БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ . BULGARIAN ACADEMY OF SCIENCES

ПРОБЛЕМИ НА ТЕХНИЧЕСКАТА КИБЕРНЕТИКА И РОБОТИКАТА, 54
PROBLEMS OF ENGINEERING CYBERNETICS AND ROBOTICS, 54

София . 2004 . Sofia

Computer Analysis and Visualization of Kinematics and Dynamics Sensibility Parameters

Daniela Vassileva

Institute of Mechanics, 1113 Sofia

1. Introduction

In comparison with usual mechanisms and machines, robots work and solve more complex problems. That influences not only their construction, but also the characteristics needed to be known for successful description, control and practical application at all. Generally, the complexity of robot-manipulators design involves more fields of mathematical knowledge. But some other possibilities are not paid enough attention concerning the inner resources, the additional storage of the manipulative structures already designed. The last traces the way of new constructions and considerations resulting in very interesting characteristics and parameters improving theoretical understanding as well as helpful for optimizing the practical applications as a whole.

For instance, to solve some practical tasks with high precision it is necessary to improve their accuracy. The sensibility theory could be used to solve that problem. It is a very important quality characteristic [5] of robot-manipulators, which is expressed by the sensibility coefficients and directions [1, 2, 3] in the working zone. The sensibility depends on the robot-manipulator system state. In the case of contact tasks the notion of dynamics sensibility [15] is introduced taking into account the different forces not only between the robot and the environment but also between the robot-manipulator parts.

The presence of redundant joints [7, 10] has important influence on such quality characteristics as accuracy and sensibility, especially on sensibility parameters – coefficients and directions of kinematics [1, 2] and dynamics sensibility.

The dynamics sensibility parameters are determined on the basis of the kinematics ones and on the basis of the dynamic model of the manipulator. To describe robot-manipulators motion it is necessary to create their dynamic models. Different approaches exist for that. One of them is the method based on the graph theory and the Orthogonality principle [13, 14, 15], which is very convenient for hybrid systems analysis. Such is the method used in that work for dynamic modeling and computer analysis.

To do successful analysis of kinematics and dynamics sensibility parameters, allowing their visualization and calculation also for structures with more d.o.f., more than six, it is necessary to create some programs using existing software. In that work MATLAB package is used to create an appropriate program for sensibility parameters visualization and investigation. This developed software makes easy the theoretical understanding and makes possible the sensibility analysis of robot-manipulators with more degrees of freedom.

The subject of the paper is computer analysis and visualization of kinematics and dynamics sensibility parameters variation in dependence on the system state as well as visualization of the kinematics and dynamics sensibility ellipsoids interaction. The kinematics and dynamics sensibility parameters will be studied in order to achieve new quality characteristics – higher accuracy and energy optimization.

2. Theoretical background

2.1. Kinematics sensibility

The kinematics sensibility is a system quality characteristic having as quantity parameters the corresponding sensibility coefficients and directions. It can be described mathematically by transformation τ mapping the configuration robot space $Q\in R^n$ into its working one R^3. The transformation τ, is a homomorphism, consisting of two different ones τ_p and τ_r. They map the neighbourhood ΔQ around the point (configuration) $q\in Q$ into the sensibility position and orientation ellipsoids. The center of each one of them coincides with the point q and their semi-axes, following the sensibility directions, are equal to the sensibility coefficients by absolute values. The coefficients and directions are obtained as solutions of the general task of eigenvectors for both homomorphisms. Obviously, the rank of τ_p and τ_r does not exceed the dimension of R^3. The presence of redundancy reflects the dimension of Q, i.e. it becomes bigger.

2.1.1. Positioning. Tree-like manipulative structures are considered with n degrees of freedom, where contiguous bodies are connected by translation and rotational joints. The joint parameters q_i ($i = 1, \dots, n$) are chosen as generalized coordinates. The vectors $\boldsymbol{q} = (q_1, \dots, q_n)^{\mathrm{T}}$ belong to the configuration space. An arbitrary point H is fixed in the last structure body. Two coordinate systems are fixed in the support and in the last structure body. Usually in practice the needed state realizes some deviations δR, and $\delta\theta$ having probability behaviour. This is due to various reasons – errors in geometry, errors in calculations, compliance, sensing, calculations, etc.

In the case of position the deviations are described by the following expression:

$$\delta R = A(q)\delta q \,. \tag{1}$$

Let us consider an ε – neighbourhood around a configuration q and assume that the vectors δq belong there. It is proved [8, 12] that the transformation (1) maps n-dimensoinal ball ε in k-dimensional sensibility ellipsoid $E_p\in R^3$, where $k = \mathrm{rank}A$. It is also shown [4] that the ellipsoid's semi-axes' lengths are upper borders of δR on k orthogonal directions and they are obtained as eigenvalues of general task of eigenvectors: $(B_p - \lambda C)X = 0$. For every state q the matrix $A(q)$ from (1) defines a homomorphism

τ_p between configuration space Q and working zone Z. The image of τ_p is the sensibility ellipsoid for positioning [5] and the kernel is its orthogonal completing.

2.1.2. Orientation. In the case of orientation the deviations are described by the following expression, which is equivalent to (1):

(2) $$\delta\theta = L(q)\delta q.$$

For every state $q \in Q$, the matrix $L(q)$ from (2) defines a homomorphism τ_r between the configuration space Q and the working zone Z. Its image is called kinematics sensibility ellipsoid for orientation and the kernel is its orthogonal compleating.

2.2. Dynamics sensibility

If a force F is supposed to act at the characteristic point when a moment M is applied to the last structure link, the dynamics sensibility coefficients and directions can be defined for position and orientation respectively:

(3) $$\alpha_p = F.\delta R; \quad \alpha_r = M.\delta\theta; \quad \beta_p = F.\delta R; \quad \beta_r = M.\delta\theta.$$

They are related to the additional energy, forces and moments have to be compensated to assure optimal energy environment interaction.

Let at first the position dynamics sensibility coefficients be considered [2, 10]. The first one – $\alpha_p = F.\delta R$, is a scalar and has a dimension of energy. Here the question of maximal and minimal values of α_p is important for practice. It is clear that when force direction is perpendicular to some of the ellipsoid axes then the corresponding component of δR disappears and α_p takes lower value. Generally the following cases are possible.

The kinematics sensibility ellipsoid is three-dimensional. In relation to force direction one or two components of δR could be eliminated, but α_p is always positive. Its minimum value is obtained when the force is collinear to the minimal ellipsoid axis, i.e. the direction with minimal length.

When the kinematics sensibility ellipsoid is one- or two-dimensional it is possible to minimize the coefficient α_p up to zero. Here the role of redundancy is important because it is related to the problem concerning the realization of sensibility directions, following preliminarily given orientations.

The upper border of α_p, i.e. its maximal value is equal to the product of the upper border of F and the maximal sensibility coefficient for position.

The coefficient β_p expresses the additional moment caused by force F in the presence of δR. In the same way the kinematics sensibility ellipsoid is modified, i.e. any of its axis changes its direction in perpendicular plane. All the moments belong to that ellipsoid which will be called dynamics sensibility ellipsoid for position. Generally the following cases are possible.

The dynamics sensibility ellipsoid is three-dimensional when the force direction is non-collinear to its three axes.

If the force direction is collinear to some axes, the dimension of the dynamics sensibility ellipsoid decreases by 1 in comparison with the kinematics sensibility ellipsoid. The most interesting case is when the kinematics ellipsoid is a segment, collinear to the force – then the dynamics ellipsoid disappears, i.e. β_p takes its minimal value – zero.

The maximal value of β_p is obtained when the force direction is perpendicular to the biggest kinematics sensibility ellipsoid axis.

In the same way the dynamics sensibility ellipsoid for orientation can be defined and some analogous cases could be considered.

3. Computer analysis

To make complete investigation of the sensibility parameters it is necessary not only to obtain their analytical expressions, but also to visualize their variation in dependence of the system state or of the applied external force, or changing the geometrical parameters to analyze the sensibility parameters variation. Sometimes even it is not possible to calculate them without using some appropriate software, because for mechanical structures with more degrees of freedom, more than six for instance the calculations are long and hard. In order to achieve better and more precise results from our analysis, we have developed some software, using MATLAB package, to visualize the kinematics and dynamics sensibility parameters, their variation in dependence of the configuration of robot-manipulator. With the help of this software it is also possible to visualize the kinematics and dynamics sensibility ellipsoids mutual disposition in the working zone.

At first some software has been developed allowing the kinematics sensibility coefficients analysis. Through this software the variation of each kinematics sensibility coefficient could be visualized in dependence of the system configuration. This way these configurations where the kinematics coefficients take lower values, close to zero could be easily detected. For these configurations the orientation and position errors distribution is limited in smaller region. The system accuracy is higher. As a result of this computer analysis the final result is two-dimensional graphics. The configuration vector is represented as a scalar, along the x axis. Along the y axis is the investigated coefficient. Its variation as a function of the configuration vector Q is shown.

The kinematics sensibility coefficients represent the sensibility ellipsoid axes lengths. Another software has been developed which is helpful for kinematics sensibility ellipsoids' visualization in the case of position and orientation respectively. Their mutual disposition in the working zone is investigated and visualized, as well as their dimension variation in dependence of the system state. As a result we have obtained three-dimensional graphics where the kinematics sensibility ellipsoids for orientation and position are shown. With the help of this software their variation in the whole working zone could be visualized, or in some parts of it.

In the same time a very interesting question is related with the dynamics sensibility ellipsoid variation in comparison with the kinematics sensibility ellipsoid. Its dimension could decrease in dependence of the external force direction with respect to the kinematics sensibility ellipsoid axes. When the applied external force is parallel to some of the axes of the kinematics sensibility ellipsoid, then the dynamics sensibility ellipsoid dimension decrease by one in comparison with the kinematics sensibility ellipsoid dimension. Also its axes directions are the kinematics sensibility ellipsoid axes rotated to 90°. The last software investigation is exactly related with the above-enumerated problems' solution. Varying the external force direction and also changing the generalized coordinates values, we could visualized the kinematics and dynamics sensibility ellipsoid variation in the working zone of the robot-manipulator. That is helpful to view their mutual disposition, dimension and orientation. Of course all these computer analyses are convenient to be applied in the case of orientation as well as in the case of position. They could be done in the whole working zone, or in some part of it, for all the kinematics parameters. The most precious thing is that this computer analysis is appropriate also for structures with many d.o.f., when it is not simple and even possible to obtain the sensibility parameters analytical expressions. Using this software it is easy to choose these configurations where the error distribution is in smaller regions or in preliminary given directions. It is simple to investigate the redundant joint influence on the sensibility ellipsoid dimensions. It is proved [6, 10] the redundancy is helpful to control the sensibility parameters. And not in the last place, this way of analysis makes the theoretical understanding easier and is useful for better quality characteristic achievement.

4. Application

The software developed for investigating and visualizing the kinematics and dynamics sensibility parameters finds its concrete application during the conception of a mechatronic system, which will be used in medicine for drilling operations. The kinematics scheme and the obtained results are presented bellow.

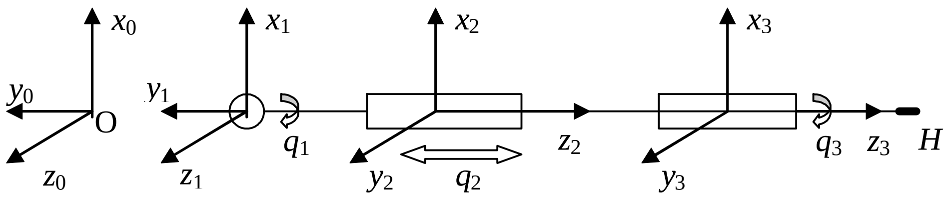

Fig. 1. Kinematics scheme of the manipulative structure $R\perp T\|R$

4.1. Kinematics sensibility parameters investigation for the manipulative structure $R\perp T\|R$

First the kinematics sensibility parameters are investigated. Their analytical expressions are obtained and presented.

The homomorphism τ_p is described by matrix A and matrix B_p having the form respectively:

$$(4)\qquad A=\begin{pmatrix}(p_3+q_2)\cos q_1 & \sin q_1 & 0\\ (p_3+q_2)\sin q_1 & -\cos q_1 & 0\\ 0 & 0 & 0\end{pmatrix},\quad B_p=\begin{pmatrix}(p_3+q_2)^2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}.$$

The Ker B_p = Ker A is described by one basic eigenvector:

$$(5)\qquad X^{(1)}=\begin{bmatrix}0 & 0 & 1\end{bmatrix}^T.$$

Here Ker A is one-dimensional. And Im A is two-dimensional derived by the eigenvectors $X^{(2)}=\begin{bmatrix}1 & 0 & 0\end{bmatrix}^T$ and $X^{(3)}=\begin{bmatrix}0 & 1 & 0\end{bmatrix}^T$ corresponding to the positive eigenvalues $\lambda_2 = (p_3 + q_2)^2$, $\lambda_3 = 1$ respectively. Therefore we have two-dimensional kinematics sensibility ellipsoid in the case of position.

In the case of orientation the homomorphism τ_r is described by matrix L and the matrix B_r having the following form respectively:

$$L = \frac{1}{2}\begin{pmatrix} cq_1 sq_3 & 0 & sq_1 cq_3 \\ cq_1(1+cq_3) & 0 & -sq_1 sq_3 \\ -sq_1 sq_3 & 0 & cq_3(1+cq_1) \end{pmatrix}, \tag{6}$$

$$B_r = \begin{pmatrix} 1+c^2q_1+2cq_3 & 0 & -sq_1 sq_3\left(cq_1(1+cq_3)+cq_3\right) \\ 0 & 0 & 0 \\ -sq_1 sq_3\left(cq_1(1+cq_3)+cq_3\right) & 0 & 1+c^2q_3+2cq_1 \end{pmatrix}. \tag{7}$$

The Ker B_r = Ker L is described by one basic eigenvector:

$$X^{(1)} = [0 \quad 1 \quad 0]^T. \tag{8}$$

Here Ker L is one-dimensional. And Im L is two-dimensional described by the eigenvectors $X^{(2)} = [1 \quad 0 \quad 0]^T$ and $X^{(3)} = [0 \quad 0 \quad 1]^T$ corresponding to the positive eigenvalues $\lambda_2 = \lambda_3 = 1$ (in first approximation). Here the sensibility ellipsoid for orientation is also two-dimensional.

Here it is interesting to know which are the configurations where the kinematics sensibility coefficients take lower values, close to zero. One way to detect easily these configurations is to develop some software giving the possibility to visualize them. Such software is developed allowing the visualization of the kinematics sensibility coefficients variation in dependence of the system state (configuration). The n-dimensional zone Q is interpreted as one-dimensional numerical axis and the sensibility coefficients can be graphically presented as a function of one argument. The concrete results for the considered manipulative structure could be seen on the next Fig. 2.

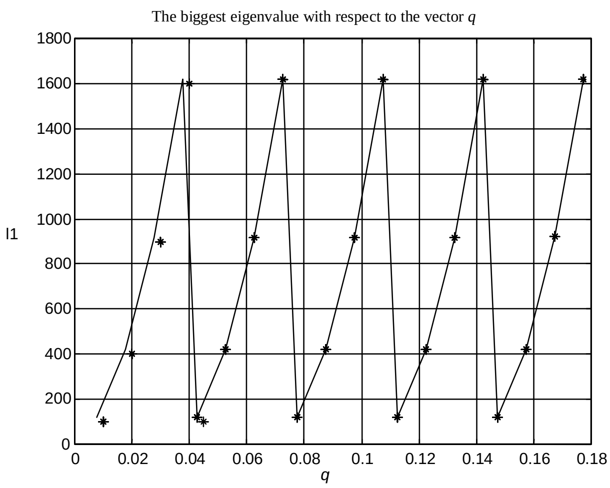

Fig. 2. Maximal kinematics sensibility coefficient variation for the manipulative structure $R\perp T\|R$ with respect to the normalized configurational vector Q, in the case of position

Here the maximal kinematics sensibility coefficient is visualized, in the case of positioning, as a function of Q (configuration vector). With "*" the coefficients calculated values are marked (without normalizing them).

The kinematics sensibility coefficients represent the sensibility ellipsoids axes lengths. When they decrease or become zero, the sensibility ellipsoid dimension decrease therefore the orientation and position errors are distributed in smaller regions. In the case of the concrete structure *R⊥T//R* we have two-dimensional kinematics sensibility ellipsoids in the case of position and orientation respectively. It is interesting to know their mutual disposition in the working zone of the robot-manipulator. It is useful also to investigate the sensibility ellipsoid dimension variation in dependence of the system state. For these configurations where the sensibility ellipsoid axes lengths decrease or one of them is close to zero, one could say that the errors are distributed in smaller region and the system accuracy increase. For some practical tasks it is useful to choose some configurations where the kinematics sensibility directions are oriented in a preliminary given direction. For instance for drilling operations the most useful case is when the sensibility ellipsoid is one-dimensional and its axis follows the drilling direction. That is the reason why it is important to investigate the sensibility ellipsoid dimension variation in dependence of the vector Q, as well as its orientation in the working zone. Software is developed visualizing the sensibility ellipsoids variation in the case of position and orientation simultaneously. The results for the considered structure *R⊥T//R* are presented below for two arbitrary chosen configurations. We could see that in the first case the sensibility ellipsoid for orientation (with gray color) is nearly a segment, and its remaining axes lengths are close to zero. Its minimal axis length is shorter than the minimal axis length for the sensibility ellipsoid in the case of position. The last (with black color) is also two-dimensional and there are some configurations where its axes lengths decrease visible.

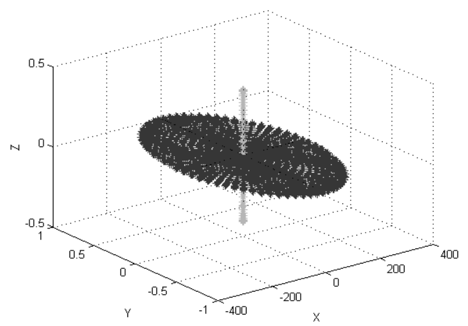

Fig. 3. Kinematics sensibility ellipses interaction in the case of position and orientation, for some arbitrary chosen configurations. Grey – kinematics sensibility ellipsoid for orientation which is approximately a segment. The eigenvalues are: $\lambda_1 = 0$; $\lambda_2 = 0.4132$; $\lambda_3 = 0.8214$. With black color the kinematics sensibility ellipse for position is marked. Its axis lengths (eigenvalues) are: $\lambda_1 = 0$; $\lambda_2 = 1$, $\lambda_3 = 225$. The configuration for which these ellipsoids are visualized is $q_1 = 0$ rad, $q_2 = 15$ mm, $q_3 = 0.8727$ rad

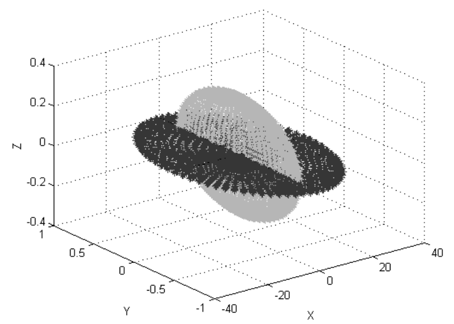

Fig. 4. Kinematics sensibility ellipses interaction in the case of position and orientation, for another configuration ($q_1 = 0.5585$ rad; $q_2 = 5$ mm; $q_3 = 0.8727$ rad). In the case of orientation the eigenvalues corresponding to this configuration are: $\lambda_1 = 0$; $\lambda_2 = 0.2959$; $\lambda_3 = 0.7590$. In the case of position they are: $\lambda_1 = 0$; $\lambda_2 = 1$; $\lambda_3 = 25$

4.2. Dynamics sensibility parameters investigation for the manipulative structure $R\perp T\|R$.

Another element of this investigation is the dynamics sensibility parameters analysis. They are investigated when we have contact task. Let the following notations are made**:** *F*, *T* – main vector and main moment of external action on the last body; λ_p, λ_r – the non-zero (positive) eigenvalues for position and orientation. Following the definitions for dynamics sensibility coefficients, it can be written for the maximal coefficient values $\alpha_p = \sqrt{\lambda_p}|F|$; $\alpha_r = \sqrt{\lambda_r}|T|$. For the considered structure the main vector of external forces consists of gravity force, resistant force (due the contact with the bones). The last is on the translation direction and the first depends on the concrete drilling position. Both coefficients express the additional power, i.e. energy for unit time the structure needs to compensate the system error, so that it can be minimized in the case when the gravity is perpendicular to the drilling direction. Finally, the maximal values of α_p and α_r are obtained by evaluation of upper borders of *F* and *T*, which could be taken from real experimental results. In the same way the coefficients β_p and β_r can be analyzed for the considered structure. As general, they are related to the additional moments, the system needs to compensate due to corresponding errors. In our case β_r is different from zero only for the gravity force component F^t, i.e. the additional moment appears when F^t is collinear to drilling direction. There is dependence between α_p and β_p due to mutual vectors disposition. When α_p increases, β_p decreases at the same time by absolute value. By analogy, the minimal value (zero) of β_r is obtained when the main moment is collinear to the orientation error vector. The maximal values for β_p and β_r also depend on the appropriate evaluations of main force and moment absolute values. In our case only α_p and β_p can be minimized for the sake of λ_3 (especially the geometric parameter). But the remaining coefficients depend entirely on the force and moment evaluation.

Two cases are vizualized in the case of orientation The first one is when the external force (Fig. 5) is perpendicular to the drilling direction. In that case the dynamics sensibility ellipsoid dimension decrease by one with respect to the kinematics sensibility ellipsoid dimension. In the second case the external force is parallel to the drilling direction (Fig. 6). In that case the dynamics sensibility ellipsoid dimension do not decrease with respect to the kinematics sensibility ellipsoid dimensions. Both ellipsoids are two-dimentional here. In the figures bellow the kinematics and dynamics sensibility ellipsoids are visualized for two arbitrary chosen configurations. Their mutual disposition and variation in the working zone in dependence of the configuration vector could be visualized with the help of the developed software. The last makes the theoretical understanding easier and traces new ways of investigation and accuracy and energy optimization.

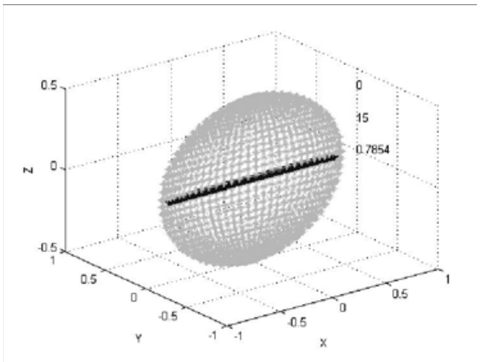

Fig. 5. Manipulative structure $R\perp T||R$. Case of orientation. With grey – kinematics sensibility ellipse, with black – dynamics sensibility ellipse (in that case – segment). When the external force F is perpendicular to the drilling direction. In that case the dynamics sensibility ellipsoid dimension decrease by one in comparison with the kinematics sensibility ellipsoid

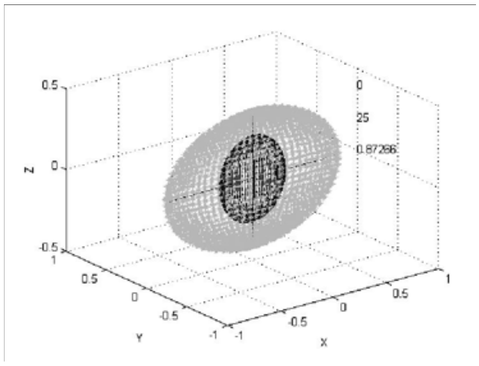

Fig. 6. Manipulative structure $R\perp T||R$. Case of orientation. With grey – kinematics sensibility ellipse, with black – dynamics sensibility ellipse. When the external force F is parallel to the drilling direction. In that case the dynamics sensibility ellipsoid dimension do not decrease

5. Conclusion

Sensibility parameters investigation is very important for achievement of better accuracy and for energy optimization. In that sense it is important the obtained results to be very clear and precise. But during the investigation of manipulative structures with many degrees of freedom, in most of the cases, it is impossible to obtain explicit analytical expressions for the sensibility parameters. In that case it is convenient to use some software, able to calculate the sensibility parameters values in the whole working zone, or round some chosen configurations. Such software has been developed in that work and the obtained results are visualized. The last makes the obtained results clearer and offers a faster way to detect the most appropriate configurations for better execution of the concrete robot task. These are the configurations for which the kinematics sensibility coefficients take lower values, close to zero, or the sensibility ellipsoids axes are oriented along preliminary given directions. As well as these configurations, where the position and orientation errors are distributed in smaller regions and the additional moments and energy which have to be compensated due to corresponding errors are minimal. To control the sensibility parameters values and the sensibility directions state as well as the sensibility ellipsoids dimensions an important role have the additional degrees of freedom. Using the developed software there is no problem to add as many additional d.o.f. as we want. The geometric parameters also have influence on the sensibility parameters. As future work it will be useful to investigate their influence and that could be used for mechanical structure optimization.

References

1. Boiadjiev, G. Kinematics sensible directions of manipulating systems. – Iv: Proc. of the Int. Symposium on Measirment and Control in Robotics, Brussels, 1996, 77-82.

2. B o i a d j i e v, G., K. K o s t a d i n o v. Coefficients and directions of dynamic sensibility for robot manipulators. – In: Int. Workshop on Adv. Robotics and Inteligent Machins, Manchester, ISSN 11363-2698, PN7, 1997, 1-11.
3. V a s s i l e v a, D., G. B o i a d j i e v. Coefficients and directions of senibility for orientation of redundant robot-manipulators. – SMU, Vol. 3, Drjanovo, 1999, ISSN 1310-3946, 1.43-1.49.
4. V a s s i l e v a, D., G. B o i a d j i e v. Control of kinematics senibility parameters for planar robot-manipulators with redundancy. – Edition of Scientific Machine Union, Vol. 3, Drjanovo, ISSN 1310-3946, 2000, 3.27-3.33 (in Bulgarian).
5. V a s s i l e v a, D., K. H r i s t o v, G. B o i a d j i e v. Mathematical techniques at sensibility analyses of mechanical manipulative structures. – Int. Journal Information – Theories&Applications, **7**, ISSN 1310-0513, 2000, No 2, 69-83.
6. V a s s i l e v a, D. Sensibility of Mechatronic Systems with planar manipulative structures. – In: Proc. First Baltic-Bulgarian Conf. on Mechanics, Biomechanics and Bionics, vol. 1, Varna, 2000, ISBN 9984-681-74-2, 71-76.
7. V a s s i l e v a, D., G. B o i a d j i e v. Sensibility analysis of manipulative structures with redundancy. – SMU, vol. 3 (54), Year VII, 1310-3946, 2000, 211-216 (in Bulgarian).
8. V a s s i l e v a, D., G. B o i a d j i e v. Kinematics sensibility parameters interaction for manipulative structures with redundancy. – In: Proc. of 9th National Congress of Theoretical and Applied Mechanics, Varna, September 19-22, 2001.
9. B o i a d j i e v, T., G. B o i a d j i e v. Force control synthesis during interaction with variable density environment. – In: Proc. of 9th National Congress of Theoretical and Applied Mechanics, Varna, September 19-22, 2001.

Компьютерный анализ и визуальное представление параметров кинематической и динамической чувствительности

Даниела Василева

Институт механики, 1113 София

(Р е з ю м е)

Рассматриваются проблемы, связанные с анализом параметров кинематической и динамической чувствительности манипуляционных роботов. Чувствительность является качественной характеристикой системы, опысивающей ее “внутренных” свойств в каждой конфигурации. Математически она представляется гомоморфизмом, изображающий окрестность произвольной конфигурации в рабочей зоне манипулятора. Количественные параметры этой характеристики называются коеффициентами и направлениями кинематической и динамической чувствительности, а образы гомоморфизма – эллипсоиды чувствительности. Исследование параметров чувствительности связанно с повышением точности и оптимизации энергии, необходимой для выполнении рабочей задачи манипулятора. Аналитическое решение, особенно для манипуляторов с избыточными степенями подвижности (более чем шести), требует громоздкие вычисления и в результате получаются слишком сложные математические выражения. Поэтому на основе MATLAB создана программная система, с помощью которой исследуются параметры чувствительности и получается их визуальное представление. В работе приведенны примеры анализа структур,

показывающие соответствие с теоретическими результатами, а также и исследование системы с большим числом степеней подвижности. Показано также визуальное представление взаимодействия эллипсоидов чувствительности. Значение этой работы связанно с возможностями нахождений оптимальных состояний манипулятора для выполнении соответствующей рабочей задачи в смысле повышения его точность и минимизации компенсирующей энергии из-за вероятных отклонений. Указывается также возможность применения созданной програмной системы для будущего исследования параметров чувствительности в результате варирования геометрических параметров исследуемой системы.