БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ . BULGARIAN ACADEMY OF SCIENCES

ПРОБЛЕМИ НА ТЕХНИЧЕСКАТА КИБЕРНЕТИКА И РОБОТИКАТА, 55
PROBLEMS OF ENGINEERING CYBERNETICS AND ROBOTICS, 55

София . 2005 . Sofia

Energy-Saving Ferrite Transformers

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Introduction

A general optimality criterion when designing ferrite transformers (FTr) is the reach of maximum output power of the transformer without overheating. This is expedient when the device is not a big electricity consumer, if it works for a short period of time within twenty-four hours and its price is of major importance. Such are, for example, the various types of power adapters for peripheral devices, battery charger adapters for mobile phones, digital cameras and other similar devices. There is, however, a wide range of another type of devices, like the various types of communication servers, powerful rooters and devices for their uninterruptable power supply (UPS) whose energy consumption may reach up to several kilowatt-hours and, in practice, work uninterruptedly for several years. The price of these devices is multiple times higher than the price of the inbuilt ferrite transformer which is the reason why the problem with energy saving is of an essence. That is why the design of a ferrite transformer based on an optimality criterion that secures minimal power loss will lead to electricity savings, which is an extremely topical issue at the moment.

The object of the present article is to determine such magnetic regime of the FTr that would minimize the power loss of the transformer without significantly increasing its size or price.

Legend of Symbols

The following symbols are used in the paper:

f – commutation frequency of the transformer, Hz;
δ – pulse width coefficient of input voltage;
E – inducted electric voltage in the winding, V;
I_{eff} – efficient electric current in the winding, A;

$\hat{I}$ – electric current amplitude in the winding, A;
P_{tr} – transformer power, W;
U_0 – power supply output voltage, V;
I_0 – power supply output current, A;
P_0 – power supply output power, W;
P_{tot} – transformer total losses, W;
P_{Fe} – core losses, W;
P_{dFe} – specific core losses, W/cm^3;
P_{Cu} – losses in the windings, W;
T – transformer temperature, °C;
ΔT – transformer over-temperature, °C;
R_{th} – transformer thermal resistance, °C per W;
$\hat{B}$ – magnetic induction maximum value equal to half of the magnetic induction swing ΔB, T;
V_{eff} – core efficient volume, cm^3;
A_{eff} – core efficient cross section, m^2;
S_{w} – cross section of the winding window, mm^2;
l_{Cu} – average winding length, m;
w – number of winding turns,
r – DC resistance in one winding (w), Ω;
ρ_{Cu} – copper wire specific resistance, (Ω.mm^2)/m;
q_{Cu} – winding wire cross section, mm^2;
k_{Cu} – filling coefficient of S_{w} with pure copper for one winding (w),
ξ – skin effect and proximity effect coefficient.

Magnetic mode for ferrite transformers with minimized losses

This research has been made with the following presuppositions:

- The supply voltage pulses are rectangular;
- The primary and secondary transformer windings have equal number of turns (w);
- The RMS value of the efficient electric current (I_{eff}) in the two windings is the same, which corresponds to the most frequently applied power supply circuitries: forward converter with two transistors and full bridge converter.
- Reactance of winding leakage is ignored.
- The puls width coefficient of input voltage have maximum value

The total amount of transformer active losses (P_{tot}) is a sum of the losses in the ferrite core (P_{Fe}) and the windings (P_{Cu}), that is

$$P_{\mathrm{tot}} = P_{\mathrm{Fe}} + P_{\mathrm{Cu}}. \tag{1}$$

The losses in the windings are represented by the expression

$$P_{\mathrm{Cu}} = 2rI^2_{\mathrm{eff}}, \quad \text{where } r = \xi \frac{\rho_{\mathrm{Cu}} l_{\mathrm{Cu}} w}{q_{\mathrm{Cu}}}. \tag{2}$$

Taking into account that $q_{\mathrm{Cu}} = \dfrac{k_{\mathrm{Cu}} S_{\mathrm{w}}}{w}$, it follows that

$$r = \xi \frac{\rho_{\mathrm{Cu}} l_{\mathrm{Cu}} w^2}{k_{\mathrm{Cu}} S_w}. \tag{3}$$

From the expression for the voltage inducted in the winding

$$E = 4\hat{B} f w A_{\mathrm{e}} \tag{4}$$

we can determine the number of windings

$$w = \frac{E}{4\hat{B} f A_{\mathrm{e}}}. \tag{5}$$

Then, by substituting w from (5) into (3) and the resulting expression for r in (2), the losses in the windings can be calculated by the following formula:

$$P_{\mathrm{Cu}} = k_{\mathrm{w}} \frac{E^2 I_{\mathrm{eff}}^2}{f^2 \hat{B}^2}$$

or

$$P_{\mathrm{Cu}} = k_{\mathrm{w}} \frac{P_{\mathrm{tr}}^2}{f^2 \hat{B}^2} \tag{6}$$

where

$$k_{\mathrm{w}} = \frac{\xi \rho_{\mathrm{Cu}} l_{\mathrm{Cu}}}{8 k_{\mathrm{Cu}} S_{\mathrm{w}} A_{\mathrm{e}}^2}. \tag{7}$$

The losses in the ferrite core are determined by the expression

$$P_{\mathrm{Fe}} = V_{\mathrm{e}} P_{\mathrm{dFe}}. \tag{8}$$

The calculation of P_{dFe} for the specific ferrite materials can be performed through the means of analytical formulas, determined by the producers of the materials.

The *Ferroxube* company presents the following formula [3] for its ferrite materials:

$$P_{\mathrm{dFe}} = C_{\mathrm{m}} C_T f^m \hat{B}^n 10^{-3}, \text{ where } C_T = C_{t_0} - C_{t_1} T + C_{t_2} T^2. \tag{9}$$

In this case, T is measured in $^{\mathrm{o}}$C. The coefficients have been chosen and adjusted to the experimental characteristics of the ferrite materials in such a way that at T=100 $^{\mathrm{o}}$C the value of C_T is equal to one.

The values of coefficients: $C_{\mathrm{m}}, m, n, C_{t_0}, C_{t_1}, C_{t_2}$ for the more widely spread ferrite materials of *Ferroxcube* are shown in Table 1.

Table 1. Fit parameters to calculate the power loss density of the *Feroxcube* material 3F3

f, kHz	C_m	m	n	C_{t_0}	C_{t_1}	C_{t_2}
20-300	0.25×10^{-3}	1.6	2.5	0.79	1.05×10^{-2}	1.26×10^{-4}
300-500	2×10^{-5}	1.8	2.5	0.77	1.05×10^{-2}	1.28×10^{-4}
500-1000	3.6×10^{-9}	2.4	2.25	0.67	0.81×10^{-2}	1.14×10^{-4}

An analytical expression similar to (9) is valid for the ferrite materials of *Magnetics – USA* [2], namely

$$P_{\mathrm{dFe}} = af^{c}\hat{B}^{d}\,. \tag{10}$$

The values of coefficients: *a, c, d* for the more widely spread ferrite materials of *Magnetics* are shown in Table 2.

Table 2. Parameters of the low profile E-cores

E-core	V_e, cm^3	A_e, mm^2	S_w, mm^2	l_{Cu}, mm	k_w	R_{th}, °C per W	$P_{tot\ max}$,W
EILP22	2.04	78.5	18.88	65.20	32.36×10^3	38	1.316
EELP22	2.55	78.5	37.76	65.20	16.18×10^3	35	1.429
EILP32	4.56	129	29.50	90.44	10.64×10^3	26	1.923
EELP32	5.38	129	59.00	90.44	5.32×10^3	24	2.083
EILP38	8.46	194	50.30	111.26	3.392×10^3	20	2.500
EELP38	10.20	194	100.60	111.26	1.696×10^3	18	2.778
EILP43	11.50	225	71.82	125.20	1.988×10^3	16	3.125
EELP43	13.90	225	143.64	125.20	0.994×10^3	15	3.333

By substituting P_{dFe} from (9) into (8) and the resulting expression for P_{Fe} and P_{Cu} from (6) are substituted in (1), the total losses can be calculated by the following formula:

$$P_{\mathrm{tot}} = k_{\mathrm{w}}\frac{P_{\mathrm{tr}}^{2}}{f^{2}\hat{B}^{2}} + V_{\mathrm{e}}C_{\mathrm{m}}C_{T}f^{m}\hat{B}^{n}10^{-3}. \tag{11}$$

The optimal value of magnetic induction with minimum losses in the transformer (B_{opt}) at a defined frequency of transformer commutation can be calculated by equating to zero the private derivative of P_{tot} with respect to $\hat{B}$. From (11) we get

$$\frac{\partial P_{\mathrm{tot}}}{\partial\hat{B}} = \frac{k_{\mathrm{w}}P_{\mathrm{tr}}^{2}}{f^{2}}\left(\frac{-2}{\hat{B}^{3}}\right) + V_{\mathrm{e}}C_{\mathrm{m}}C_{T}f^{m}n\hat{B}^{n-1}10^{-3}.$$

Then with $\frac{\partial P_{\mathrm{tot}}}{\partial\hat{B}} = 0$ we can calculate:

$$\hat{B}_{\mathrm{opt}} = \left[\frac{2000\,k_{\mathrm{w}}P_{\mathrm{tr}}^{2}}{V_{\mathrm{e}}C_{\mathrm{m}}C_{T}f^{m+2}n}\right]^{\frac{1}{n+2}}. \tag{12}$$

By substituting B_{opt} from (12) into (11) the minimum value of the active total losses in the transformer can be calculated by the following formula:

$$P_{\text{tot min}} = k_{\text{w}} \frac{P_{\text{tr}}^{2}}{f^{2}\hat{B}_{\text{opt}}^{2}} + V_{\text{e}} C_{\text{m}} C_{T} f^{m} \hat{B}_{\text{opt}}^{n} 10^{-3}. \tag{13}$$

Application of the criterion for minimum total losses in the transformer

With the purpose of decreasing of the losses and for bigger nominal power of the transformer, the next typesize from the standard family of ferrite cores are usually used, for which the active losses are minimized in the suggested way. Since the size of the ferrite cores from the standard family change in leaps, the increase in the size and price for each particular case will be different and the efficiency should be assessed based on the specific application.

The materials of the popular European company *Ferrocube* are the object of discussion in the exposition, more specifically, the widely spread ferrite material 3F3, whose coefficient values from (9) are shown in Table 1.

To demonstrate the application of the results from the analytical study in practice the data from tables 2 and 3 have been used. These tables refer to low profile E-cores whose length is 22, 32, 38 and 43 mm. E-cores comprised of two low profile E-components are denoted with EELPx, while E-cores comprised of one low profile E – component and one normal I-component is denoted with EILPx, where x represents the length of the core.

Table 2 includes the geometric parameters of the cores (V_{e}, A_{e}, S_{w}, l_{Cu}), the values of coefficient k_{w} from (7), as well as the thermal resistance (R_{th}) [3] and the maximum allowed transformer losses at ΔT=50 °C ($P_{\text{tot max}}=50/R_{\text{th}}$) corresponding to the core. Since the windings in the low profile transformers are usually printed, it is generally accepted that $k_{\text{Cu}}=0.05$ (for one winding) and $\zeta=1$, because the influence of the Skin effect and the Proximity effect can be ignored.

Table 3 shows the values of the maximum allowed power ($P_{\text{tr max}}$) of the transformer, built on the different cores for two frequency values: 300 kHz and 500 kHz at ΔT=50 °C.

As an example for the construction of ferrite transformer with minimum losses we have taken a transformer whose power is P_{tr}=210 W and commutation frequency f=300 kHz. Table 3 proves that such a transformer can be designed on an EILP32 core, and maximum power of the transformer with maximum admissible total losses is reached: P_{tot}=1.92 W (Table 2).

To decrease losses, the transformer is constructed on the next bigger in size EILP38 core.

By substituting in (11) P_{tr}=210 W, f = 300 000 Hz, from Table 2 $k_{\text{w}}=3.392\times10^{3}$ and V_{e}= 8.46 cm^3, from Table 1 $C_{\text{m}}=2\times10^{-5}$, m = 1.8, n = 2.5, and assuming that C_T= 0.7, we can calculate the losses in the transformer by the following formula:

$$P_{\text{tot}} = \frac{0.001662}{\hat{B}^{2}} + 855.69\hat{B}^{2.5}.$$

In the Fig. 1 is shown the dependency $P_{\text{tot}} = f(\hat{B})$, that has been determined in a program way. As can be seen, the minimum losses $P_{\text{tot min}} \approx 1.15$ W are incurred at

Table 3. Maximum power of the low profile E-cores transformers.

E-core	f = 300 kHz	f = 500 kHz
	$P_{\text{tr max}}$	$P_{\text{tr max}}$
EILP22	118	136
EELP22	165	190
EILP32	210	243
EELP32	299	345
EILP38	368	425
EELP38	532	613
EILP43	520	601
EELP43	774	833

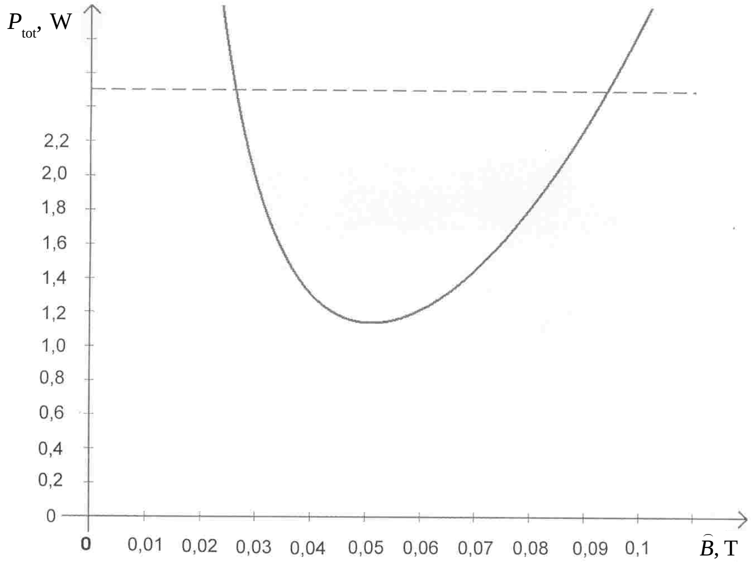

Fig. 1. P_{tot} versus flux density (--------- $P_{\text{tot limit}}$)

$\hat{B}_{\text{opt}} \approx 50$ mT. The horizontal dotted line shows the value of the admissible losses in a transformer built on an EILP38 core with R_{th}=20 °C per W (Table 2), namely:

$$P_{\text{tot limit}} = \frac{\Delta T_{\text{limit}}}{R_{\text{th}}} = \frac{50}{20} = 2.5\,\text{W}.$$

The exact calculation of $\hat{B}_{\text{opt}}$ and $P_{\text{tot min}}$ is performed using (12) and (13), respectively, and the resulting values are the following:

$$\hat{B}_{\mathrm{opt}} = 51\ \mathrm{mT};\quad P_{\mathrm{tot\,min}} = 1.14\ \mathrm{W}.$$

Next , we must check if the value of C_T is equal to the assumed $C_T = 0.7$. From Table 1 we read the following values for the frequency range 300-500 kHz: $C_{t_0} = 0.77$; $C_{t_1} = 1.05\times10^{-2}$ and $C_{t_2} = 1.28\times10^{-4}$. The overheating is determined by the equation: $\Delta T = P_{\mathrm{tot\,min}} R_{\mathrm{th}}$. Then $\Delta T = 1.14\times20 \approx 23$ °C. If we assume that the maximum ambient temperature is 50 °C, the heating will be $T = 73$ °C, so the value of C_T in (9) is:

$$C_T = 0.77 - 1.05\times10^{-2}\times73 + 1.28\times10^{-4}\times73^2 \approx 0.7.$$

If there are differences in relation to the assumed value for C_T, the calculations must be repeated based on the new value.

By comparing the values for P_{tot} in the case when the criterion for reaching a maximum power of the transformer (1.92 W for EILP32 core) and the criterion for minimizing the transformer losses (1.14 W for EILP38 core) are applied, we get the percentage decrease of losses:

$$\Delta P_{\mathrm{tot}} = \frac{1.92 - 1.14}{1.92}.100 = 40.6\ \%.$$

Conclusions

- We have suggested a method for design of ferrite transformers with minimum active losses.
- We have formulated analytical expressions through which the value of the minimum active losses in ferrite transformers (13) and the respective optimal value of magnet induction (12) are determined.
- The application of the suggested methods has been demonstrated with a low profile ferrite transformer built on an EILP38 core. It has been proved that the active losses of the transformer are reduced with more than 40% compared to the losses in a transformer with the same power, constructed in conformity with the criterion for the maximum attainable power.

References

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Энергоэкономные ферритовые трансформаторы

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(Резюме)

В статье предложена методика проектирования ферритовых трансформаторов с минимальными потерями. Определена зависимость суммарных потерь ферритовых трансформаторов от магнитной индукции и частоты коммутации, от параметров ферритового материала, геометрии сердечника и свойств теплоотдачи трансформатора. Функция исследована для определения минимума суммарных потерь. Доказано конкретным примером, что применение предложенной методики приводит к уменьшению потерь на 40%.