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Determination of the Flow Rate of Different Fluids by a Rotameter*

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1. Introduction

Rotameters are devices for direct measuring of the flow of moving fluid. An accounting element of the value measured is an efflux rotational float, moving vertically in a conic transparent pipe as a result of the action of the fluid running through the pipe. The value of the quantity measured is defined by the height of float going up. The rotameters have found wide application in practice due to their simple construction, distinct indications along a linear scale, possibility to measure small flows of fluids and gases, including aggressive ones, sufficiently wide range of measuring, etc.

The principal diagram of a classic rotameter is shown in Fig. 1. The main elements of its construction are a transparent conic pipe 1 and a float, made of a conic tip 2, a cylindrical body 3 and a board 4. There are some screw-shaped channels cut along the periphery of the board and due to them the float is constantly rotating around its axis, positioning in the middle of the fluid flow, not rubbing the pipe walls. The float is an efflux body, which takes on the dynamic pressure of the fluid flow and supports constant drop of the static pressure in front of and behind it. The fluid flow ascending in the conic pipe of the device lifts the float at height H, at that the annular orifice S between the float board and the inner surface of the conic pipe reaches a value, at which the forces acting on the float are balanced. The height at which the float is raised corresponds to a definite value of the flow rate of the passing fluid.

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2. Defining of a fluid flow by a rotameter

The formula for volumetric flow Q (m³/s) of the fluid running through the conic pipe of the rotameter, based on the equation for continuity of the fluid medium and on Bernoulli's equation, applied for sections a–a and b–b, has the form given in Fig. 1a,

(1)
$$Q = \alpha S_{H} \sqrt{2gL_{f}\kappa_{\rho}}, \ \mathrm{m}^{3}/\mathrm{s}$$

where: α is a discharge coefficient of the fluid flow through the conic pipe;

 S_{H} – the least surface of the annular orifice of the fluid flow, corresponding to the height of the float H with respect to the initial section of the conic pipe, m²;

g – acceleration of the float mass, m/s²;

 $L_{\rm f}$ – float length, m;

(2)
$$\kappa_{\rho} = \frac{\rho_{\rm f}}{\rho} - 1,$$

 $\rho_{\rm f}$, is float thickness, kg/m³; $\rho_{\rm g}$ – fluid thickness, kg/m³.

The discharge coefficient α in (1) is analogical to the discharge coefficient of a fluid flow with a narrowing. It depends on the float shape and on the resistance the fluid flow meets against the float:

(3)
$$\alpha = \sqrt{\frac{\beta}{\chi_H}},$$

where β is a coefficient of the float shape, determined by the relation

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$$\beta = \frac{V_{\rm f}}{S_{\rm f} L_{\rm f}}$$

where $V_{\rm f}$ is the float volume, m³;

 $S_{\rm f} = \pi D^2/4$ – surface area, (m²) of the cross section of the board of a float with a diameter *D* (Fig.1), equal to the inner diameter of the conic pipe at the starting position of the float at *H*=0;

 χ_{H} – drag coefficient of the float, depending on the float shape and on Reynolds criterion:

(5)
$$\chi_{H} = \frac{2gV_{f}}{S_{f}\omega^{2}}\kappa_{\rho},$$

where ω is the rate of the fluid flow.

Replacing (4) in (5) and expressing the speed of the fluid flow by $\omega = Q/S_H$, and the formula is obtained of the basic functional relation of the parameters in a rotameter with free movement of a float and a pipe of an arbitrary profile (m²):

(6)
$$S_{H} = Q_{\sqrt{\frac{\chi_{H}}{2gL_{f}\beta\kappa_{\rho}}}}$$

This formula indicates that at established equilibrium position of the float, the surface area of the smallest annular orifice between it and the pipe wall is directly proportional to the volumetric flow of the passing fluid.

The geometric dependence of the surface area of the throttling section S_H on the height of float lifting *H* is defined according to the scheme in Fig. 1 (m²),

(7)
$$S_{H} = \pi (D t g \theta H + t g^{2} \theta H^{2}),$$

where θ is the angle of the cone-forming line with relation to the pipe axis.

A generalized static characteristic is obtained from equations (1) and (7) for determination of the volumetric flow with the help of a rotameter with a conic pipe and a float is (m^3/s)

(8)
$$Q = K\alpha \sqrt{\kappa_{\rho}} \left(aH + bH^2 \right),$$

where:

(9)
$$K = \pi \sqrt{2gL_{f}}$$

(10)
$$a = Dtg\theta$$

(11)
$$b = tg^2\theta.$$

As seen from equation (8), the value of the flow rate of a given fluid running through a rotameter with known parameters of the pipe and the float, is a function of the discharge coefficient α , the thickness of the fluid and of the float and the height of float lifting *H*. The discharge coefficient α in this equation is analogical to the discharge

coefficient for throttling devices measuring the flow rate in a vertical pipeline. It depends on the float shape and on the critical Reynolds number Re. For the float, shown in Fig. 1, that is most frequently used in practice, at Re = 8000 up to Re =120000, α is approximately equal to 0.99.

The relation of the float rate Q and the float travel H, as seen in equation (8), is not proportional. The rotameter scale division accomplished according to this relation, is non-linear. The measuring of the flow rate in this case will be difficult and imprecise due to the possibility for coarse errors in its reading. For pipes with small conicity (from the order of 1:100) and not long float travel, the expression (bH^2) in equation (8) gets insignificant values and it can be ignored, at that the error will be within the limits of the admissible in practice values – up to 2%. Then the generalized characteristic of the rotameter for measuring a volumetric flow, will obtain the form (m³/s):

(12)
$$Q = Ka\alpha \sqrt{\kappa_{\rho} \cdot H} \cdot H$$

Multiplying the two sides of this equality by the thickness of the fluid ρ (kg/m³), we will get the generalized characteristic of the rotameter for measuring the mass flow *G* (kg/s):

(13)
$$G = Ka\alpha \sqrt{\rho(\rho_{f} - \rho)H}$$

For the purpose of comparison Fig. 2 shows in a graphic form the relations (8) and (12) of a rotameter often used in practice for measuring the volumetric flow of air. The measuring range is from 2 up to 20 m³/h, at temperature 20 °C and nominal operating pressure of 101.3215 kPa. The rotameter pipe is with conicity of 1:100, and the float is made of X10CrNi18.9 alloy with thickness $\rho_f = 7.8 \times 10^3 \text{ kg/m}^3$. The shape and the size of the float are given in Fig. 1b. As seen in the diagram, at scale division of the rotameter with respect to linear equation (12), the ma ximal error of the measuring range is 0.6%, which is considerably smaller than the admissible error in flow rate measuring. This gives reason for the practical use of this equation in the scale division of rotameters of the type discussed in measuring flow rates of different fluids.



 \rightarrow by equation (8) \rightarrow by equation (12)

Fig. 2. Relations of the rotameter gauge according to equations (8) and (12)

3. Gauge transformation of a rotameter for flow rate of different fluids

The rotameters offered by the companies are scaled only for measuring the flow rate of water or air. Their direct application for other fluids, without any corrections, leads to measuring errors. Hence, the problem of gauge transformation of the rotameters proposed for measuring flow rates of other fluids, has got great practical significance.

The gauge transformation of a given rotameter from the type discussed for another fluid (the flow rate of which is to be measured) may be realized according to equation (12) or equation (13). Fig. 3 shows the characteristics of equation (12) for measuring the volumetric flow of several different fluids with the help of the rotameter considered. The calculations are made for nominal pressure and temperature of the fluids. Using these characteristics, the scale of a rotameter for air can be transformed into a scale measuring the volumetric flow of other fluids can be directly determined from the diagram, if this is accomplished without gauge transformation of the rotameter for air. It can be seen, that even for oxygen, which is with thickness close to that of air, the error would be greater than 5 %. In measuring gases, the greater the difference is between the thickness of the gas measured and the air thickness, the greater the inaccuracy is.

The gauge transformation of a rotameter for another fluid may be realized also directly [7], multiplying the flow rate values according to the existing scale, adjusted for a given fluid (air or water), by a correction coefficient C. This coefficient may be defined using equation (12) or (13). The correction coefficient C_{ϱ} of a scale for volumetric flow is obtained from (12):



(14)
where:
$$C_{\varrho} = C_{\varrho\alpha} C_{\varrho\rho} = \frac{\alpha''}{\alpha'} \sqrt{\frac{\rho'}{\rho''} \left(\frac{\rho_{\rm f} - \rho''}{\rho_{\rm f} - \rho'}\right)},$$

where:

(15)

$$C_{Q\alpha} = \frac{\alpha'}{Q\alpha'} \rho \qquad \alpha' \sqrt{\rho''} \left(\rho_{f} - C_{Q\alpha} = \frac{\alpha''}{\alpha'} \right)$$

is a correction coefficient of fluids viscosities;

(16)
$$C_{\varrho\rho} = \sqrt{\frac{\rho'}{\rho''}} \left(\frac{\rho_{\rm f} - \rho''}{\rho_{\rm f} - \rho'}\right)$$

is a correction coefficient of fluids thickness;

 $\alpha'\alpha''$ and ρ' , ρ'' are the coefficients of the flow rate and thickness respectively of the fluid, for which the scale is divided and the fluid, for which gauge transformation is to be made.

For most cases in practice, the coefficients for fluids flow rates - the one for which the rotameter is designed for and the one subject to gauge transformation, are of very close values and in these cases the coefficient $C_{Q\alpha}$ may be accepted as 1 and then $C = C_{Q\rho}$. For gases, due to their smaller thickness compared to that of the float, formula (16) is simplified, the expression in the brackets receiving also the value 1 and then for the correction coefficient C_o it is obtained:

(17)
$$C_{\varrho} = \sqrt{\frac{\rho'}{\rho''}}$$

The correction coefficient C_{g} of a scale for mass flow is obtained from (13):

(18)
$$C_{_{G}} = C_{_{G\alpha}}C_{_{G\rho}} = \frac{\alpha''}{\alpha'}\sqrt{\frac{\rho''(\rho_{_{\rm f}} - \rho'')}{\rho'(\rho_{_{\rm f}} - \rho')}},$$

where:

(19)
$$C_{G\alpha} = \frac{\alpha''}{\alpha'},$$

(20)
$$C_{G\rho} = \sqrt{\frac{\rho''(\rho_{\rm f} - \rho'')}{\rho'(\rho_{\rm f} - \rho')}} \,.$$

Accepting the same assumptions, as for the scale for volumetric flow above mentioned, the simplified expression for the correction coefficient $C_{_G}$ of the scale for mass flow will take the form:

(21)
$$C_{_{G}} = \sqrt{\frac{\rho^{\prime\prime}}{\rho^{\prime}}}.$$

If the absolute pressure P'' and the absolute temperature T'' of the fluid, which flow rate will be measured, differ from the nominal values P' and T', for which the rotameter scale is divided, the correction coefficients C_o and C_g will be respectively:

(22)
$$C_{\varrho} = \sqrt{\frac{\rho'}{\rho''}} \sqrt{\frac{P'}{P''}} \sqrt{\frac{T''}{T'}},$$

(23)
$$C_{_{G}} = \sqrt{\frac{\rho''}{\rho'}} \sqrt{\frac{P''}{P'}} \sqrt{\frac{T'}{T''}}$$

Fig. 4 is an illustration of the graphic relation of the gauge transformation by the correction coefficient (22) of the rotameter scale for air into a scale measuring the volumetric flow of Argon at parameters different than the nominal ones – pressure of 2 bar and temperature of 20 $^{\circ}$ C.



Fig. 4. An example of the gauge transformation of a rotameter for air into a scale for measuring Argon flow with the help of a correction coefficient.[°]

4. Determination of the error in flow measuring by a rotameter

The general mean square relative error in measuring the volumetric flow by a rotameter, derived for equation (1), has the form

(24)
$$\sigma_{\varrho} = \sqrt{\sigma_{\alpha}^{2} + \sigma_{s_{\mu}}^{2} + \frac{\sigma_{l_{f}}^{2}}{4} + \frac{\rho_{f}^{2}}{4(\rho_{f} - \rho)^{2}} \left(\sigma_{\rho_{f}}^{2} + \sigma_{\rho}^{2}\right)},$$

where: $\sigma_{\alpha}, \sigma_{S_{\mu}}, \sigma_{l_{t}}, \sigma_{\rho_{t}}, \sigma_{\rho}$ are the mean square relative errors of the flow coefficient, of the annular orifice, of the float length, of the material thickness, and of the thickness

of the fluid being measured. The formula determining the general mean square relative error in measuring mass flow by a rotameter, is analogical.

In rotameter scale division, if the values of the float length and its thickness are carefully determined, the errors from these values can be avoided. The error σ_{S_H} is obtained from inaccurate positioning of the float in its rising due to its friction with the pipe walls. For the type of rotameters considered, with a free moving float in a conic pipe, there is not any considerable friction practically and this error is also ignored.

The error σ_{α} is a result of the alteration of the turbulence degree of the fluid flow in the rotameter. In many references [1, 3, 9] the value of 0.25% is accepted for this error. The error σ_{ρ} is a result of the alteration in the fluid thickness due to alteration in its pressure and temperature. The value of this error is accepted to be 0.5% for fluids and 0.75% – for gases. Taking into consideration the measuring error, it can be accepted that in measuring a flow by a rotameter of the type discussed, the general relative error will be smaller than 1.5%.

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Определение дебита любых флюидов ротаметром

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(Резюме)

Выведены теоретические зависимости для определения объемного и массового дебита любых флюидов, измеряемых ротаметром из конусной трубки и поплавка. Предложен метод преградуирования скалы ротаметра, используя упрощенные зависимости дебита от поднимания поплавка в трубе и метод прямого перечисления скалы ротаметра посредством коррекционного коэффициента. Анализированны относительные ошибки при измерении дебита ротаметром.