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Methods for Finding Non-Dominated Solutions of the Problem of a Multiobjective Flow in a Network*

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1. Introduction

The problem of a multiobjective flow in a network (MNF) is formulated as follows:

Let a network $G=\{N, A, f\}$ be given, where $N=\{1, 2, ..., n\}$ is the set of its nodes, the node $l=s$ being a source and the node $n=t$ – a sink of the flow f and $A=\{(i, j): i, j \in N\}$ is a set of m arcs. The function f defined on the set A is called a flow in G and $f=\{f_{ij}: (i, j) \in A\}$. Each of the arcs $(i, j) \in A$ is assigned k in number parameters a_{ij}^{1}, $l \in I_k$, called "prices". A flow is searched for under these conditions satisfying the conditions (1)–(4).

(1) MNF:
$$\min A_1(f) = \sum_{(i,j)\in A} a_{ij}^{1} f_{ij};$$
$$.........;$$
$$\min A_k(f) = \sum_{(i,j)\in A} a_{ij}^{k} f_{ij};$$

(2)
$$\max v$$

under the constraints (3) and (4) defining the set F of the feasible solutions

(3)
$$\sum_{j\in N} f_{ij} - \sum_{j\in N} f_{ji} = \begin{cases} v, \text{ if } i=s, \\ 0, \text{ if } i\neq s, t, \\ -v, \text{ if } i=t. \end{cases}$$

(4)
$$0 \leq f_{ij} \leq c_{ij}, \ (i, j) \in A.$$

The parameter c_{ij} is the upper bound of the flow value along the arc $(i, j) \in A$.

The objective functions (1) are mutually excluding in the general case, which means that there does not exist a flow which minimizes all the functions simultaneously, such a case is trivial and it is not further discussed.

Usually the approaches of multiobjective programming require solving of a sequence (or of one) of the single objective problems with additional criteria. The matrix of constraints however is of specific structure, as it is for the flows in networks, then this structure is violated and the direct application of the efficient flow methods

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is impossible.

The paper describes three adaptations of simplex methods realizing the separate support and alteration of the bases of the flow and additional constraints and thus allowing the use of the advantages of the structure of the flow constraints matrix.

The assumption that the rank of the matrix $|a_{ij}^1|$ of the objective functions is of dimension $k\times m$ is k, where $k\le m-n+3$ is common for the algorithms described.

2.1. Let the scalarizing problem be defined as follows:

$$\text{S1MNF: } \max \sum_{c\in I_k} \varepsilon_l$$

under the constraints

$$A_i(f) + \varepsilon_i = A_i(f^*),$$
$$\varepsilon_i \ge 0,$$
$$f\in F,\ f^*\in F.$$

It is appropriate to apply in this case the adaptation of the decomposition method of Dantzig-Wolf, which consists of the following:

F is represented as a convex combination of q in number extremal points $f^1, f^2, ..., f^q$. They can be defined solving the problem:

$$\min \sum_{(i,j)\in A} a_{ij}^1 f_{ij};\ \text{at } f\in F.$$

Then $f=\sum_{i\in I_q} \lambda^i f^i; \sum \lambda^i = 1.$

The main program of the problem is written in the form:

$$\text{GMNF: } \max \sum_{i\in I_k} \varepsilon_i$$

under the constraints:

(5) $$\sum_{l\in I_q} \Big(\sum_{(i,j)\in A} a_{ij}^p f_{ij}^l\Big)\lambda^l + \varepsilon_p = A_p(f^*),\ p\in I_k.$$

(6) $$\sum \lambda^l = 1.$$

GMNF is a linear programming problem with k rows and a rank, equal to k also. The number of the problem columns is equal to the number of the extremal points of the set F which could be very large. The modified simplex method is applied to solve it. The estimation of each out-of-basis column of the problem is done solving the optimization single criterion problem for minimal flow in the network:

(7) $$a^* = \min \sum_{(i,j)\in A} \Big(\sum_{l\in I_q} -\pi_l a_{ij}^l\Big) f_{ij},$$
$$f\in F,$$

where π_l, $l\in I_q$, are Lagrange multipliers, corresponding to the rows (5).

Let σ be a Lagrange multiplier corresponding to the row (6). If it is satisfied that $a^* < \sigma$ then replacement is accomplished in the basis and the iteration described is repeated, otherwise th nondominated solution which is searched for, is found.

2.2. In case no maximality of the flow is required, i.e., the criterion (2) is not included, is is appropriate to solve scalarizing problems built on the basis of the global criterion method.

Let S2MNF be a problem for minimization of the distance upto any reference point:

$$\text{S2MNF: } \min \alpha$$

under the constraints

$$A_i(f) - \alpha/\varpi_i \leq A_i^*, \ i \in I_k;$$
$$f \in F,$$
$$\sum \varpi_i = 1, \ \varpi_i > 0, \ \alpha \geq 0.$$

The numbers ϖ_i are weighting coefficients reflecting the priority of the criteria $A_i(f)$.

The additional arcs (s, t) and (t, s) are constructed in the network G, for which $c_{st}=c_{ts}=M$, M is a sufficiently large number. The network obtained is denoted by $G^*=\{N, A^*, f\}$ where $|A^*| = m+2$. The problem S2MNF takes the form:

$$\text{S2MNF: } \min f_{st}$$

under the constraints

$$\sum_{j \in N} f_{ij} - \sum_{j \in N} f_{ji} = 0,$$
$$f_{st} - f_{ts} = 0,$$
$$0 \leq f_{ij} \leq c_{ij}, \ (i, j) \in A,$$
$$\sum_{(i,j) \in A} a_{ij}^1 f_{ij} - f_{st} \leq A^*_l, \ l \in I_k.$$

It is proved in [3] that the basis solution of such a problem corresponds to the covering tree of the graph $\{N, A^*\}$, the number of the arcs being $n-1$, plus k arcs, which form together with the arcs of this tree independent cycles σ_p, $p \in I_k$. The cycles are independent in this sense that each one of them contains at least one arc, not belonging to the remaining cycles. The value

$$\sigma_p(A_l) = \sum_{(i,j) \in \sigma_p} \varepsilon_{ij} a_{ij}^1,$$

is defined for each cycle, where $\varepsilon_{ij}=1$ or -1 depending on this whether the arc (i, j) is direct or inverse with respect to an apriori selected direction of movement along the cycle.

It is proved that the rank of the matrix $|\sigma_p(A_l)|$, $l \in I_k$, $p \in I_{m-n+3}$ is k.

The set of constraints of problem S2MNF is assigned the set of constraints

(8)
$$\sigma_l(A_l) f^{\sigma}_l + \sigma_2(A_l) f^{\sigma}_2 + \ldots + \sigma_q(A_l) f^{\sigma}_q \leq A^*_l,$$
$$\cdots\cdots\cdots$$
$$\sigma_l(A_k) f^{\sigma}_l + \sigma_2(A_k) f^{\sigma}_2 + \ldots + \sigma_q(A_k) f^{\sigma}_q \leq A^*_k.$$

where f^{σ}_l is a flow along the corresponding cycle σ_i.

For each basis solution of the problem the arcs (s, t) and (t, s) are included into the basis and the optimality conditions are checked by the system of constraints (7), and if they are not satisfied a neighbouring basis solution is defined.

2.3. The scalarizing flow problem S3MNF defined by the method of ε-constraints, looks like:

$$\text{S3MNF: } \min A_p(f)$$

under the constraints

$$A_i(f) \leq \varepsilon_i, \ i \in I_k, \ i \neq p;$$
$$f \in F.$$

The rank of the constraints matrix (9) is $k-1$. Not limiting the considerations area, $p = k$ is assumed.

The problem for an optimal flow under additional linear constraints of the flow along the network arcs is investigated in [4]. It is proved that its basis solution corresponds to a covering tree plus $k-1$ arcs more, forming with the tree arcs $k-1$ oriented paths and cycles, denoted below as μ_p, $p \in I_{k-1}$. The estimation of each out-of-basis arc (r, q) is realized, computing the price of the path or cycle, which it forms together with the basis tree arcs. In case this price is denoted by $A(\mu_{rq})$ it is found with the help of the formula

$$A(\mu_{rq}) = \sum_{l \in I_{k-1}} \lambda_l A^l(\mu_{rq})$$

where

$$A^l(\mu_{rq}) = \sum_{(i,j)\in\mu_{rq}} a_{ij}^{\,l},$$

and λ_l, $l \in I_{k-1}$, are Lagrange resolving multipliers for the respective basis solution of the problem

$$\min A_l(f)$$

under the constraints

$$(10)\qquad \begin{array}{l} A^l(\mu_l)\, f_l^{\mu} + A^l(\mu_2)\, f_2^{\mu} + \ldots + A^l(\mu_{k-l})\, f_{k-l}^{\mu} \le \varepsilon_l \\ \ldots\ldots\ldots\ldots \\ A^k(\mu_l)\, f_l^{\mu} + A^k(\mu_2)\, f_2^{\mu} + \ldots + A^k(\mu_{k-l})\, f_{k-l}^{\mu} \le \varepsilon_k . \end{array}$$

3. Conclusion

The first of the methods described finds a non-dominated solution of the problem set as a linear convex combination of solutions of single objective flow problems. In the third method increase of the flow is applied along paths formed by the arcs corresponding to the basis solution and it can be categorized as an analog of the method of the minimal paths in the single objective flow problem. The second one of the methods proposed changes the flow along the cycles, created by the basis solution arcs and has also its analog in the single objective case.

References

1. Miettinen, K. On the Methodology of Multiobjective Optimization with Applications. Javaskula, 1994.
2. Papadimitriu, H., K. Stiglitz. Combinatornaja optimizatzia. Moskva, Mir, 1985 (in Russian).
3. Gabassov, P., F. Kirilova. Metodi lineinovo programmirovania. Minsk, 1980 (in Russian).
4. Djelatova, M. PhD Thesis, 1991.
5. Jelatova, M. Properties of the efficient solutions of the multicriteria network flow problem. – Problems of Eng. Cybernetics and Robotics, 1998, **47**, 104-110.

Методы определения недетерминированных решений для мультикритериальной сетевой потоковой задачи

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(Резюме)

Исследуется задача мультикритериального сетевого потока. Представлены три метода определения недоминированных решений задачи. Первый из них соответствует декомпозиционному методу Данцига–Вольфа. Второй и третий являются спецификациями первичного симплекс–метода и соответствуют потоковым методам негативных циклов и минимальной пути.