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Workspace of a Hybrid (Parallel-Serial) Robot Manipulator

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1. Introduction

A hybrid type manipulation system is a combination of closed-chain and open-chain mechanisms or it is a sequence of parallel mechanisms. Parallel manipulators have high stiffness and large load capacity, but suffer from a very limited workspace volume. The hybrid manipulators overcome the limited workspace of the parallel manipulators and at the same time they have high stiffness.

The determination of the workspace of the hybrid manipulators is a challenging problem due to the complexity of the solutions of the kinematic problems. Workspace of parallel manipulators has been studied by several researchers (Kumar [2]; Luh et al. [3]; Merlet et al. [4]). However, the determination of the workspace of hybrid manipulation systems has been presented in few papers (e.g. Ricard and Gosselin [5]). Different types of workspaces have been used, e.g. Kumar and Waldron (Kumar and Waldron [1]) introduced the terms reachable and dextrous workspace. Since then, some more terms have been used: workspace with constant orientation, maximal workspace, inclusive maximal workspace, total orientation workspace [3].

2. Determination of the workspace of the hybrid manipulator

This paper presents workspace determination of a new type of hybrid manipulator with six degrees of freedom. The algorithm for the determination of the boundaries of the workspace is based on the solution of the inverse kinematic problem of the consider hybrid manipulator.

2.1. Inverse position analysis

The considered hybrid manipulator (Fig.1) consists of two serially connected parallel mechanisms. Each mechanism has 3 degrees of freedom. The overall degrees of freedom for the considered hybrid robot are 6. The lower parallel mechanism has three legs with different structures: *SPS*, $R_2 \perp P \perp S$ and $R_1 \perp P \perp R_3$. The upper parallel mechanism has two identical legs with *SPS* structure and a $R_4 \perp R_5 \perp R_6$ leg. In this case, the axes of the R_3 and R_4 revolute joints are perpendicular. The three prismatic joints of the first mechanism and the two prismatic joints and the R_5 revolute joint of the upper mechanism are active (actuated). For details considering the kinematics of this hybrid manipulator see (T a n e v [6]).

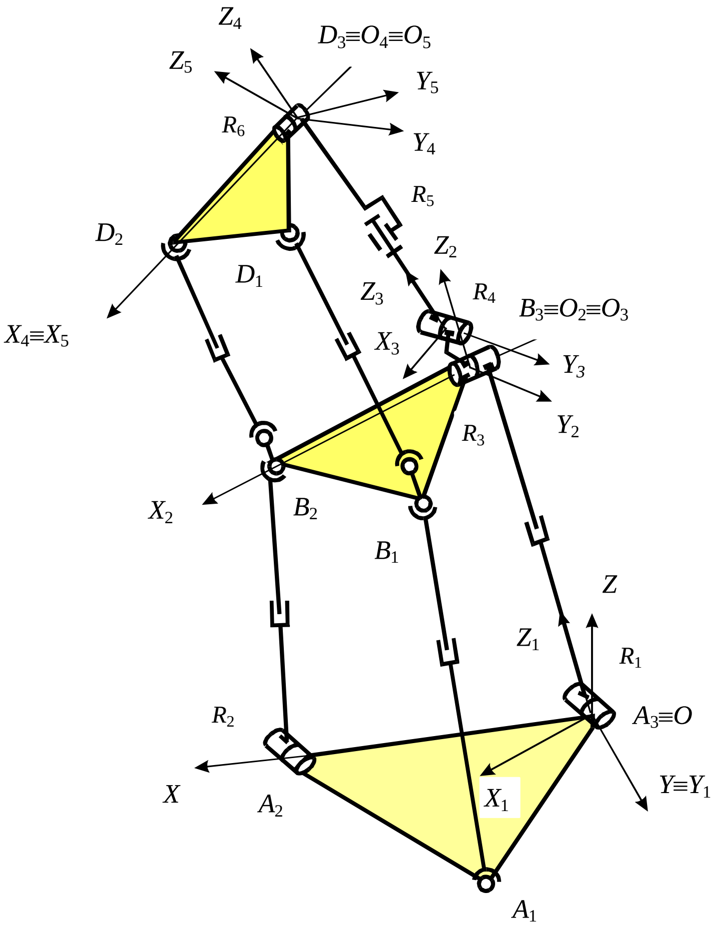

Fig. 1. The hybrid robot manipulator

The axes of the revolute joints R_3 and R_6 are perpendicular to the legs A_3B_3 and B_3D_3, respectively. The reference coordinate frame *XYZ* is attached to the base and its origin is at point A_3. The *OX* axis is collinear with the line A_3A_2. The coordinate frame $O_1X_1Y_1Z_1$ is attached to the A_3B_3 leg and $O_2X_2Y_2Z_2$ is attached to the middle moving platform. The O_2X_2 axis is collinear with the line B_3B_2. The coordinate frames $O_3X_3Y_3Z_3$ and $O_4X_4Y_4Z_4$ are fixed to the two links of the B_3D_3 leg, respectively. In this case, the axes of the revolute joints R_3 and R_4 are perpendicular and their origins are coincident. The last coordinate frame $O_5X_5Y_5Z_5$ is attached to the upper moving platform and its origin is at the revolute joint R_6, so that the axis of rotation (O_5X_5) is directed along the line D_3D_2. Therefore, the corresponding transformation matrices can be written as:

(1) $$\mathbf{A}_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & 0 \\ -s_1 & 0 & c_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ 0 & 1 & 0 & 0 \\ -s_3 & 0 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_5 & -s_5 & 0 \\ 0 & s_5 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where s_i and c_i , i = 1, ..., 5, indicate $\sin\theta_i$ and $\cos\theta_i$, respectively; $L_3 = A_3B_3$; $L_6 = B_3D_3$.

The transformation matrix for the considered hybrid robot manipulator can be written as follows:

(2) $$\mathbf{A} = \mathbf{A}_1 \mathbf{A}_2 \mathbf{A}_3 \mathbf{A}_4 \mathbf{A}_5.$$

The inverse problem of the kinematic analysis is to obtain the required actuator coordinates for given position and orientation of the end-effector. The six actuator coordinates which are to be obtained are the leg lengths L_i (i = 1, ..., 5) and the angle θ_4 . Let the position and orientation of the end-effector be given by the transformation matrix **A**, i.e.,

(3) $$\mathbf{A} = \begin{bmatrix} l_1 & l_2 & l_3 & r_x \\ m_1 & m_2 & m_3 & r_y \\ n_1 & n_2 & n_3 & r_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where r_x , r_y , r_z are the coordinates of the centre of the $O_5X_5Y_5Z_5$ coordinate frame which is attached to the upper moving platform.

Postmultiplying both sides of equation (2) by $\mathbf{A}_5^{-1}\mathbf{A}_4^{-1}$ we obtain

(4) $$\mathbf{A}\,\mathbf{A}_5^{-1}\mathbf{A}_4^{-1} = \mathbf{A}_1\mathbf{A}_2\mathbf{A}_3.$$

Then, from equation (2), (3) and (4) can be derived the angle θ_5 and the leg length L_3, respectively:

(5) $$\theta_5 = 2\arctan\frac{m_2 \pm \sqrt{m_2^2 + m_3^2 - q_5^2}}{m_3 + q_5},$$

where $q_5 = \dfrac{r_y}{L_6}$;

(6) $$L_3 = \sqrt{\left[r_x - L_6(l_2s_5 + l_3c_5)\right]^2 + \left[r_z - L_6(n_2s_5 + n_3c_5)\right]^2}.$$

The angle θ_1 can be obtained from equations (2), (3) and (4) after some manipulations, i.e.,

(7) $$\theta_1 = \text{Atan2}(s_1, c_1),$$

where

$$s_1 = \frac{r_x - L_6(l_2 s_5 + l_3 c_5)}{L_3}, \quad c_1 = \frac{r_z - L_6(n_2 s_5 + n_3 c_5)}{L_3}.$$

Premultiplying both sides of equation (2) by $\mathbf{A}_3^{-1}\mathbf{A}_2^{-1}\mathbf{A}_1^{-1}$ we obtain:

(8) $$\mathbf{A}_3^{-1}\mathbf{A}_2^{-1}\mathbf{A}_1^{-1}\mathbf{A} = \mathbf{A}_4\mathbf{A}_5.$$

Now, equating the (2,4) elements from both sides of equation (8) we get:

(9) $$-(L_3 - r_x s_1 - r_z c_1)s_2 + r_y c_2 = 0.$$

The solution of equation (9) gives the angle θ_2 , i.e.,

(10) $$\theta_2 = \text{Atan2}\left[\pm r_y, \pm(L_3 - r_x s_1 - r_z c_1)\right].$$

Expressing r_x through equation (2) leads to the following:

(11) $$f_1 s_3 + f_2 c_3 = f_3,$$

where

$$f_1 = c_1, \quad f_2 = s_1 c_2, \quad f_3 = \frac{r_x - L_3 s_1}{L_6}.$$

The solution of equation (11) gives the angle θ_3, i.e.,

(12) $$\theta_3 = 2\arctan\frac{f_1 \pm \sqrt{f_1^2 + f_2^2 - f_3^2}}{f_2 + f_3}.$$

Now, equating the (1,1) and the (2,1) elements, respectively from both sides of equation (8), we get the following two equations:

(13) $$c_4 = l_1(c_1 c_3 - s_1 c_2 s_3) + m_1 s_2 s_3 - n_1(s_1 c_3 + c_1 c_2 s_3),$$

(14) $$s_4 = l_1 s_1 s_2 + m_1 c_2 + n_1 c_1 s_2.$$

Therefore, the angle θ_4 can be obtained from equations (13) and (14), i.e.,

(15) $$\theta_4 = \text{Atan2}(s_4, c_4),$$

where s_4 and c_4 are the right sides of equations (13) and (14), respectively.

Having obtained the angles θ_i , the matrices $\mathbf{A}_i$ can be formed and the leg lengths can be obtained from the following equations:

(16) $$L_i = \left\|\mathbf{OB}_i - \mathbf{OA}_i\right\|, \; i = 1, 2;$$

(17) $$L_j = \left\|\mathbf{OD}_j - \mathbf{OB}_j\right\|, \; j = 4, 5,$$

where

$$\mathbf{OB}_i = \mathbf{A}_1\mathbf{A}_2\mathbf{A}_3\ {}^3\mathbf{O}_3\mathbf{B}_i\ ;\quad \mathbf{OD}_j = \mathbf{A}\ {}^5\mathbf{O}_5\mathbf{D}_j\ .$$

This completes the solution of the inverse position problem for the considered hybrid robot manipulator.

2.2. Algorithm and graphical representation of the workspace

The main objective of this section is to obtain the envelope of the workspace and present it graphically. The following types of workspace are discussed in this section:

- reachable workspace is defined as a volume within which every point can be reached by the manipulator end-effector with at least one orientation;
- dextrous workspace: this is the volume within which every point can be reached by the manipulator end-effector with any desired orientation;
- workspace with constant orientation – this is a volume which consists of all the points which can be reached by the end-effector with constant orientation.

Obviously the union of all workspaces with constant orientations will give the reachable workspace, while their intersection will determine the dextrous workspace. In this section of the paper the reachable workspace and the workspace with constant orientation of the considered hybrid manipulator are presented. The algorithm comprises the following steps:

- find a boundary point for a given section of the workspace;
- follow the boundary curve until it closes;
- graphically present the envelope;
- repeat the above steps for another section of the workspace (e.g. another *z*-level).

Boundary curve is formed when at least one of the joint coordinates is at one of its limits.

The workspace of the hybrid manipulator is obtained using the above-mentioned algorithm and the results are shown in Figs. 2-5. Figs. 2, 3 and 4 show different views of the workspace with constant orientation (the orientation matrix is the identity matrix). Fig. 5 shows a *z*-slice of the reachable workspace and workspaces with different constant orientations.

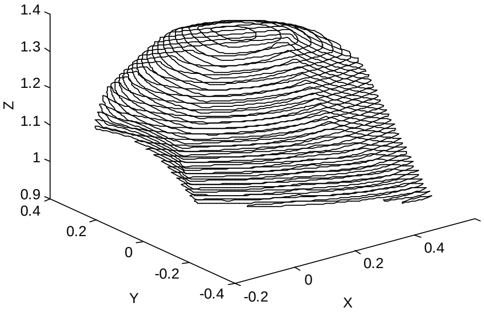

Fig. 2. 3-D view of the workspace with a constant orientation

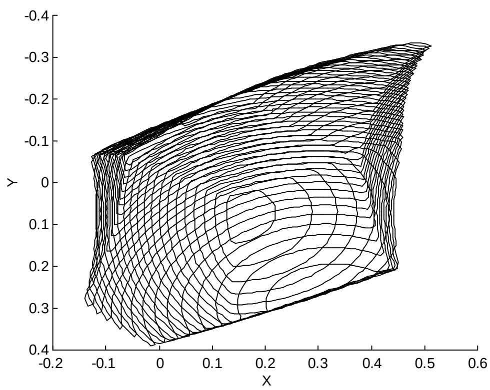

Fig. 3. 2-D view (*Z*-view) of the workspace with a constant orientation

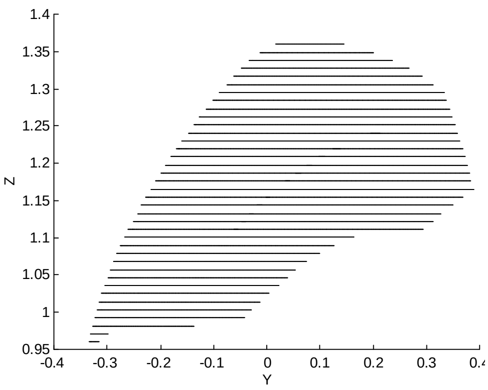

Fig. 4. 2-D view (*X*-view) of the workspace with a constant orientation

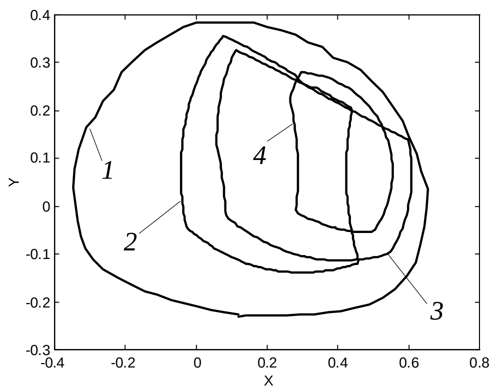

Fig. 5. Workspace contours for a given *Z*-height (*Z*=1.25): *1*– reachable workspace, *2, 3, 4* – workspaces with constant orientation for the following roll, pitch and yaw angles: *2* – [0,0,0]; *3* – [12°,10°,5°]; *4* – [0,20°,0]

The workspace has been obtained for the following design parameters of the manipulator: $\mathbf{OA}_1 = (0.2,\ 0.2\sqrt{3},\ 0)$, $\mathbf{OA}_2 = (0.4,\ 0,\ 0)$, $\mathbf{OA}_3 = (0,\ 0,\ 0)$, $\mathbf{O}_3\mathbf{B}_1 = (0.15, 0.15\sqrt{3}, 0)^T$, $\mathbf{O}_3\mathbf{B}_2 = (0.3, 0, 0)$, $\mathbf{O}_3\mathbf{B}_3 = (0, 0, 0)$, $\mathbf{O}_5\mathbf{D}_1 = (0.125, 0.125\sqrt{3}, 0)$, $\mathbf{O}_5\mathbf{D}_2 = (0.25, 0, 0)$, $\mathbf{O}_5\mathbf{D}_3 = (0, 0, 0)$, $L_6 = 0.525$. The joint limits are: $L_1 \in [0.65, 0.86]$, $L_2 \in [0.5, 0.84]$, $L_3 \in [0.5, 0.84]$, $L_4 \in [0.42, 0.64]$, $L_5 \in [0.42, 0.64]$, $\theta_4 \in [-30°, 90°]$. The dimensions are given in meters.

3. Conclusion

An algorithm for the determination of the workspace of a new type of hybrid manipulation system is proposed in the paper. This algorithm is based on the obtained closed form solution of the inverse kinematic problem for the hybrid manipulator. The determined reachable and dextrous workspaces are graphically presented. The knowledge of the workspace is important for the design, trajectory planning and application of the manipulators.

R e f e r e n c e s

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Рабочее пространство хибридного (паралельно-последовательного) робота манипулятора

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(Р е з ю м е)

В статье рассматривается новый тип хибридного манипулятора с шестью степенями свободы. Этот манипулятор составлен из двух последовательно связанных параллельных механизмов. В настоящей работе предлагается алгоритм определения рабочего пространства хибридного манипулятора.