K. Deb proposed a methodology to design MOPs that is widely used [9]. Later on, an alternative set of test functions was proposed, but this time, due to their characteristics, no enumerative process was required to generate their true Pareto front [11, 12]. These test functions are also scalable, their use has become spread. Researchers in the field normally validate their MOEAs with problems having three or more objective functions, and 10 or more decision variables.

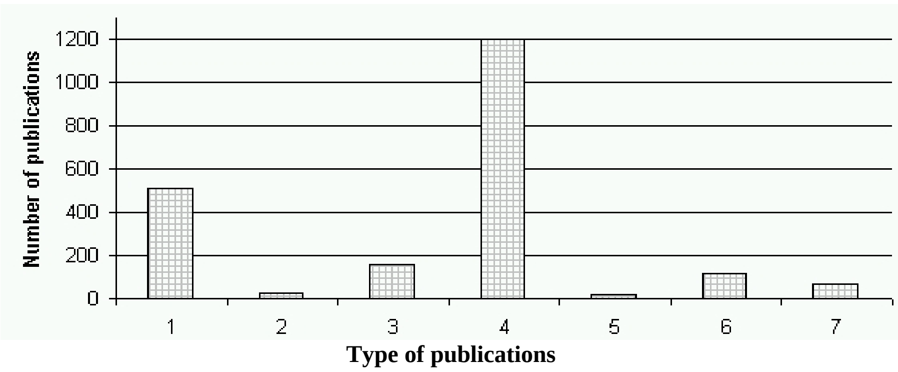

Fig. 1

For the enhancing development of the scientific investigations in thus direction (MOEA) the basic proof is the continuously increasing number of references and applications in the last ten years. In a paper of his Coello [7] represents approximate graphics of the publications according their type. Fig. 1 represents the distribution of the publications, depending on the issue: 1 – journal papers, 2 – books, 3 – book chapters, 4 – conference papers, 5 – master's theses, 6 – Ph.D. theses, 7 – technical reports. Fig. 2 gives the distribution in years.

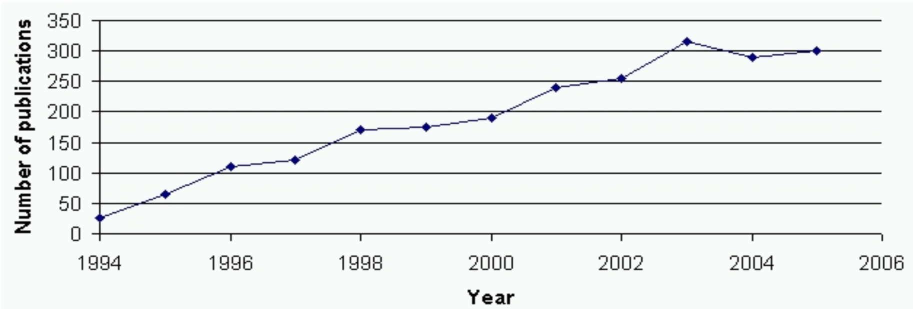

Fig. 2

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Развитие эволюционных подходов в многокритериальной оптимизации

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(Р е з ю м е)

В работе обсуждается развитие эволюционных подходов и генетических методов при решении задач многокритериальной оптимизации. Представленны некоторые из самых фундаментальных алгоритмов последных десятилетий. Показаны также и направления, связаны с использованием многокритериальных эволюционных алгоритмов и их применения. Показано развитие научных исследований в этой области в последных годах.