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Orientation Possibilities of Redundant Robots Based on Sensibility Analysis

George Boiadjiev[1], *Daniela Vassileva*[2]

[1]*Institute of Mechanics, 1113 Sofia*
URL: http://www.imbm.bg
E-mail: George@imbm.bas.bg
[2]*Central Laboratory of Mechatronics and Instrumentation, 1113 Sofia*
URL: http://www.clmi.bg
E-mail: Daniela@clmi.bas.bg

1. Introduction

Redundant robot-manipulators are considered in the paper. Their orientation capabilities are analyzed, following two approaches.

The first one is based on the sensibility theory and the orientation factor-group. In the most arbitrary case the position and orientation states are realized by a subset of six active drives. The approach treats the question how any of these subsets have to be chosen to maintain the needed state of the robot in the sense of accuracy according to the sensibility criterion (realization of needed orientation for instance). The criterion itself shows what kind of sensibility characteristics the system must reveal following a task of execution.

The second approach for evaluation of robot orientation capabilities is based on Riemann measure. A manifold which is hyper-surface in R^9 is considered. It contains all 3×3 transformation matrices between the two bases fixed on the robot origin and end-effector, i.e. it describes all possible orientations of end-effector with respect to the support. Considering all the tangent vectors of this hyper-surface, we determine the Riemann measure and its surface. And we can make comparison between the orientation capabilities of different manipulative structures, taking into account the calculated area realized by the considered structure on this surface. The larger the calculated area, the better the orientation capabilities. Two examples are given in the paper.

The aim of the first approach is to find out what subset of drives has to be chosen to realise the vectors of the kernel of the homomorphism for orientation. This subset does not affect the position state and the orientation could be changed without changing the reached position. This is useful for errors correction and control laws developing for accurate control.

The aim of the second approach evaluating the orientation capabilities by Riemann measure is to choose that one between different structures, which realizes better the corresponding motion of solid body with fixed point.

2. Theoretical background

The purpose of the work is: Description of orientation capability of redundant robots and evaluation of redundancy influence.

Let us consider a robot-manipulator with n degrees of freedom. Every specific state can be realised by a set of n parameters (q_1, ..., q_n), i.e., a vector $q_n \in R^n$, which usually belongs to some set Q, called configurational robot space. In part, when we have a tree-like manipulative structure with five class kinematics joints, then joint parameters could be chosen as generalised coordinates. Every change of q_n reflects to some specific robot state in $Z \in R^3$ – its working zone. The last can be described as a homomorphism between linear spaces R^n and R^3 because the elements of Q and Z are vectors. That homomorphism by definition is called kinematics sensibility [4, 7, 9]. More precisely, it transforms the vectors from the configurational space into a field in the working zone called sensibility ellipsoid. It is also shown [7, 8] that the ellipsoid's semi-axes' lengths are the sensibility coefficients. In that sensibility field (ellipsoid) robot errors are distributed in random way [2, 8].

Due to different reasons – inexactness in drives, errors in generalised coordinates execution, compliance, errors in calculations, errors in the sensor system etc., some deviations from the theoretical position and orientation appear which are described by the vectors δR and $\delta\theta$ [8, 9]. This motion deviation of the real object is expressed by some small deviations of the generalised coordinates with respect to the coordinates of a free-way chosen characteristic point on the last structure body. The deviations from the needed orientation are described by the expression:

$$\delta\theta = L(q)\delta q. \tag{1}$$

For each configuration q the matrix $L(q)$ from (1) defines a homomorphism τ_r between the configuration space Q and the working zone Z. The image of τ_r is the sensibility ellipsoid in the case of orientation and the kernel is its orthogonal completing. As the dimension of Q depends on q_s, i.e. – the generalised coordinates number, the redundancy in the system leads to kernel dimension increasing (the image dimension is not greater than 3 where the robot moves).

3. Factor-group description and mechanical interpretation

Each kinematics chain generates specific homomorphism by the operator $L(q)$ from (1). It is natural that the opposite question arises – if the homomorphism is given then how to construct corresponding chain and how to choose the configuration vectors realising that homomorphism. But for that reason it is necessary to know the possible homomorphism number in dependence of the generalised coordinates number. To answer this question some elements of the group theory are introduced below.

It is known [1] that a set of elements forms a group if an associative operation is defined inside between the elements, and its result belongs also to that set. There the neutral element exists, which means the inverse operation is defined as well. Moreover, if the operation is commutative one, the group is Abelian group. An example for infinite Abelian groups is a linear space considered over an arbitrary field. Under some conditions the homomorphisms between two linear spaces could be interpreted as transformations between groups.

On the other hand, when the considerations are related with real robot-manipulators (in the sense of comparing them with the theoretical models), several restrictions have to be taken into account. For instance, it is clear the real robot-manipulator can not realise completely all the linear space as configurational one but only some part of it. The same can be said about its working zone. But a limited subset of the given linear space, taken under consideration, could change its features as a group concerning the vector addition operation, which is defined in all the linear space. It is enough to mention just one example – if a non-zero vector belonging to a limited subset of vectors (the restriction concerns only the vector length) is added to itself n times, then exists some number n_0, so that the result will exceed the preliminarily considered length. The last means this vector will not belong to the set having vectors whose length is smaller than the fixed value.

Let us consider an n-dimensional linear space R^n over the field of real numbers. Its elements are the vectors q, and for them the standard operation addition is defined, and the neutral element here is the zero vector. Let by q_p a fixed vector is denoted, and by Q_ε^p – a set of vectors q_s^ε, which difference from q_p is not greater by length than a preliminarily given number ε, i. e.

$$Q_\varepsilon^p = \left\{ q_s^\varepsilon : \left| q_s^\varepsilon - q_p \right| \le \varepsilon \right\}. \tag{2}$$

This set is an n-dimensional sphere with a radius ε. If we introduce the notation $\delta q_s^\varepsilon = q_s^\varepsilon - q_p$, then we can write Q_ε^p in the form

$$Q_\varepsilon^p = \left\{ \delta q_s^\varepsilon : \left| \delta q_s^\varepsilon \right| \le \varepsilon \right\}. \tag{3}$$

In the set $Q_\varepsilon^p \subset R^n$ the following operation is defined, called an ε-addition, in the following manner. For each two vectors $\delta q_\alpha^\varepsilon$ and $\delta q_\beta^\varepsilon$:

$$\delta q_\alpha^\varepsilon + \delta q_\beta^\varepsilon = \begin{cases} \delta q_\alpha^\varepsilon + \delta q_\beta^\varepsilon = \delta q_\gamma^\varepsilon \text{ if } \left| \delta q_\gamma^\varepsilon \right| \le \varepsilon, \\ \dfrac{1}{2}(\delta q_\alpha^\varepsilon + \delta q_\beta^\varepsilon) = \dfrac{1}{2}\delta q_\gamma^\varepsilon \text{ if } \left| \delta q_\gamma^\varepsilon \right| > \varepsilon. \end{cases} \tag{4}$$

Then the set Q_ε^p is a group with respect to the introduced operation, i.e. in the first case it can be seen easily that the result $\delta q_\gamma^\varepsilon$ belongs to Q_ε^p, but in the second one using the triangle inequality it can be written

(5) $$\left|\frac{1}{2}\delta\, q^{\varepsilon}_{\gamma}\right| = \left|\frac{1}{2}\left(\delta q^{\varepsilon}_{\alpha} + \delta q^{\varepsilon}_{\beta}\right)\right| \le \frac{1}{2}\left|\delta q^{\varepsilon}_{\alpha}\right| + \frac{1}{2}\left|\delta q^{\varepsilon}_{\beta}\right| \le \frac{1}{2}\varepsilon + \frac{1}{2}\varepsilon = \varepsilon,$$

i.e. the obtained result is also an element from Q^{p}_{ε}. The role of a neutral element in Q^{p}_{ε} is played by the zero vector, i. e. a neutral element in R^n, and every non-zero vector $\delta q^{\varepsilon}_{\alpha}$ has an inverse element, the vector $-\delta q^{\varepsilon}_{\alpha}$, for which it is shown it belongs to Q^{p}_{ε}, i.e.

(6) $$\left|-\delta q^{\varepsilon}_{\alpha}\right| = |-1|\left|\delta q^{\varepsilon}_{\alpha}\right| = \left|\delta q^{\varepsilon}_{\alpha}\right| \le \varepsilon.$$

Taking into account the definition about a group operation in Q^{p}_{ε}, it is not difficult to see it is commutative, i. e. Q^{p}_{ε} is Abelian group. Analogously the group $E^{H}_{\varepsilon} \subset R^{k}$ is defined, where the dimension of the *k*-dimensional linear space is not greater than three (everywhere $k < n$). But the elements of E^{H}_{ε} and Q^{p}_{ε} are vectors, i. e. they are elements from the corresponding linear spaces, so for that reason there exists orthogonal bases inside these sets which dimension is equal of that of the linear space. Then the following homomorphism τ could be considered between Q^{p}_{ε} and E^{H}_{ε}, which makes correspondence between k basic vectors from Q^{p}_{ε} and k basic vectors from E^{H}_{ε}, and the remaining $n - k$ vectors from Q^{p}_{ε} – correspond to the zero vector in E^{H}_{ε}. According to the definition of a transformation kernel, in that case the kernel $\mathrm{Ker}\,\tau$ has a dimension $n - k$. As $k < n$, the kernel elements could be chosen in $C^{n-k}_{n} = \binom{n}{n-k} = \frac{n!}{(n-k)!k!}$ different manners, equal to the combinations between "n" elements of class $n - k$. Telling in another way, there exist C^{k}_{n} homomorphisms between Q^{p}_{ε} and E^{H}_{ε} having the above-described features. Besides, it is interesting to point out that if we calculate the different ways of correspondence of k basic vectors from Q^{p}_{ε} to k basic vectors from E^{H}_{ε} respectively, i. e. calculating the combinations $C^{k}_{n} = \binom{n}{k} = \frac{n!}{k!(n-k)!}$, we obtain the well known equality $C^{n-k}_{n} = C^{k}_{n}$, reached here as a result of mechanical interpretation.

Next, it is known [1], that the kernel of each homomorphism, in part the kernel of $\tau : Q^{p}_{\varepsilon} \to E^{H}_{\varepsilon}$, is a normal divisor (invariant sub-group) of the group Q^{p}_{ε}, i. e. the left and right neighbour classes in the prime factorisation of Q^{p}_{ε} with respect to $\mathrm{Ker}\,\tau$ coincide. Then the following theorem about homomorphisms holds [1]:

$$\begin{tikzcd} Q_\varepsilon^p \arrow[rr, "\tau"] \arrow[dr, "\psi"'] & & E_\varepsilon^H \\ & Q_\varepsilon^p/\mathrm{Ker}\,\tau \arrow[ur, "\sigma"'] & \end{tikzcd}$$

Fig. 1. Isomorphism between the factor-group and the sensibility ellipsoid

Theorem. Let a homomorphism φ of the group G above the group G' be given and let A be the kernel of that homomorphism. Then the group G' is an isomorphic one to the factor-group *G/A*, and in the same time there exist such an isomorphic image σ of the first group in the second, that the result of the composition of transformations φ and σ (i. e. the composition $\sigma\circ\varphi$) coincides with the natural homomorphism of the group G to the factor-group *G/A*.

Let denote by ψ the natural homomorphism of the group Q_ε^p to the factor-group $Q_\varepsilon^p/\mathrm{Ker}\,\tau$. Then from the theorem it follows that there exists a homomorphism σ between the factor-group and E_ε^H , which is illustrated by the following scheme (Fig. 1).

The conclusion can be made that the pre-image of the sensibility ellipsoid is just the factor-group of the homomorphism, describing the system sensibility in the generating element q_p. Moreover all the evaluations of the sensibility in the working zone (considering the ellipsoid form – three-axed, rotational, sphere or their dimensions – three-, two-dimensional (ellipse) or one-dimensional) can be done only in the configurational space with the help of the factor-group $Q_\varepsilon^p/\mathrm{Ker}\,\tau$. We can mention here that there always exists a single-valued homothetic transformation [1], which maps every ellipsoid in a sphere with the same dimension as the ellipsoid, i.e. such a transformation exists between every two ellipsoids. The reason for transforming an ε-sphere in an ellipsoid is caused by the linear transformations characteristics where the multiplication with scalars is defined; in the case of the considered homomorphisms until now no restriction was put on the length keeping.

Another important conclusion is that here the role of redundancy and its influence on the system sensibility is clarified. When the number of the generalised coordinates increase, (the number *n*) and taking into account the physical restrictions on the number *k*, the number of the combinations C_n^k increases and all the possible homomorphisms *S* that can be realised in the generating element q_p are given by the expression:

$$S=\sum_{k=0}^{3} C_n^k \,, \tag{7}$$

when *n* is fixed. Or we can talk about “manifold” and enrich of the sensibility substance as a quality characteristic in presence of redundancy. The last expression allows making another evaluation concerning the “degrees” of system sensibility in the point q_p in dependence of the generalised coordinates number. It could be considered as a sensibility criterion.

4. Evaluation of orientation possibilities

4.1. Description of the orientation homomorphism and its kernel

Let now a concrete homomorphism be considered. It is derived generally by the matrix L from (1), i.e. from the Jacoby matrix that depends on each kinematics chain. It is well known [6] that the matrix L can be written in the form as a block matrix, which last $n - r$ columns are zeros:

(8) $$L = \begin{pmatrix} l_{11} & \dots & \dots & \dots & \dots & l_{1n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ l_{m1} & \dots & \dots & \dots & \dots & l_{mn} \end{pmatrix} = LPP^{-1} = \begin{pmatrix} p_{11} & \dots & \dots & p_{1r} & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ p_{m1} & \dots & \dots & p_{mr} & 0 & 0 \end{pmatrix} P^{-1} = \tilde{P}P^{-1} \cdot$$

Here n is the generalised coordinates number and r – the rank of considered homomorphism. Later we can write the following:

(9) $$Lq = LPP^{-1}q = \tilde{P}P^{-1}q = \tilde{P}\tilde{q} \cdot$$

The substitution $\tilde{q}$ itself consists of linear combinations of generalised coordinates, i. e. of drives in the kinematics chain. From (8) it is obvious if the first r elements of $\tilde{q}$ are zeros then the homomorphism transforms such vectors in zero vector. In other words it belongs to the homomorphism kernel. Here the question is what subset of drives has to be chosen to realise the vectors of the kernel. The answer is found in the linear non-determined homogeneous system

(10) $$\tilde{q}_s = 0, \quad s = 1, \dots, r,$$

which fundamental solutions are based on $n - r$ vectors, i.e. on $n - r$ parameters [1, 6]. In our case these parameters are system drives. In dependence of the numbers n and r here can be counted $C_n^{n-r} = \begin{pmatrix} n \\ n-r \end{pmatrix} = \dfrac{n!}{(n-r)!r!}$ combinations of drive subsets whose do not affect the system orientation. These subsets could be used for another purposes, for instance to correct position deviations etc. The remaining subsets could be chosen as active drives for orientation settings. An interesting question arises when some part of the active drives for position coincide with some “passive” drives for orientation or vice versa, i.e. the joint coordinates corresponding to active drives for position belong to the kernel of orientation homomorphism or vice versa [7].

4.2. Description of end-effector orientation by a three-dimensional hypersurface and evaluation by Riemann measure

The set $M(3, R)$ of all matrices of order 3 forms a linear space R^9 with dimension 9, the coordinates of which are the elements of the matrices. All the matrices of order 3 with determinant 1 belong to a group SL(3, R), O(3) is used for the group of all the orthogonal matrices of order 3 and SO(3) is the group of all the orthogonal matrices with order 3 and determinant 1 [11]. The group SO(3) forms a region in R^9 which could be considered as a hypersurface described by 6 scalar equations $f_i(x_1, x_2, ..., x_9) = 0, \quad i = 1, 2, ..., 6,$ where the coordinates in R^9 are denoted by x_k, $k = 1, 2, ..., 9$. Thus, the surface SO(3) has dimension 3 and three local coordinates u, v and w can be involved around every point for its parametrical description: $x_i = x_i(u, v, w)$, $i = 1, 2, ..., 9$.

The Riemann measure in some region of R^n is defined by the positive quadratic form given on the tangent vectors at every point in the region. The Riemann measure in SO(3) can be written as

(11) $$g_{ij}(u,v,w) = \sum_{k=1}^{9} \frac{\partial x_k}{\partial y_i} \frac{\partial x_k}{\partial y_j}, \quad i, j = 1, 2, 3,$$

where (y_i, y_j) means every combination of pairs of u, v and w. If U is a region of SO(3) in R^9 where the independent parameters u, v and w are changed, the area of U can be calculated by [2, 11]:

(12) $$\sigma(U) = \iiint_U \sqrt{g}\, du\, dv\, dw, \quad g = \det(g_{ij}).$$

5. Illustration

5.1. Orientation evaluation by Riemann measure for two manipulative structures.

Let us apply the orientation evaluation by Riemann measure to two types of tree-like kinematic chain. The first one is presented in Fig. 2. It has 6 DOF, which are the following – T ⊥ R ⊥ R | | R ⊥ R ⊥ R. The second one is a subset of the first one and consists of its last three joints R ⊥ R ⊥ R. Fig. 3 shows correspondence between joints parameters, spherical coordinates and Euler angles for the structure T ⊥ R ⊥ R | | R ⊥ R ⊥ R after passage to the limit of all local coordinate systems origins distance.

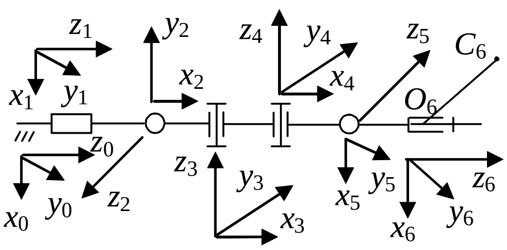

Fig.2. Kinematic structure and coordinate systems for 6 DOF tree-like chain T ⊥ R ⊥ R | | R ⊥ R ⊥ R

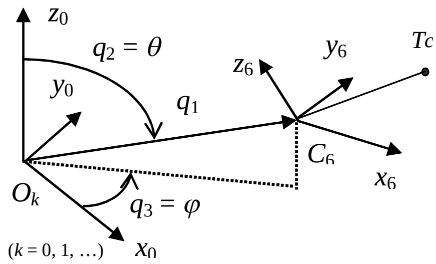

Fig.3. Spherical coordinates and Euler angles after passage to the limit

All simple chain movements are described by orthogonal matrices as well as the matrix A^{06} (the product of transformation matrices) is also orthogonal. This matrix could be rewritten in the terms of independent parameters α, β, γ, δ, where $\alpha = q_2$, $\beta = q_3 + q_4$, $\gamma = q_5$, $\delta = q_6$. Therefore, the matrix of Riemann metric tensor $g = (g_{ij})_{i,j=1}^{3}$ is obtained as

$$(13)\quad g=(g_{ij})=\begin{bmatrix} 2 & -2s\beta\, s\gamma(\partial\delta/\partial\beta) & -2c\beta \\ -2s\beta\, s\gamma(\partial\delta/\partial\beta) & 2+4c\gamma(\partial\delta/\partial\beta)+2(\partial\delta/\partial\beta)^2 & 0 \\ -2c\beta & 0 & 2 \end{bmatrix}.$$

Now, the needed area of our surface could be calculated as

$$\sigma(U)=\iiint_U \sqrt{g}\,d\alpha\, d\beta\, d\gamma = 2\sqrt{2}\int_0^{\alpha} d\alpha \iint_{\beta,\gamma} |s\beta| \left(1+c\gamma\frac{d\delta}{d\beta}\right) d\beta\, d\gamma =$$

$$= 2\sqrt{2}\alpha[(1-c\beta)\gamma + s\gamma\int_0^{\beta}|s\beta|\,d\delta].$$

Since the angles in the above equation are functions of q_1, i.e. $\alpha=\alpha(q_1)$, $\beta=\beta(q_1)$, $\gamma=\gamma(q_1)$, it gives the area at every fixed q_1. Thus, the integration over it can be executed as a final result $V=\int_{q_1^{(1)}}^{q_1^{(2)}}\sigma(U)\,dq_1$ using experimental data for these functions [10].

Next, for the structure R $\perp$ R $\perp$ R the corresponding results are [2]

$$(14)\qquad g=(g_{ij})=\begin{bmatrix} 2 & 0 & 2\cos\beta \\ 0 & 2 & 0 \\ 2\cos\beta & 0 & 2 \end{bmatrix},\quad \det(g_{ij})=8\sin^2\beta\,.$$

The area of that surface could be calculated as

$$(15)\qquad \sigma(U)=\iiint_U\sqrt{g}\,du\,dv\,dw=\int_0^{u_1}\int_0^{v_1}\int_0^{w_1}2\sqrt{2}|\sin v|\,du\,dv\,dw=2\sqrt{2}u_1w_1(1-\cos v_1).$$

In the last expression u, v and w are corresponding independent parameters which in that case interpret mechanically Euler angles, usually used for orientation description [2].

5.2. Description of orientation homomorphism and its factor-group for 5 DOF robot.

The kinematic structure R$\perp$T$\perp$R$\perp$T$\perp$R is considered which has five joints coinciding with the generalized coordinates in this case. The matrix L from (8) takes the form:

$$(16)\qquad L=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \cos(q_1+q_3+q_5) & 0 & \cos(q_1+q_3+q_5) & 0 & \cos(q_1+q_3+q_5) \end{bmatrix}.$$

The matrix $\tilde{P}$ from the expression (8) has the explicit form

(17) $$\tilde{P} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \cos(q_1 + q_3 + q_5) & 0 & 0 & 0 & 0 \end{bmatrix}$$

and the vector $\tilde{q} = [\tilde{q}_1, ..., \tilde{q}_5]^T$ is

(18) $$\tilde{q} = [q_1 + q_3 + q_5 \quad q_2 \quad q_3 \quad q_4 \quad q_5]^T .$$

The system (10) becomes:

(19) $$q_1 \cos(q_1 + q_3 + q_5) + q_3 \cos(q_1 + q_3 + q_5) + q_5 \cos(q_1 + q_3 + q_5) = 0 .$$

It consists of one equation with three variables, so that its solution is based on two independent parameters. In the case of linearization, in our example, there exist $C_3^2 = 3$ ways of choosing independent parameters. For instance, if the independent parameters are (q_1, q_3), then the fundamental solution of (19) after linearization looks like:

(20) $$\left[\begin{pmatrix} 1 \\ 0 \\ q_5 = -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ q_5 = -1 \end{pmatrix} \right] .$$

Coming back to our example, from 3 available drive subsets some would reveal minimal system sensibility, which could be seen calculating corresponding sensibility coefficients. For optimal in that sense subset the system motion close to desired orientation will be realized with maximal accuracy. Generally speaking, the whole motion can be considered as a set of such partial motions where every one of them has been executed with optimal drives subset. The drives subsets are activated consecutively during the whole motion.

But actually, (19) can be rewritten in the form

(21) $$(q_1 + q_3 + q_5)\cos(q_1 + q_3 + q_5) = 0 .$$

The first multiplier expresses just the linear case, which has already been discussed above. The second one shows the existence of another solution also based on two independent parameters, but here the connection between dependent and independent parameters is

(22) $$q_5 = k\frac{\pi}{2} - (q_1 + q_3) ,$$

where k takes the values available according to the system construction.

Analogously, for the last linear system (22) corresponding fundamental solution is based on two vectors:

(23) $$\left[\begin{pmatrix} 1 \\ 0 \\ q_5 = 0.57 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ q_5 = 0.57 \end{pmatrix}\right].$$

Finally, for our example the factor group G / A, where the homomorphism kernel is formed by (q_1, q_3), consists of two elements, i.e. two neighbour classes. The neutral element is a linear combination of (q_1, q_3), and the remaining one – the sum of q_5 with the above mentioned combination.

6. Summary

Sensibility is a system quality characteristic mathematically defined as a homomorphism between robot configuration space and working zone. It is well known that redundancy brings more inexactness in the system as a whole but, on the other hand, it can increase the accuracy in some specific system states.

A natural question is how redundant joints influence the sensibility parameters and which joints we can use to realise some precise motion following some criteria. Here such criteria are developed in the case of orientation of manipulative structures and an approach for the choice of active drives in kinematics chain is presented.

In the paper an illustrative example is presented where the explicit combinations of active drives are given. These subchains realize specific system sensibility.

The orientation capabilities of manipulative structures are analysed using an approach based on Riemann measure. Two examples are given here and considering the part of the hyper-surface in R^9 they realize one could judge about their end-effector orientation capabilities. This hyper-surface contains all possible transformation matrices between two bases. Each concrete manipulative structure end-effector goes round a part of it.

Finally the following conclusions could be formulated. The approach described here enable the robot to realize specific sensibility characteristics appropriate for different working tasks. The approach also treats the question of choosing different drive sub-systems to realize the same (desired) sensibility. Such different drive sub-systems choice comes as an answer towards changes of generating element q_p , i.e. the system state. The considered approach points out the advantages of using redundant systems where the number of combinations of drives realizing needed system state (sensibility) increase, i.e. including the case of active drive subsets changing. This approach is related to the question of simple kinematics chains synthesis which realizes specific sensibility, and later could be used in some more complex chains. Applying this method we can improve the robot accuracy in local points, fields and regimes.

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Ориентационные возможности избыточных роботов на основе анализа чувствительности

Георги Бояджиев[1], *Даниела Василева*[2]

[1]*Институт механики, 1113 София*
E-mail: George@imbm.bas.bg
[2]*Центральная лаборатория мехатроники и приборостроения, 1113 София*
E-mail: Daniela@clmi.bas.bg

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В статье рассматриваются избыточние роботы-манипуляторы. Их ориентационные возможности анализируются, используя два подхода. Первый основан на теории чувствительности и на фактор-группу ориентации. Он относится к вопросу как следовало бы выбирать разные подмножества кинематической вериги манипуляционных систем для реализации желанных параметров чувствительности, в частности, параметров ориентации. Второй подход использует Римановую метрику для оценки площади гиперповерхности, реализуемой разными структурами. В работе описываются ориентационние возможности роботов с избыточными степенями свободы и оценивается влияние дополнительных приводов на этих возможностей.