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Determination of the Workspace of a Complex Manipulation System

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1. Introduction

Most of the industrial robots have open kinematic chains and they are based on serial connections of links. These manipulators have large workspace and good dexterity, but their rigidity is poor. Parallel manipulators can be considered as an alternative of the serial manipulators. Parallel manipulators have some advantages such as higher stiffness and greater payload-to-selfweight ratio. Their major disadvantage is the limited workspace volume. The third group of manipulation systems is the group of hybrid type manipulators. Hybrid manipulation systems combine the advantages of both open and closed chain mechanisms., i.e., on one hand they have greater workspace and on the other hand good stiffness, high load-carrying capacity and higher accuracy.

Workspace of manipulators has been studied by many researchers. The methods for determination of the workspace can be divided in two groups: analytical and numerical methods. The analytical methods are very complex while the numerical ones are relatively simple. Analytical methods give closed form descriptions of the workspace boundary but they can apply only to a certain specific manipulator. There are investigations of the workspace of serial as well as of parallel manipulators [1-6]. However, the workspace of different kinds of hybrid manipulators is still to be determined. Presentation of the workspace of a manipulator not only gives a clear idea of the geometric characteristic of the manipulator but can be used for evaluation of different performance characteristics. The knowledge of the workspace helps to determine the possible applications of the manipulator.

This paper presents workspace investigation of a hybrid type robot manipulator.

2 Workspace of a hybrid type robot manipulator

The hybrid type manipulator under consideration consists of two variable-geometry modules, which are serially connected. (Fig.1). A lot of hybrid manipulation systems can be obtained by combining such modules (see [8]). We will consider one of the possible structures here (Fig.1). The first module has two actuated links, i.e., they are with variable lengths, while the second module has one actuated link. In addition to these modules there are two revolute joints situated at the input and at the output of the manipulator. This manipulation system has five degrees of freedom.

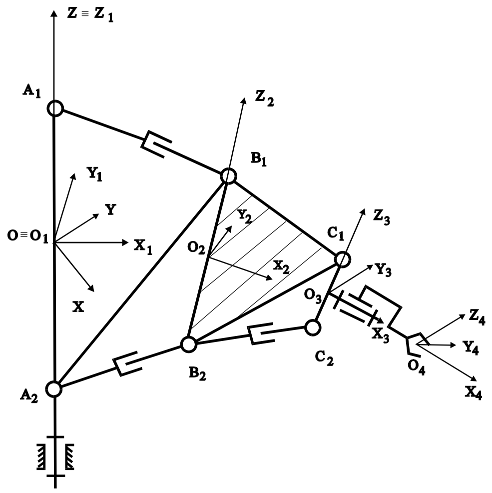

Fig.1. The hybrid type manipulation system

For the determination of the workspace of this manipulator we have used the forward and inverse kinematics. That is why the closed form solution of the forward and inverse kinematic problems are given below (for more details see [7]).

2.1. Forward and inverse kinematics

For the forward position problem we can write:

(1) $$\boldsymbol{r} = {}^0\boldsymbol{A}_1\,{}^1\boldsymbol{A}_1\,{}^2\boldsymbol{A}_1\,{}^3\boldsymbol{A}_1\,{}^4\,r = \boldsymbol{A}\,{}^4\boldsymbol{r},$$

where $${}^{i-1}\boldsymbol{A}_i = \begin{bmatrix} {}^{i-1}\boldsymbol{R}_i & {}^{i-1}\boldsymbol{r}_i \\ 0 & 1 \end{bmatrix};$$

${}^{i-1}\boldsymbol{R}_i$ is a 3x3 rotation matrix representing the orientation of the i-th coordinate system with respect to the (i-1)th coordinate system; ${}^{i-1}\boldsymbol{r}_i$ is a 3x1 matrix denoting the position vector of point O_i with respect to the (i-1)th co-ordinate system; **0** is a 1x3 zero matrix; ${}^4\boldsymbol{r}$ is a 4x1 position vector written in the $O_4X_4Y_4Z_4$ coordinate system; the left leading index denotes the coordinate system with respect to which the vectors and matrices are written. Let ${}^1\boldsymbol{a}_i$ and ${}^1\boldsymbol{b}_i$ (i=1,2,3) be vectors relative to the $O_1X_1Y_1Z_1$ coordinate frame, i.e.,

(2) $${}^1\boldsymbol{a}_i = {}^0\boldsymbol{O}_1{}^1\boldsymbol{A}_1 = (0,\ 0,\ \rho)^T;$$

(3) $${}^1\boldsymbol{a}_2 = {}^1\boldsymbol{a}_3 = {}^1\boldsymbol{O}_1{}^1\boldsymbol{A}_2 = (0,\ 0,\ -\rho)^T;$$

(4) $$^1\boldsymbol{b}_1 = {}^1\boldsymbol{b}_2 = {}^1\boldsymbol{O}_1{}^1\boldsymbol{B}_1 = (v_1,\ 0,\ u_1)^T;$$

(5) $$^1\boldsymbol{b}_3 = {}^1\boldsymbol{O}_1{}^1\boldsymbol{B}_2 = (v_1,\ 0,\ u_2)^T.$$

We can write similar vectors for the second module referred to the $O_2X_2Y_2Z_2$ coordinate frame, i.e.,

(6) $$^1\boldsymbol{c}_1 = {}^2\boldsymbol{O}_2{}^1\boldsymbol{C}_1 = (v_3,\ 0,\ u_3)^T;$$

(7) $$^2\boldsymbol{c}_2 = {}^2\boldsymbol{O}_2{}^1\boldsymbol{C}_2 = (v_4,\ 0,\ u_4)^T.$$

The rotation matrices $^1\boldsymbol{R}_2$ and $^2\boldsymbol{R}_3$, and the position vectors $^1\boldsymbol{r}_2$, $^2\boldsymbol{r}_3$ can be written as follows:

(8) $$^1\boldsymbol{R}_2 = 1/b \begin{bmatrix} (u_1 - u_2) & 0 & (v_1 - v_2) \\ 0 & b & 0 \\ (v_2 - v_1) & 0 & (u_1 - u_2) \end{bmatrix},$$

(9) $$^2\boldsymbol{R}_3 = 1/c \begin{bmatrix} (u_3 - u_4) & 0 & (v_3 - v_4) \\ 0 & c & 0 \\ (v_4 - v_3) & 0 & (u_3 - u_4) \end{bmatrix},$$

(10) $$^1\boldsymbol{r}_2 = \big((v_1 + v_2)/2,\ 0,\ (u_1 + u_2)/2 \big),$$

(11) $$^2\boldsymbol{r}_3 = \big((v_3 + v_4)/2,\ 0,\ (u_3 + u_4)/2 \big).$$

The other two rotation matrices are:]

(12) $$\boldsymbol{R}_2 = \begin{bmatrix} c_1 & -s_1 & 0 \\ c_1 & -s_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \boldsymbol{R}_2 = \begin{bmatrix} c_1 & -s_1 & 0 \\ c_1 & -s_1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where s_i and c_i (i=1,4) denote $\sin\theta_i$ and $\cos\theta_i$, respectively.

The inverse position problem is to determine the actuated joint variables (angles θ_1 and θ_2, and leg lengths L_1, L_2, L_4) for the given position and orientation of the end-effector of the manipulator. Let the orientation of the end-effector relative to the base be defined by a rotation matrix, $\boldsymbol{R}$, i.e.,

,
(13) $$\boldsymbol{R} = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}.$$

Referring to Fig.1 we can easily obtain the angles θ_1 and θ_4, i.e.,

(14) $$\theta_1 = \mathrm{arctg2}(m_1,\ l_1),$$

(15) $$\theta_4 = \mathrm{arctg2}(n_2,\ n_3).$$

Then for the rotation matrix representing the orientation of the $O_3X_3Y_3Z_3$ coordinate system with respect to the fixed coordinate system can be written:

(16) $$^0\boldsymbol{R}_3 = \boldsymbol{R}\,{}^3\boldsymbol{R}^{-1}_4,$$

and respectively:

(17) $$^0\boldsymbol{A}_3 = \begin{bmatrix} ^0\boldsymbol{R}_3 & ^0\boldsymbol{r}_{OO_3} \\ 0 & 1 \end{bmatrix},$$

where $\boldsymbol{r}_{OO_3} = \boldsymbol{r}_{OO_4} - {}^0\boldsymbol{R}_3\ \boldsymbol{r}_{O_3O_4}$ is a position vector written in $O_4X_4Y_4Z_4$ coordinate system.

Therefore for the Cartesian coordinates of points C_i ($i = 1, 2$) relative to the base coordinate system and $O_1X_1Y_1Z_1$ coordinate system, respectively, can be written as follows:

(18) $$^0\boldsymbol{OC}_i = {}^0\boldsymbol{A}_3 \, . \, {}^3\boldsymbol{OC}_i ,$$

(19) $$^1\boldsymbol{OC}_i = {}^0\boldsymbol{R}_1^{-1} \, . \, {}^0\boldsymbol{OC}_i ,$$

where $^1\boldsymbol{OC}_i = (x_{Ci}, 0, z_{Ci})^T$.

Now using vectors $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ given by equations (2)-(5) we obtain the following:

(20) $$v_1^2 + \rho^2 + 2\rho u_1 + u_1^2 = L_3^2 .$$

From the following vector equation

(21) $$^1\boldsymbol{OC}_1 - {}^1\boldsymbol{OB}_1 = {}^1B_1C_1$$

we can obtain:

(22) $$(x_{C1} - v_1)^2 + (x_{C1} - u_1)^2 = L_5^2 ,$$

where $L_5^2 = \| B_1 C_1 \|$.

Solving equations (20) and (22) together leads to the following:

(23) $$u_1 = \frac{(PQ - \rho) \pm \sqrt{(\rho - PQ)^2 - (Q^2+1)(P^2 + \rho^2 - L_3^2)}}{Q^2+1},$$

(24) $$v_1 = P - Q\left(\frac{(PQ - \rho) \pm \sqrt{(\rho - PQ)^2 - (Q^2+1)(P^2 + \rho^2 - L_3^2)}}{Q^2+1}\right),$$

where

$$P = \frac{L_3^2 - L_5^2 + x^2_{C1} + x^2_{C1} - \rho^2}{2\,x_{C1}}; \qquad Q = \frac{\rho + z_{C1}}{x_{C1}}.$$

Then the variable length L_1 is given by the following equation:

(25) $$L_1 = \sqrt{v_1^2 + (\rho - u_1)^2}.$$

Let $^1\boldsymbol{e}_5 = (^1\boldsymbol{e}_{5x}, 0, {}^1\boldsymbol{e}_{5z})^T$ and $^2\boldsymbol{e}_5 = (^2\boldsymbol{e}_{5x}, 0, {}^2\boldsymbol{e}_{5z})^T$ be unit vectors along the line $\mathbf{B}_1\mathbf{C}_1$ written in $O_1X_1Y_1Z_1$ and $O_2X_2Y_2Z_2$ coordinate systems, respectively. The vector $^2\boldsymbol{e}_5$ is constant and depends only on the design of the manipulator, while the components of the vector $^1\boldsymbol{e}_5$ can by obtained using the following vector equations:

(26) $$\mathbf{BC} = \mathbf{O}_1\mathbf{C}_1 - \mathbf{O}_1\mathbf{B}_1,$$

which leads to the following:

(27) $$^1\boldsymbol{e}_{5x} = \frac{x_{C1} - v_1}{||\mathbf{B}_1\mathbf{C}_1||},$$

(28) $$^1\boldsymbol{e}_{5z} = \frac{z_{C1} - u_1}{||\mathbf{B}_1\mathbf{C}_1||}.$$

Obviously, the dot product of these unit vectors represents the rotation of the $O_2X_2Y_2Z_2$ coordinate system with respect to the $O_1X_1Y_1Z_1$ coordinate frame, i.e.,

(29) $$^2\boldsymbol{e}_5 . {}^1\boldsymbol{e}_5 = \cos\theta_2.$$

Then keeping in mind the components of the rotation matrix given by equation (8), equation (29) leads to:

(30) $$u_2 = u_1 - b({}^2\mathbf{e}_5 . {}^1\mathbf{e}_5).$$

Now, in order to find v_2 we will write the following equation:

(31) $$\|{}^1\mathbf{OB}_2 - {}^1\mathbf{OC}_1\| = L_6,$$

where

$L_6 = \|\mathbf{C}_1\mathbf{B}_2\|$.

Equations (31) lead to the following:

(32) $$v_2 = \frac{L_6^2 - b^2 - x_{C_1}^2 - z_{C_1}^2 + 2(z_{C_1} - u_1)u_2 + u_1^2 + v_1^2}{2(v_1 - x_{C_1})}.$$

Then the variable length L_2 is given by:

(33) $$L_2 = \sqrt{v_2^2 + (r + u_2)^2}.$$

The unknown rotation matrix can be obtained by the following equation;

(34) $${}^2\mathbf{R}_3 = {}^1\mathbf{R}_2 \cdot {}^0\mathbf{R}_1 \cdot \mathbf{R} \cdot {}^3\mathbf{R}_4^{-1}.$$

Let $${}^2\mathbf{R}_3 = \begin{bmatrix} r_{11} & 0 & r_{13} \\ 0 & 1 & 0 \\ r_{31} & 0 & r_{33} \end{bmatrix}$$

denotes the components of the rotation matrix ${}^2\mathbf{R}_3$ obtained by equation (34). Then equations (9) and (34) lead to the following:

(35) $$u_4 = u_4 - cr_{11},$$

(36) $$v_4 = v_4 + cr_{31},$$

where $c = \|\mathbf{C}_1\mathbf{C}_2\|$.

Then for the variable length $L_4 = \|\mathbf{B}_2\mathbf{C}_2\|$ can be written:

(37) $$L_4 = \sqrt{\left(u_4 + b/2\right)^2 + v_4^2}.$$

Equations (14), (15), (25), (33) and (37) give the solution of the inverse position problem for the considered manipulation system.

2.2. Determination and representation of the workspace

Two workspaces of manipulators can be defined:

- ♦ Reachable workspace: this is the volume within which every point can be reached by the manipulator end-effector.
- ♦ Dextrous workspace: this is the volume within which every point can be reached by the manipulator end-effector with any desired orientation.

In addition to these two definitions given by Kumar and Waldron [5] another workspace can be defined:

- ♦ Workspace with constant orientation - this is a volume which consists of all the points which can be reached by the end-effector with constant orientation.

Obviously the union of all workspaces with constant orientations will give the reach-

able workspace, while their intersection will determine the dextrous workspace.
In this section of the paper the reachable and dextrous workspaces of the considered hybrid type manipulation system is presented. The algorithm consists of the following steps:
-Determination of the reachable workspace by using the Monte Carlo method: i) the computer programme uses random sampling for the joint coordinates, ii) computes the Cartesian coordinates of the end-effector using forward position problem and iii) plots the points;
-Scanning the boundaries of different areas with constant orientation by using the inverse kinematics;
-Obtaining the dextrous workspace for a given range of orientations by applying the intersection of the workspaces with constant orientations.
Using the above-mentioned algorithm a radial slice of the reachable workspace for the considered hybrid manipulator is presented in Fig.2.

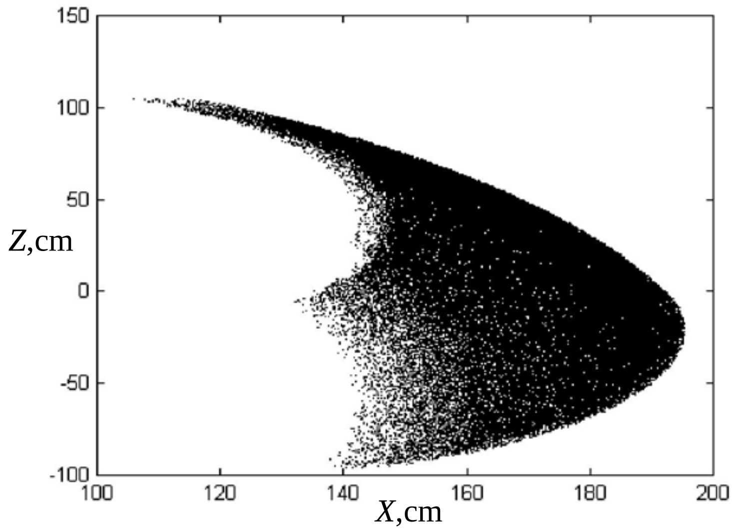

Fig.2. The reachable workspace

The other radial slices are identical in shape and dimensions for the whole workspace and the union of all radial slices will produce the 3-dimensional reachable workspace. The shown workspace was obtained using 100 000 sample points.

In Fig.3 are shown two workspaces with constant orientations (the first orientation=10° and the second=-10°, i.e., this is the angle b - rotation about the Y axis, the other two angles of rotations are zero in this case). The intersection of all the workspaces with constant orientations with the range of (10° ë –10°) gives the dextrous workspace. Other two workspaces with constant orientation (the first orientation=30° and the second=-30°) are shown in Fig. 4. It is clear that in this case the dextrous workspace for the orientation range of (30°ë–30°) is only the common line for the two workspaces.

The design parameters of the manipulators are as follows: $a \equiv A_1A_2$=1200 mm, $b \equiv B_1B_2$=600 mm, $c \equiv C_1C_2$=300 mm, $L_3 \equiv A_2B_1$=1200 mm, $L_5 \equiv B_1C_1$=570 mm, $L_6 \equiv B_2C_1$=700 mm, O_3O_4=200 mm. The range of motion of the joints are as follows: $q_1=0^o \div 360^o$, $q_4=0^o \div 360^o$, L_1=700÷1400 mm, L_2=700÷1400 mm, L_4=500÷900 mm.

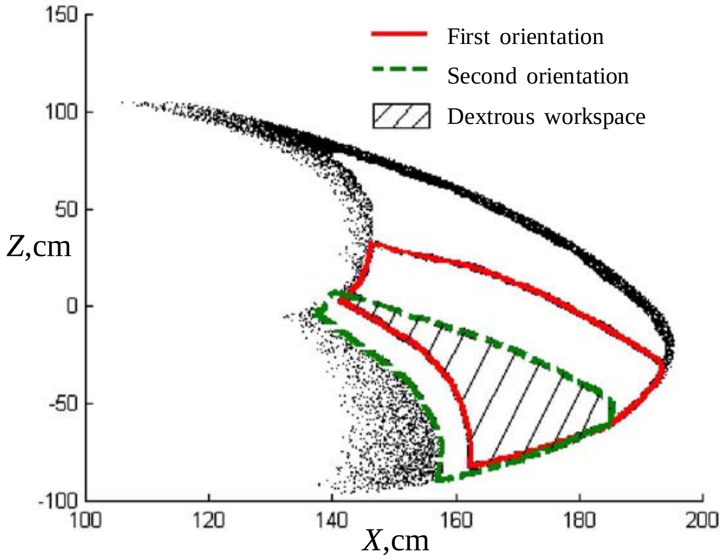

Fig.3. Workspace for the two fixed orientations
(first = 10^o ; second = -10^o)

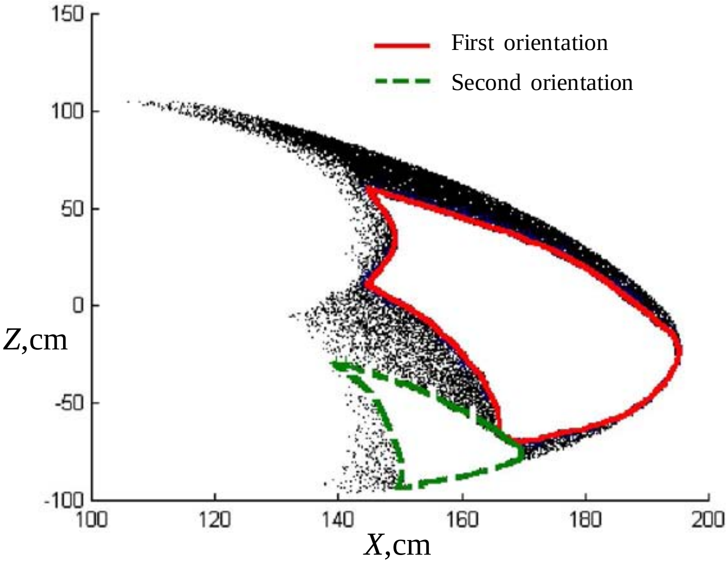

Fig.4. Workspace for the two fixed orientations
(first = 30^o ; second = -30^o)

3. Conclusion

An algorithm for the determination of the workspace of a hybrid type manipulation system is proposed in the paper. This algorithm is based on the obtained closed form solutions of the forward and inverse kinematic problems for the hybrid manipulator. The determined different kinds of workspaces are graphically presented.

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Определение рабочего пространства комплексной манипуляционной системы

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(Р е з ю м е)

Исследуется рабочее пространство манипуляционной сиситемы гибридного типа. Представлен алгоритм для определения этого пространства. Результаты показаны в форме графики. Работа предлагает тоже решение правой и инверсной кинематической задачи.