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Generalized Interactive Algorithm of Multicriteria Optimization*

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1. Introduction

In the interactive algorithms for multicriteria optimization the decision maker (DM) may express his/her preferences among the separate Pareto optimal solutions with the help of the values of the scalarizing problem parameters. The DM must choose the final (most preferred) solution and be responsible for this selection. The interactive methods are the best developed and wide spread methods. This is due to the following advantages:

- a small part of the Pareto optimal solutions is generated and evaluated by the DM;
- the DM has the possibility to learn with respect to the problem during the process of multicriteria problem solving;
- the DM may change (correct) his/her preferences in the process of the multicriteria problem solving;
- the DM feels more confident in his/her preferences concerning the final solution of the multicriteria problem.

The interactive algorithms developed for solving multicriteria optimization problems are characterized by the following features:

- each interactive method is designed for the solving of a given type of problems – linear, nonlinear, integer, etc.;
- each interactive method is built on the basis of a certain type of scalarizing problem and possesses the specific features of this problem, connected with the possibilities for setting DM's preferences as well as the type and time for obtaining the currently preferred Pareto optimal solution;
- the larger part of the interactive algorithms are algorithms, oriented towards learning, and this means that the DM controls the process of the multicriteria problem solving;

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• each interactive algorithm is the starting point for a software system of general purpose developed for the solution of the corresponding class of multicriteria problems and it defines to a high extent the characteristics of this system.

The interactive algorithms for multicriteria optimization use (M i e t t i n e n [7], M i e t t i n e n and M a k e l a [8], M i e t t i n e n and M a k e l a [9] different scalarizing problems like the MinMax scalarizing problem (S t e u e r and C h o o [12]), the weighted scalarizing problem WS (G a s s and S a a t y [3]), Z a d e h [20]), the scalarizing problem of a – constraint method EC (H a i m e s et al. [4]), STEM scalarizing problem (B e n a y o u n at al. [2]), STOM scalarizing problem (N a k a y a m a and S a w a r a g i [10]), the scalarizing problem of the reference point RP (W i e r z b i c k i [19]), GUESS scalarizing problem (B u c h a n a n [1]), the scalarizing problem of the modified reference point MRP (V a s s i l e v et al. [14]), the scalarizing problem of the external reference direction RD1 (K o r h o n e n and L a a k s o [5], Korhonen [6]), the scalarizing problem of the reference direction RD2 (V a s s i l e v and N a r u l a [13]), the scalarizing problem of the internal reference direction RD3 (N a r u l a et al. [11])), the classification-oriented scalarizing problem NIMBUS (M i e t t i n e n [7], M i e t t i n e n and M a k e l a [8], M i e t t i n e n and M a k e l a [9]) the classification-oriented scalarizing problem DALDI (V a s s i l e v a [17]). These problems approximate to a great extent the whole set of well-known scalarizing problems.

In the weighted scalarizing problems the DM defines his/her preferences by the values of the criteria weights, while in the scalarizing problems of the *a*-constraint method – by the selection of one function for optimization (maximization or minimization with respect to the initial multicriteria problem being solved) and defining the lower or upper bounds (with respect to the form of optimization) of change of the remaining criteria. In the scalarizing problem of the reference point the reference point is determined by the aspiration levels of the criteria and these levels are the ones that the DM wishes or agrees to be obtained in the new solution. These aspiration levels of the criteria are parameters of the scalarizing problems of the reference point. In the classification-oriented scalarizing problems not only the aspiration levels could be parameters of the multicriteria problem being solved, but also the directions and intervals of alteration in the criteria values that the DM wishes or agrees to be obtained in the new solution.

2. Generalized intercative algorithm GENWS-IM

The generalized interactive algorithm GENWS-IM with variable scalarization and parameterization is designed on the basis of the generalized scalarizing problem GENWS (V a s s i l e v a et al. [18]). This scalarizing problem is used to obtain a (weak) Pareto optimal solution, starting from the current (weak) Pareto optimal solution.

The GENWS scalarizing problem has the following form:

To minimize:

$$(1)\qquad S(x) = \max\Big(\big(\max_{k\in K^{\geq}} \big(F_k^1 - f_k(x)\big) G_k^1\ R_1 \max_{k\in K^{\leq}} \big(F_k^2 - f_k(x)\big) G_k^2\ R_2\,,$$

$$\max_{k\in K^{<}} \big(F_k^3 - f_k(x)\big) G_k^3 R_3 \max_{k\in K^{>}} \big(F_k^4 - f_k(x)\big) G_k^4 \Big) + \sum_{k\in K^0} \big(F_k^5 - f_k(x)\big) G_k^5$$

under the constraints:

(2) $$f_k(x) \ge f_k,\ k \in K^{>} \cup K^{=},$$

(3) $$f_k(x) \ge f_k - D_k,\ k \in K^{\le},$$

(4) $$f_k(x) \ge f_k - t_k^{-},\ k \in K^{><},$$

(5) $$f_k(x) \le f_k + t_k^{+},\ k \in K^{><},$$

(6) $$x \in X,$$

where:

– $G_k^1, G_k^2, G_k^3, G_k^4, G_k^5$ are scaling, normalizing or weighting coefficients;

– $F_k^1, F_k^2, F_k^3, F_k^4, F_k^5$ are parameters, connected with aspiration, current and other levels of the criteria values;

– R_1, R_2, R_3 are equal to the arithmetic operation "+" or to the separator " ,";

– D_k is a value, by which the DM agrees the criterion with an index, $k \in K^{\le}$ to be deteriorated D_k>0;

– t_k^- and t_k^+ are the upper and lower limits of the acceptable for the DM interval of alteration of the criterion with an index $k \in K^{><}$; t_k^->0, t_k^+>0;

– f_k is the value of the criterion with an index $k \in K$ in the current preferred solution;

– K is the set of all the criteria;

– $K^{\ge}$ is the set of the criteria, the current values of which the DM wishes to improve up to set by him/her desired levels F_k^1;

– $K^{>}$ is the set of the criteria, the current values of which the DM wishes to improve;

– $K^{\le}$ is the set of the criteria, the current values of which the DM agrees to be deteriorated to set by him/her acceptable levels F_k^2, but not more than given values D_k; D_k>0;

– $K^{<}$ is the set of criteria, the current values of which the DM agrees to be deteriorated;

– $K^{=}$ is the set of criteria, the current values of which the DM does not wish to be deteriorated;

– $K^{><}$ is the set of criteria, the values of which the DM agrees to be altered in given intervals;

– K^0 is the set of criteria, for which the DM does not explicit preferences about the alterations of the criteria values.

The scalarizing problem GENWS has got sense, when $K^{\ge} \ne \emptyset$ or $K^{>} \ne \emptyset$ or $K^0 = K$. That is why, we assume that $K^{\ge}$ and/or $K^{>} \ne \emptyset$ or $K^0 = K$.

Altering some parameters of the generalized sclarizing problem GENWS, (V a s s i l e v a et al. [18]), the greater part of the known scalarizing problems can be obtained. Moreover, new scalarizing problems with definite characteristics can be generated as well.

The generalized intercative algorithm GENWS-IM is developed on the basis of the GENWS scalarizing problem and has the following characteristics:

- the DM may set his/her preferences with the help of the criteria weights, ε- constraints, desired and acceptable levels of change of the criteria values, desired and acceptable levels, directions and intervals of alteration in the criteria values, etc.;
- during the process of the multicriteria problems solving, the DM may change the way of presenting his/her preferences;
- starting from one and the same current Pareto optimal solution and applying different scalarizing problems (with respective alteration of GENWS parameters), the DM may obtain different new Pareto optimal solutions at a given iteration, and this opportunity is especially useful in education and in comparison of different scalarizing problems.

In the generalized interactive method, called GENWS-IM, the steps for setting DM's preferences and for evaluation of the information obtained are complicated, depending on what type of a problem is solved, whether at one iteration one or more solutions are sought, whether real-life problems are being solved or the DM is learning the specifics of different scalarizing problems, searching for solutions at one and the same DM's preferences, defined in a different way. The step, concerning the solving of the single-criterion problem is also complicated with respect to the type of the multicriteria problem being solved (linear, nonlinear, integer), depending on the type and number of the solutions obtained (exact or approximate).

The main steps of the generalized interactive algoriyhm GENWS-IM are as follows:

Step 1. Determining the type of the problem being solved – linear, nonlinear, linear integer and nonlinear integer.

Step 2. Finding an initial Pareto optimal solution and passing to Step 4.

Step 3. In case the DM wishes to obtain a new solution at another type of preferences, Step 5 is executed.

Step 4. Evaluation of the Pareto optimal solution(s) obtained. If the DM approves this solution (one of these solutions) as the most preferred solution, the algorithm passes to Step 9.

Step 5. Selection of the type of DM's current preferences which are going to be set by the DM.

Step 6. Choice of the corresponding scalarizing problem and setting of the current preferences.

Step 7. Selection of the type and number of the new Pareto optimal solutions searched for.

Step 8. Solution of the corresponding scalarizing problem and passing to Step 3.

Step 9. Stop of the operation of the generalized interactive method.

The DM defines the type of the problem being solved at Step 1 – linear or nonlinear, continuous or integer, etc. At Step 2, independently on the DM, an initial Pareto optimal solution is found for the corresponding multicriteria optimization problem being solved. At Step 3, the DM must decide whether before the evaluation of the current Pareto optimal solution(s) obtained, he/she wishes to obtain a new solution with another type of preferences and using other scalarizing problems. Otherwise, the evaluation of the Pareto optimal solution(s) obtained is accomplished at Step 4. The evaluation is done depending on DM's purpose – whether a multicriteria problem is practically solved, whether the DM is educated or whether he/she investigates different scalarizing problems. An extended description will be presented for the next four steps (fifth, sixth, seventh and eighth) of the following form:

Step 5.1. In case the DM chooses to set his/her preferences by criteria weights, Step 6.1 is executed.

Step 5.2. If the DM decides to define his/her preferences with the help of ε-constraints, the algorithm passes to Step 6.2.

Step 5.3. In case the DM chooses to set his/her preferences by desired and acceptable levels of change of the criteria values, Step 6.3 follows.

Step 5.4. If the DM decides to set his/her preferences by desired or acceptable levels of alteration of the criteria values with respect to a reference direction, the algorithm passes to Step 6.4.

Step 5.5. In case the DM chooses to define his/her preferences by desired and acceptable levels, directions and intervals of alteration in the criteria values, Step 6.5 is executed.

Step 6.1. Selection of the weighted scalarazing problem WS or the MinMax scalarzing problem (WS by default) by the DM, setting of the current criteria weights and passing to Step 7.1.

Step 6.2. Selection of the scalarizing problem of ε-constraint method EC by the DM, selection of a criterion for optimization, setting the current ε-values of the remaining criteria and passing to Step 7.1.

Step 6.3. Selection of one scalarizing problem by the DM among the group of scalarizing problems of the reference point – STEM, STOM, RP, GUESS and MRP (RP by default), setting the current desired and acceptable levels of alteration of the criteria values and passing to Step 7.1.

Step 6.4. Selection of one scalarizing problem by the DM among the group of the scalarizing problems of the reference directions – RD1, RD2 and RD3 (RD1 by default), setting the current desired and acceptable levels of change in the criteria values and passing to Step 7.1.

Step 6.5. Selection of one scalarizing problem by the DM among the group of the classification-oriented scalarizing problems – NIMBUS and DALDI (DALDI by default), setting the current desired and acceptable levels, directions and intervals of change in the criteria values.

Step 7.1. In case the multicriteria problem is a continuous problem, Step 7.3 follows.

Step 7.2. The DM determines the number of the integer solutions sought. In case their number is greater than one, Step 8.4 is executed, otherwise – Step 8.3.

Step 7.3. The DM defines the number of the continuous solutions being searched for. If their number is greater than one, Step 8.2 is executed.

Step 8.1. Solving the continuous scalarizing problem selected and passing to Step 3.

Step 8.2. Parametric solving of the continuous scalarizing problem selected and passing to Step 3.

Step 8.3. Solving the integer scalarizing problem selected and passing to Step 3.

Step 8.4. Approximate solving of the integer scalarizing problem selected and passing to Step 3.

The most wide-spread interactive algorithms are the algorithms of the reference point, the algorithms of the reference direction and the classification-oriented algorithms. GENWS-IM interactive algorithms is an algorithm with variable scalarization and parametrization and it is a generalization of a large number of these algorithms. This generalization is with regard to the classes of the problems solved, the type of defined preferences, the number and type of the scalarizing problems used, the strategies utilized in the search for new Pareto optimal solutions. GENWS-IM interactive algorithm is a good basis for the development of a software system with improved DM's interface, in relation both to the class of the multicriteria optimization problems being solved and also to the possibilities for setting his/her preferences.

MKO-2 software system is developed on the basis of the generalized interactive algorithm GENWS–IM. The first version of the system is designed to support the solution of linear and linear integer multicriteria problems only. MKO-2 system is an extension of the software system MKO-1 (Vassilev et al. [15], (Vassilev et al. [16]). All the functions of MKO-1 system referring to the entry, correction, visualization, storing and printing of the input and intermediate results, are preserved in MKO-2 system. The expansions of MKO-2 system are connected with the extension of DM's possibilities to set his/her preferences with the help of criteria weights, ε-constraints, desired and acceptable levels of alteration in the criteria values, desired and acceptable directions of change of the criteria values, as well as desired and acceptable levels, directions and intervals of alteration of the criteria values. Thirteen different types of scalarizing problems are generated in MKO-2 system in order to realize these possibilities. Depending on DM's preferences, these scalarizing problems are automatically generated by the generalized scalarizing problem GENWS by changing the structure and the parameters. The structure and the organization of the access to the file storing the data about the process of multicriteria problems solution are complicated, as well. From the viewpoint of the DM, the differences between MKO-1 and MKO-2 software systems consist only in the presence of additional windows in MKO-2 system, intended for the definition of his/her preferences. Fig. 1 and Fig. 2 show two of these windows. The window, presented in Fig. 1, is designed to identify the type of DM's preferences. The DM may select among five types of preferences and let assume that he/she has selected to set the preferences by aspiration levels (or reference point). The window, shown in Fig. 2, is intended for selection of the scalarizing problem from the set of already known scalarizing problems of the reference point.

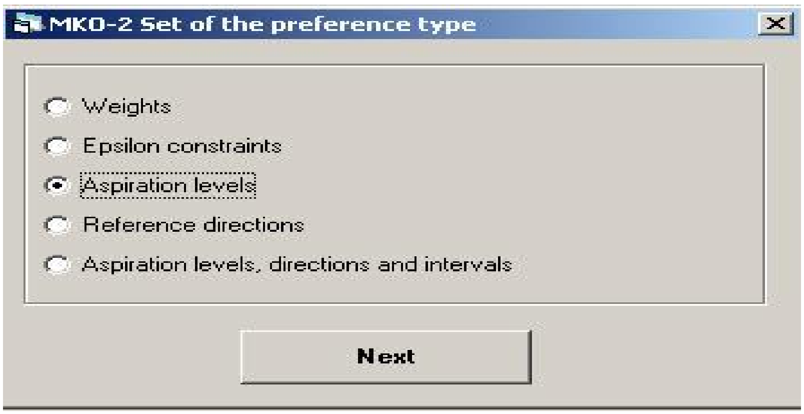

Fig. 1

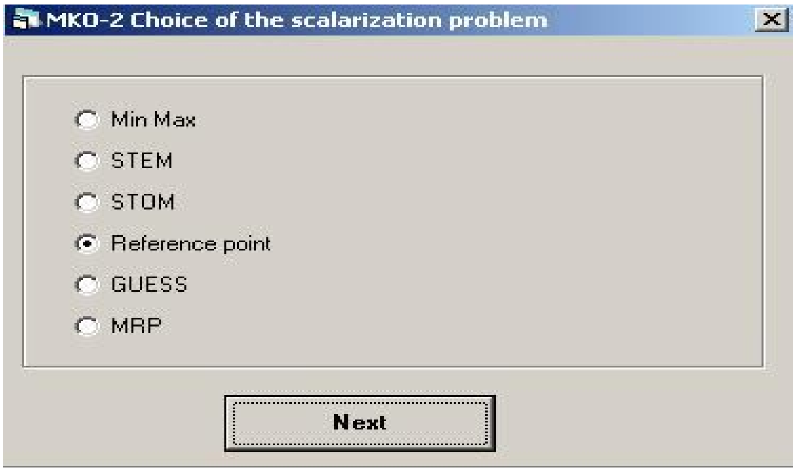

Fig. 2

Conclusion

The generalized interactive algorithm GENWS-IM is an interactive algorithm with variable scalarization and parametrization. It is a generalization of a large part of the interactive algorithms developed up to the present moment. This generalization is with respect to the classes of the problems solved, the type of the defined preferences, the number and type of the applied scalarizing problems, the strategies used in the search for new Pareto optimal solutions. GENWS-IM interactive algorithm is a good basis for the development of MKO-2 software system with improved user's interface in relation both to the class of the multicriteria optimization problems solved and also to the possibilities for setting his/her preferences.

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Интерактивный алгоритм многокритериальной оптимизации с променливой скаляризацией и параметризацией

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(Р е з ю м е)

В статье описан обобщенный алгоритм GENWS-IM, который представляет интерактивный алгоритм с променливой скаляризацией и параметризацией. Обобщение относится к классам решаемых задач, к типу задаваемых предпочитаний, к числу и типу используемых скаляризирующих задач, также как и к используемым стратегиям для поиска новых Парето оптимальных решений. Рассмотрены и основные характеристики разработанной на основе интерактивного алгоритма GENWS-IM программной системы МКО-2 для ассистирования решения задач многокритериальной оптимизации. Система отличается улучшенным интерфейсом как в отношении классов многокритериальных задач, которые могут решаться при помощи этой системы, так и в отношении возможностей, предоставленных лицу, принимающему решение (ЛВР) при задании его/ее предпочитаний в процессе поиска наиболее предпочитаемого им/ей Парето оптимального решения.