БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ . BULGARIAN ACADEMY OF SCIENCES

ПРОБЛЕМИ НА ТЕХНИЧЕСКАТА КИБЕРНЕТИКА И РОБОТИКАТА, 50
PROBLEMS OF ENGINEERING CYBERNETICS AND ROBOTICS, 50

София . 2000 . Sofia

Scalarizing Problems of Multiobjective Linear Integer Programming

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1. Introduction

The interactive algorithms are the most widely spread algorithms solving problems of multiobjective linear and nonlinear programming. Each iteration of such an algorithm consists of two phases: a computing one and a dialogue one. During the computing phase one or more (weak) nondominated solutions are generated with the help of a scalarizing problem. In the dialogue phase these (weak) nondominated solutions are represented for evaluation to the decision maker (DM). In case the DM does not approve any of these solutions as a final solution (the most preferred solution), the he gives information, concerning his local preferences, that improve these solutions. This information is used to formulate a new scalarizing problem, which is solved at the next iteration.

The quality of each interactive algorithm is defined to a great extent by the quality of the dialogue with the DM. On its side the quality of the dialogue with the DM depends on:

– the type of information, required from the DM in order to improve the local nondominated solution preferred. The clearer the desired information is for him, the more realistically he can express his preferences. The abilities to improve the information required from the DM are connected with the possibilities to formulate the respective scalarizing problems, the parameters of which express this information;

– the time for solution of the scalarizing problem. The smaller the time for evaluation of local (weak) nondominated solutions by the DM is, the greater his desire is to solve the multiobjective problem;

– the possibilities to train the DM with respect to the multiobjective problem solved. When the freedom of movement in the feasible (weak) nondominated set of the DM is greater on one hand and when more (weak) nondominated solutions can be evaluated at one iteration on the other hand, the DM can choose faster the most preferred solution;

– the type and the number of the new (weak) nondominated solutions compared with the local preferred solution. The more distant the new (weak) nondominated solutions are from the local preferred solution, the fewer (weak) nondominated solutions the DM can evaluate at one iteration.

When solving problems of multiobjective linear programming as scalarizing problems, linear programming problems are used. These problems belong to the class of P-problems (Garey and Johnson [2]). They are easily solved problems. That is why in the interactive algorithms for solving multiobjective linear problems the time for solution of the scalarizing problems does not play a significant role. Particular attention in the development of these algorithms is paid to the type of information, which is required from the DM to improve the locally preferred (weak) nondominated solution. Up to now (Wierzbicki [10]) mainly the aspiration levels of the criteria, that the DM wants to achieve, have been used as such information. These levels define in the criteria space the so called local reference point. Especial attention is paid to the possibilities for training the DM, expressed in the defining during the computing phase of more than one (weak) nondominated solution. These solutions are shown for evaluation to the DM (Korhonen and Laasko [6]). It should be noted nevertheless that in modern interactive algorithms for solving multiobjective linear problems it is accepted by default that the DM can easily estimate more than two (weak) nondominated solutions. Anyway, when comparing and evaluating more than two (weak) nondominated solutions, especially when the criteria number is large and when the (weak) nondominated solutions differ considerably, the DM may encounter difficulties in the selection of the local (global) preferred (weak) nondominated solution (Jashkiewich and Slowinski).

The problems of linear integer programming are NP-difficult problems (Garey and Johnson [2]). The exact algorithms, solving these problems have exponential complexity. Moreover, the finding of a feasible integer solution in them is so difficult as the finding of an optimal solution. That is why in the design of interactive algorithms solving multiobjective linear integer problems, it is obligatory to take into account the time for solving the scalarizing problems. If this time is too long, the dialogue with the DM, though quite convenient, may not occur. This can happen in case the DM does not want to wait too long for the solution of the scalarizing problem

In modern interactive algorithms solving multiobjective linear integer problems (Gabbani and Magazine [3]; Ramesh, Karwan and Zionts [9]: Hajela and Shin [4]; Eswarn, Ravindran and Moskowitz [1]; Narula and Vassilev [8]; Karaivanova, Korhonen et al. [7], in a smaller or greater extent, the factor “time” of scalarizing problem solving is taken intoconsideration. For this purpose, the number of the integer problems solved is decreased; approximate algorithms are used to solve the integer problems or a possibility is provided to interrupt the exact algorithms in solving these problems; instead of integer problems (especially in the process of DM’s learning), continuous problems are solved and the (weak) nondominated solutions obtained are represented to the DM for evaluation.

Some of the interactive algorithms operate with aspiration levels of the criteria, others use weight coefficients for the relative significance of the criteria. The greater part of them show the DM for evaluation one (weak) nondominated solution at each iteration, the remaining ones – several (weak) nondominated solutions (sometimes hardly compared solutions).

In the paper presented, on the basis of new scalarizing problems, interactive algorithms are suggested, which to a large extent include the positive aspects of the interactive algorithms solving multiobjective linear integer problems, realized up to the present moment. The main features of these interactive algorithms are as follows:

– the information required from the DM refers to the desired values of alteration or the desired directions of change in any of the criteria. This information is easily set by the DM;

– a possibility to obtain continuous solutions and also approximate integer solutions, which decreases the wating time of the DM;

– reduction in the number of the integer problems solved;

– a possibility for comparatively quick learning of the DM with respect to the multiobjective linear integer problems, providing at each iteration more (weak) nondominated solutions for evaluation or approximate (weak) nondominated solutions, as well as free movement of the DM in the whole domain of these solutions;

– comparatively easy evaluation of the problems by the DM due to the fact that they are near one to another.

2. Problem formulation

The problem of multiobjective linear integer programming (we shall denote it as problem (I) can be formulated as:

(1) $$\max \{f_k(x),\ k\in K\}$$

subject to the constraints:

(2) $$\sum_{j\in N} a_{ij} x_j \le b_i,\ i\in M,$$

(3) $$0 \le x_j \le d_j,\ j\in N,$$

(4) $$x_j - \text{integer},\ j\in N,$$

where

– the symbol max means that all the objective functions have to be simultaneously maximized;

– $K=\{1, 2, ..., p\}$, $M=(1, 2, ..., m\}$, $N=\{1, 2, ..., n\}$ are the index sets respectively of the linear objective functions (criteria), of the linear constraints and the variables (solutions):

– $f_k(x)$, $k\in K$, are linear objective functions (criteria):

$$f_k(x) = \sum_{j\in N} c_j^k x_j;$$

– $x=(x_1, x_2, ..., x_j, ..., x_n)^T$ is the vector of the variables (solutions).

The constraints (2)–(4) determine the feasible set of the integer variables (solutions). We shall denote this set by X_1.

The problem (1)–(3) is a problem of multiobjective linear integer programming. We shall denote it as problem (P). The feasible set of the continuous variables is denoted by X_2. The problem (P) is a relaxation of problem (I).

We shall introduce several definitions below for greater clarity and are going to use new denotations.

Definition 1. The solution x is called efficient solution of problem (I) or (P), if there does not exist any other solution $\bar{x}$, so that the following inequalities are satisfied:

$$f_k(\bar{x}) \ge f_k(x) \text{ for every } k\in K \text{ and}$$

$$f_k(\bar{x}) > f_k(x) \text{ at least for one index } k\in K.$$

Definition 2. The solution x is called a weak efficient solution of problem (1) or (P) if there does not exist another solution $\bar{x}$ such that the following inequalities are fulfilled:

$$f_k(\bar{x}) > f_k(x) \text{ for every } k\in K$$

Definition 3. The solution x is called a (weak) efficient solution, if x is either an efficient solution, or a weak efficient solution.

Definition 4. The vector $f(x)=(f_1(x),...,f_p(x))^T$ is called a (weak) nondominated solution in the criteria space, if x is a (weak) efficient solution in the variables space.

Remark. (Weak) efficient solutions in the space of the variables and (weak) nondominated solutions in the space of the criteria are (weak) Pareto optimal solutions.

Definition 5. An approximate (weak) nondominated solution is a feasible solution in the criteria space, located comparatively close to the (weak) nondominated solutions.

Definition 6. Desired alterations of the criteria at each iteration are the values, by which the DM wishes to increase the values of some criteria in the last (weak) nondominated solution obtained with the purpose to improve this solution according to the local preferences of the DM.

Definition 7. A reference (local reference) point in the criteria space is the point, determined by the last point obtained and the desired alterations of the criteria.

Definition 8. A reference direction in the criteria space is the direction, defined by the reference point and the last point obtained.

Problems (I) and (P) do not possess an optimal solution. Hence it is necessary to select one solution among the (weak) nondominated solutions, which fits best the global DM's preferences. This choice is subjective and depends entirely on the DM.

3. Scalarizing problems

As already pointed out, each interactive algorithm consists of two phases: a computing one and a dialogue one. In the computing phase a scalarizing problem is solved, with the help of which new (weak) nondominated solutions are found, that the DM expects to improve (with respect to his local preferences) in comparison with the current solution preferred (the last solution found).

Depending on the values of the criteria in the current preferred solution and the local preferences of the DM, the criteria set can be separated into three groups. Let us denote them by K_1, K_2 and K_3 respectively. The set K_1, contains the indices $k\in K$ of those criteria, which values the DM agrees to be improved at the current preferred solution (their values to be incremented by given values Δ_k). The set K_2 includes the indices $k\in K$ of the criteria that the DM does not take into account. Their values may be worsened. The set K_3 contains the indices $k\in K$ of the criteria, the values of which the DM wants to preserve.

Let us denote also by f_k, $k\in K$, the values of the criteria in the current preferred solution and by $\bar{f}_k$, $k\in K$, the values of the criteria in the reference point. These values are determined as follows:

$$\bar{f}_k = \begin{cases} f_k+\Delta_k, & k\in K_1, \\ f_k, & k\in K_2\cup K_3, \end{cases}$$

where Δ_k is the value, by which the DM wants to improve the value of the criterion with an index k.

In order to find better (weak) nondominated solutions of the multiobjective linear integer problem, taking into account the local preferences of the DM, the following scalarizing problem is proposed (let us denote it as (A1)).

Let us minimize:

$$S(x) = \max\,[\max_{k\in K_1}(\bar{f}_k - f_k(x))/|f'_k|,\ \max_{k\in K_2}(\bar{f}_k - f_k(x))/|f'_k|] \tag{5}$$

subject to

$$f_k(x)\geq f_k,\ k\in K_3, \tag{6}$$

$$x\in X_1, \tag{7}$$

where

$$f'_k = \begin{cases} f_k, \text{ if } k \neq 0, \\ 1, \text{ if } k = 0. \end{cases}$$

Problem (5)–(7) has a feasible solution if the feasible set X_1 is not empty and has an optimal solution, if the feasible set X_1 is limited.

The main advantage of the scalarizing problem (AI) is the minimization of the maximal component standard deviation of the solution searched $f(x) = (f_1(x), ..., f_p(x))^T$, from the reference point $\bar{f}(x) = (\bar{f}_1(x), ..., \bar{f}_p(x))^T$ in the criteria space.

In order to find better (weak) nondominated solutions of the multiobjective problem (P), considering the DM's local preferences, analogous to (AI) scalarizing problem is suggested (denoted as (A2)). The scalarizing problem (A2) is obtained from the scalarizing problem (AI), replacing the constraint (7) by the following constraint:

(8) $$x \in X_2.$$

Theorem 1. The optimal solution of the scalarizing problem (A1) is a weak efficient solution of the multiobjective linear integer problem (I).

Proof. The scalarizing problem (A1) has any sense, when $K_1 \neq \varnothing$. That is why we accept that $K_1 \neq \varnothing$. Let x^* be an optimal solution of the problem (A1). Then the following inequality is valid:

(9) $$S(x^*) \leq S(x) \text{ for each } x \in X_1 \text{ and } f_k(x^*) \geq f_k,\ k \in K_3.$$

Let us assume that x^* is not a weak efficient solution of problem (1). Then there must exist another point x' in the variables space, which satisfies the condition:

(10) $$f_k(x^*) < f_k(x') \text{ for } k \in K \text{ and } f_k(x^*) \geq f_k,\ k \in K_3.$$

After the transformation of the objective function $S(x)$ of problem (A1), using inequalities (10), the following relation is obtained:

(11) $$S(x') = \max\,[\max_{k \in K_1}(\bar{f}_k - f_k(x'))/|f'_k|,\ \max_{k \in K_2}(f_k - f_k(x'))/|f'_k|] =$$
$$= \max\,[\max_{k \in K_1}((\bar{f}_k - f_k(x^*)) + (f_k(x^*) - f_k(x')))/|f'_k|,$$
$$\max_{k \in K_2}((f_k - f_k(x^*)) + (f_k(x^*) - f_k(x')))/|f'_k|] <$$
$$< \max\,[\max_{k \in K_1}(\bar{f}_k - f_k(x^*))/|f'_k|,\ \max_{k \in K_2}(f_k - f_k(x^*))/|f'_k|] =$$
$$= S(x^*).$$

It follows from (11) that $S(x') < S(x^*)$ and $f_k(x^*)) \geq f_k$, $k \in K_3$, which contradicts to (9). Hence x^* is a weak nondominated solution of problem (I).

It is obvious, that the corresponding solution in the criteria space $f(x)$ is a weak nondominated solution of problem (I).

Consequence 1. Theorem 1 is true for arbitrary values of f_k, $k \in K$.

The proof of the consequence is elementary, since the proof of Theorem 1 does not take in mind what values the criteria f_k, $k \in K$, will have in the last solution obtained (the current preferred solution), In other words, $f = (f_1, ..., f_p)^T$ can be a feasible or unfeasible solution of the problem (I) in the criteria space; a nondominated, a weak nondominated or even a dominated solution.

Theorem 2. The optimal solution of the scalarizing problem (A2) is a weak efficient solution of the multiobjective linear problem (P).

The proof of Theorem 2 is analogous to the proof of Theorem 1, where it is not accounted whether x^* is an integer or continuous solution.

The objective function of the scalarizing problem (AI) is a nonlinear, even indifferentiable function. The problem (AI) is equivalent to the following standard problem of mixed linear integer programming (we shall denote it by (A1′)):

(12) $$\min \alpha$$

subject to:

(13) $$\alpha \geq (f_k - f_k(x))/|f'_k|,\ k \in K_1,$$

(14) $$\alpha \geq (f_k - f_k(x))/|f'_k|,\ k \in K_2,$$

(15) $$f_k(x) \geq f_k,\ k \in K_3,$$

(16) $$x \in X_1,$$

(17) $$\alpha - \text{arbitrary}.$$

When problem (A1) has a solution, problem (A1′) has too. This is so, since the two problems have one and the same constraints, defining their feasible sets. The value of the objective function in the optimal solution of problem (A1) is equal to the value of the objective function in the optimal solution of problem (A1′). This follows from:

Theorem 3. The optimal values of the objective functions of problems (A1) and (A1') are equal:

$$\min_{x \in X_1} \alpha = \min_{x \in X_1} \max\, [\max_{k \in K_1}(\bar{f}_k - f_k(x))/|f'_k|,\ \max_{k \in K_2}(f_k - f_k(x))/|f'_k|].$$

Proof. It follows from (13) that

$$\alpha \geq (\bar{f}_k - f_k(x))/|f'_k| \text{ for } k \in K_1.$$

Since the upper inequality is valid for each $k \in K_1$, then it follows that

(18) $$\alpha \geq \max_{k \in K_1}(\bar{f}_k - f_k(x))/|f'_k|.$$

From (14) it follows that

$$\alpha \geq (f_k - f_k(x))/|f'_k|,\ k \in K_2.$$

Since this inequality is in power for every $k \in K_2$, it follows that

(19) $$\alpha \geq \max_{k \in K_2}(f_k - f_k(x))/|f'_k|.$$

It can be written from (18) and (19) that

(20) $$\alpha \geq \max[\max_{k \in K_1}(\bar{f}_k - f_k(x))/|f'_k|,\ \max_{k \in K_2}(f_k - f_k(x))/|f'_k|].$$

If x^* is an optimal solution of (A1'), then:

(21) $$\min_{x \in X_1} \alpha = \max[\max_{k \in K_1}(\bar{f}_k - f_k(x^*))/|f'_k|,\ \max_{k \in K_2}(f_k - f_k(x^*))/|f'_k|],$$

because otherwise α could be still decreased.

The right side of equality (21) can also be written as:

$$\min_{x \in X_1} \max[\max_{k \in K_1}(\bar{f}_k - f_k(x))/|f'_k|,\ \max_{k \in K_2}(f_k - f_k(x))/|f'_k|],$$

which proves the theorem.

The scalarizing problem (A1') has three properties, which enable the overcoming to a great extent of the computing difficulties, connected with its solution, and also the decrease in DM’s tension when comparing new solutions. The first property is connected with the fact, that the current integer preferred solution (found at the previous iteration) is a feasible integer solution of problem (A1′). This facilitates the exact, as well as the approximate algorithms solving problem (A1′), because they start with a feasible integer solution. The second property is that the feasible solutions of problem (A1′) in the criteria space, found with the help of an exact or approximate algorithm, lie near to the nondominated surface of problem (I). The obtaining and use of such approximate (weak) nondominated solutions can decrease considerably the time, the DM is expecting to evaluate the new solutions. Hence, with insignificant decrease in the quality of the solutions obtained in the criteria space, the dialogue with the DM can be considerably improved. The third property is connected with the realized strategy of DM's search, and namely - "not large benefits – little losses". The solutions obtained along the reference direction, defined by a current preferred solution and the reference point, are comparatively close one to another, which enables the DM evaluate them more easily and choose the next local preferred solution, maybe a global preferred solution also. In other words, the influence of the so called "restrained comparability" of the (weak) nondominated solutions decreases.

The scalarizing problem (A2) is equivalent to the following standard problem of linear programming (we denote it by (A2′)):

(22) $$\min \alpha$$

subject to:

(23) $$\alpha \geq (\bar{f}_k - f_k(x))/|f'_k|,\ k \in K_1,$$

(24) $$\alpha \geq (f_k - f_k(x))/|f'_k|,\ k \in K_2,$$

(25) $$f_k(x) \geq f_k,\ k \in K_3,$$

(26) $$x \in X_2,$$

(27) $$\alpha - \text{arbitrary}.$$

Problem (A2′) has similar properties as problem (A1′), but it concerns continuos solutions here, not integer. The relation between problems (A2′) and (A2) is identical to that between problem (A1′) and (A1), which can be easily proved.

Let us assume that we have found a (weak) nondominated solution of problem (P) with the help of problem (A2′) and wish to find a (weak) nondominated solution of problem (I), which is near to the (weak) nondominated solution of problem (P). Let us denote by $\tilde{f} = (\tilde{f}_1, ..., \tilde{f}_p)^T$ a (weak) nondominated solution of problem (P). In case we assume that $\tilde{f} = (\tilde{f}_1, ..., \tilde{f}_p)^T$ is the reference point, in order to find a near (weak) nondominated solution of problem (I) (the criteria values in this solution do not differ much from $\tilde{f}_k$, $k \in K$), then we can use a standard Chebyshev's problem (Wierzbicki (1980)). It has the following type (we shall denote it by (A3)):

Let us minimize

(28) $$S(x) = \max_{k \in K} \{(\tilde{f}_k - f_k(x))/|f'_k|\},$$

subject to:

(29) $$x \in X_1,$$

where

$$\tilde{f}'_k = \begin{cases} \tilde{f}_k, & \text{if } f_k \neq 0, \\ 1, & \text{if } k = 0. \end{cases}$$

The following problem of mixed integer programming is equivalent to this problem:

(30) $$\min \alpha$$

under the constraints:

(31) $$\alpha \geq (\tilde{f}_k - f_k(x))/|\tilde{f}'_k|,\ k \in K,$$

(32) $$x \in X_1,$$

(33) $$\alpha - \text{arbitrary}.$$

Problem (30)–(33) will be denoted as (A3′).

With the help of scalarizing problems (A1) and (A2) ((A1′) and (A2′) respectively), (weak) efficient solutions of the multiobjective problems (I) and (P) are found. If needed to obtain only efficient solutions, then modified scalarizing problems can be solved (denoted by (B1) and (B2) respectively). The problem (B1), with which efficient solutions of problem (I) are found, has the form:

Minimize

(34) $$T(x) = \max\,[\max_{k \in K_1} (\tilde{f}_k - f_k(x))/|f'_k|,\ \max_{k \in K_2} (f_k - f_k(x))/|f'_k|] + $$
$$+ \beta\,[\sum_{k \in K_1} (\tilde{f}_k - f_k(x)) + \sum_{k \in K_2} (f_k - f_k(x))]$$

subject to:

(35) $$f_k(x) \geq f_k,\ k \in K_3,$$

(36) $$x \in X_1,$$

where β is an arbitrary small number.

Problem (B2), with the help of which efficient solutions of problem (P) are found, is the same type as problem (B1), but constraint (36) is replaced by the following constraint:

$$x \in X_2.$$

Theorem 4. The optimal solution of the scalarizing problem (B1) is an efficient solution of the multiobjective linear integer problem (I).

Proof. The scalarizing problem has sense when the set K_1 is not an empty set. Let $K_1 \neq \emptyset$.

Let x^* be an optimal solution of problem B1. Then for any $x \in X_1$, the following condition is satisfied:

(37) $$T(x^*) \leq T(x).$$

Let us assume that x^* is not an efficient solution of problem (I). Then there must exist another x (another point in the variables space), for which the condition below given is satisfied:

(38) $$f_k(x') \geq f_k(x^*) \text{ for } k \in K$$

and at least for one index $l \neq k$,

$$f_l(x') > f_l(x^*).$$

After the transformation of the objective function $T(x')$ of problem (B1), using inequalities (38), the following relation is obtained:

(39) $$T(x') = \max[\max_{k \in K_1}(\bar{f}_k - f_k(x'))/|f'_k|,\ \max_{k \in K_2}(f_k - f_k(x'))/|f'_k|] +$$
$$+\beta\,[\sum_{k \in K_1} (\bar{f}_k - f_k(x')) + \sum_{k \in K_2} (f_k - f_k(x'))] =$$
$$= \max[\max_{k \in K_1}((\bar{f}_k - f_k(x^*)) + (f_k(x^*) - f_k(x')))/|f'_k|,$$

$$\max_{k\in K_2}((f_k - f_k(x^*)) + \max_{k\in K_1}(f_k(x^*) - f_k(x')))/|f'_k|]+$$

$$+\beta\,[\sum_{k\in K_1}((\bar{f}_k - f_k(x^*)) + (f_k(x^*) - f_k(x'))) +$$

$$\sum_{k\in K_2}((f_k - f_k(x^*)) + (f_k(x^*) - f_k(x')))] <$$

$$< \max[\max_{k\in K_1}(\bar{f}_k - f_k(x^*))/|f'_k|,\ \max_{k\in K_2}(f_k - f_k(x^*))/|f'_k|]+$$

$$+\beta\,[\sum_{k\in K_1}(\bar{f}_k - f_k(x^*)) + \sum_{k\in K_2}(f_k - f_k(x^*))] = T(x^*).$$

It follows from (39) that $T(x') < T(x^*)$, which contradicts to (37). Hence x^* is an efficient solution of the multiobjective linear integer problem (I).

Consequence. Theorem 4 is valid for arbitrary values of f_k, $k\in K$.

The proof of the consequence is easy, since the proof of Theorem 4 does not take into account the values of the criteria f_k, $k\in K$, in the last solution obtained (the current preferred solution). In other words $f=(f_1, \ldots, f_p)^T$ can be a feasible or unfeasible solution of problem (I) in the criteria space; a nondominated, a weak nondominated or even a dominated solution in the criteria space.

Theorem 5. The optimal solution of the scalarizing problem (B2) is an efficient solution of the multiobjective linear problem (P).

The proof of Theorem 5 is analogous to the proof of Theorem 4, where it is not taken into account that x^* is an integer or continuous solution.

Problem (B1) is equivalent to the following standard problem of mixed linear integer programming (we shall denote it by (B1′)):

(40) $$\min\ (\alpha + \beta \sum_{k\in K} y_k)$$

subject to

(41) $$\bar{f}_k - f_k(x) = y_k,\ k\in K_1,$$

(42) $$f_k - f_k(x) = y_k,\ k\in K_2,$$

(43) $$(\bar{f}_k - f_k(x))/|f'_k| \le \alpha,\ k\in K_1,$$

(44) $$(f_k - f_k(x))/|f'_k| \le \alpha,\ k\in K_2,$$

(45) $$f_k(x) \ge f_k,\ k\in K_3,$$

(46) $$x\in X_1,$$

(47) $$\alpha,\ y_k,\ k\in K - \text{arbitrary}.$$

The scalarizing problem (B1′) has the same properties as problem (A1′), but it contains many more constraints and variables. That is why it is more difficult to solve. When the initial problem (I) is a problem of larger dimension, it is more appropriate to use problem (A1′) than problem (B1′).

The scalarizing problem (B2) is equivalent to the following standard problem of linear programming (we shall denote it by (B2′)):

(48) $$\min\ (\alpha + \beta \sum_{k\in K} y_k)$$

subject to

(49) $$\bar{f}_k - f_k(x) = y_k,\ k\in K_1,$$

(50) $$\underline{f}_k - f_k(x) = y_k,\ k\in K_2,$$

(51) $$(\bar{f}_k - f_k(x))/|f'_k| \le \alpha,\ k\in K_1,$$

(52) $$(f_k - f_k(x))/|f'_k| \le \alpha,\ k\in K_2,$$

(53) $$f_k(x) \ge f_k,\ k\in K_3,$$

(54) $$x\in X_2,$$

(55) $$\alpha;\ y_k,\ k\in K - \text{arbitrary}.$$

The scalarizing problem (B2′) has identical properties as problem (A2′), but it contains more variables and constraints. Problem (B2′) helps the finding of efficient solutions, while problem (A2′) – (weak) efficient solutions. The two problems are easily solved and hence it is more appropriate to use problem (B2′) instead of problem (A2′).

Let us assume that we have found a nondominated solution of problem (P) with the help of problem (B2′) and wish to find a nondominated solution of problem (I), close to the nondominated solution of problem (P). If we denote the nondominated solution of problem (P) by $\tilde{f} = (\tilde{f}_1, \ldots, \tilde{f}_p)^T$ the finding of a nondominated solution of problem (I) can be realized solving the following problem of mixed integer programming (we denote it by (B3′), corresponding to problem (A3′):

(56) $$\min\ (\alpha + \beta \sum_{k\in K} y_k)$$

subject to:

(57) $$\tilde{f}_k - f_k(x) = y_k,\ k\in K,$$

(58) $$(\tilde{f}_k - f_k(x))/|\tilde{f}'_k| \le \alpha,\ k\in K,$$

(59) $$x\in X_1,$$

(60) $$\alpha;\ y_k,\ k\in K - \text{arbitrary},$$

where

$$\tilde{f}'_k = \begin{cases} \tilde{f}_k, & \text{if } f_k \ne 0, \\ 1, & \text{if } k = 0. \end{cases}$$

Problem (B3') contains more variables and constraints compared to problem (A3′). From a computing viewpoint problem (A3′) is more appropriate for application, though it gives (weak) nondominated solutions. This is particularly true for problems of large dimension.

4. Conclusion

Several scalarizing problems have been formulated in the work presented which lead to obtaining of (weak) nondominated solutions of the multiobjective continuous and multiobjective integer problems. As a base of these scalarizing problems the values of the criteria in the last solutions obtained are used, as well as the desired improvements of some criteria by the DM also.

These scalarizing problems enable the design of user-friendly interactive algorithms for effcicient solution of multiobjective integer problems.

5. References

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Скаляризирующие задачи многокритериального линейного целочисленного программирования

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(Р е з ю м е)

Представленны и анализированы несколько скаляризирующих задач, которые применяются в интерактивных алгоритмах для решении многокритериальных линейных целочисленных задач. Свойства скаляризирующих задач позволяют уменьшение вычислительных трудностей, связанных с их решением в интерактивном режиме, а также и улучшение диалога с лицом, принимающим решение. Приведены обосновки, утверждения и доказательства.