# An Adaptive Threshold-Gradient Method for Segmentation of Areas and Objects of Grey Scale Images 

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## 1. Introduction

The segmentation of an image, i.e., the separation of the object from its background is one of the most important procedures in image processing.

Two basic types of segmentation exist at present - realized with respect to the intensity and to the intensity gradient, and two basic types of segments - areas and borders, respectively. The term "area" usually denotes toplogically joined regions of the image which have comparatively homogeneous distribution of intensity, while the term "border" relates to zones where the intensity changes sharply, or in other words, zones with greater value of the intensity gradient. Borders may be situatedbetween an object and a background as well as between different regions of the object.

One of these two types of segmentation is usually applied for the purposes of image processing-the intensity or gradient (the latter being famous as segmentation.by form), which, finally leads to partial use of the intensity characteristics of the picture. That is why a new adaptive threshold-gradient method is proposed in the paper. This method treats the image as one indivisible structure containing areas andborders. The analysis of this structure gives as a result the segmentation of the image.

## 2. Methods of intensity segmentation

The simplest method, calledthresholdmethod, consists in associating eachelement of the scene with one of the two groups - the group of the object or the group of the background depending on whether the intensity of the element exceeds a given thresholdvalue or not $[1,2,3]$. Themainproblemin thismethoduse is the correct choice of the threshold of separation. A widespread approach to the problem defines the selection of a threshold value, corresponding to the local minimum of the intensity histogram, which has to be bimodal. Unfortunately the histogram is generally unimodal, multimodal or step-like, and the threshold has to be definedby another method, for
instance the entropy method, which analysis the entropy function of the intensity histogram, its maximum determining the optimum threshold of quantization [7, 8] .

A method with varying thresholds has been describes in [10]. The image is sectioned in small rectangular regions, a histogram is formed for each one of them. In case it isbimodal, the threshold is calculatedby its minimum, otherwise its value is obtained by interpolation of the thresholds from adjacent zones. The chief disadvantages of the thresholdmethods is the obtaining of false regions and the loss of regions aswell.

When using gradient methods of segmentation it is assumed that the shape of the object is fixedby its borders. The set of elements with sharply changing intensity is denoted as "borders" [11]. Different gradient operators for separation of borders are describedin [7] : of Roberts, Sobel, Kirsh, Walsh, Laplace.

The main problem in the gradient methods is the appearance of false contours, their splitting and loss. Their advantage consists in the avoiding of the low-frequency noise in the image, i.e., theuneven illuminance. Humanvision is sensitive to the contrast between the separate intensity areas and automatically ignores the irrgualar illuminance. The gradient methods take into account this feature of human vision and hence they are better than the threshold ones.

## 3. An adaptive threshold-gradient method for segmentation

### 3.1. Brief description of the method

In this method the connections between regions andborders in the image are described by the structural graph $\Gamma=(\mathbf{P}, G)$, where $\mathbf{P}$ is the set of graph nodes, (the image areas), andG is the set of graph arcs (borders between the areas). $\Gamma$ is an orientednon-plain graph, i.e., its arcshave directionandthegraph configuration is spatial, not plain-like.

The image is considered a tridimensional surface $F(x, y, B)=0$, from which the areas and objects are separated by a section with $n$ tridimensional cutting surfaces $A_{i}(x, y, z)=0, i=1,2, \ldots, n$.

The notions "potential" and "gradient" markers (PM and (GM) are introduced that are in fact points from the tridimensional space, which serve for the construction by approximation of the cutting surfaces. The latter can be regarded as adaptive thresholdsurfaces.

### 3.2. Definition of an optimal gradient threshold

The obtaining of a gradient vector $\vec{G}(x, y)$ in a point $(x, y)$ is done by non-linear bidimensional discretedifferentiationusingKirshoperator:
(1)

| $a 0$ | $a 1$ | $a 2$ |
| :--- | :--- | :--- |
| $a 7$ | $x y$ | $a 3$ |
| $a 6$ | $a 5$ | $a 4$ |

$$
\begin{aligned}
& \vec{G}(x, y)=\max \left\{I, \max \left[5 S_{k}-3 T_{k}\right]\right\}, \\
& \text { where } k=0, \ldots, 7 ; \\
& S_{k}=a_{k}+a_{k+1}+a_{k+2} ; \\
& T_{k}=a_{k+3}+a_{k+4}+a_{k+5}+a_{k+6}+a_{k+7},
\end{aligned}
$$

calculating the indices kby a module of 8 . The direction of the vector $\vec{G}(x, y)$ is indexed according to the scheme given below:


All the areas, which are assumedas "gradient", i.e., where $\Theta_{G}$ is the differential threshold, must be separated from the gradient field, i.e., $\overrightarrow{\mid G}(x, y) \mid \geq \Theta_{G}$, at that $\Theta_{G}$ is a differential thresholdandserves to separate the low contrasting intensitytransitions and the homogeneous areas of high contrasting intensity transitions. An heuristic approach is proposed for the automatic determination of $\Theta_{G}$, based on the analysis of the smoothed histogram $h_{1}(|\vec{G}|)$ of the gradient $|\vec{G}|$, obtained from the histogram $h(|\vec{G}|)$ according to the formula:

$$
h(i)=(h(i-1)+h(i)+h(i+1)) / 3, i=1,2, \ldots,|\vec{G}|_{\max } .
$$

$\Theta_{G}$ is selected in such a way that the first derivative $h_{1}$ ' of $h_{1}$ has maximal value in thepoint $\Theta_{G}$.

The image is divided in two zones on the basis of the threshold $\Theta_{G}$ :
a) Gradient zone: the set Gof pixels pix $(x, y)$ for which $|\vec{G}(x, y)| \geq \Theta_{G}$,
b) Potential zone : the set $P$ of pixels pix $(x, y)$ for which $|\vec{G}(x, y)|<\Theta_{G}$.

An example division into zones for an one-dimensional case is shown inFig. 1 along the axis $x$, where bdenotes the function of pixels intensity. The notion "potential" zone is intuitively conceivedafter replacement of the intensity bby the thirddimension $z$ (height), i.e., the scene is consideredas a tridimensioanal surface $F(x, y, z)=0$.

3. 3. A concept for adaptive segmentation of an image using sectioning of its surface $F$ ( $x$, $y, z)=0$ by an adaptive cutting surface $A(x, y, z)=0$

Theexisting adaptivemethods for threshold segmentation focus almost exclusively on a local area $m \times n$ and the adaptive threshold $\Theta_{G}$ is calculated for this area.

In the method described an adaptive surface $A(x, y, z)=0$ is built approximating apriori selectedpoints (Fig. 2), the selectionbeing donenot by the investigation of local areas of the image, that very often causes errors, but analyzing the complete gradientpotential structure of the scene, which gives an entire idea about the character of the connections between the potential areas $P_{1}$ and the gradients $G_{i}$. Each area $P_{1}$, $i=1,2, \ldots, N_{i}$ and $G_{i}, j=1,2, \ldots, N_{G}$, is anelement of the sets $P$ and $G$ respectively, $P_{1} \in \mathbf{P}$, and $G_{j} \in \mathbf{G}$, where $N_{i}$ and $N_{G}$ denote the number of the potential and gradient fields in the image. The sets $\boldsymbol{P}$ and $\boldsymbol{G}$ are sets, obtained from $\boldsymbol{P}^{\prime}$ and $\boldsymbol{G}^{\prime}$ with the help of the surrection operation

The physical interpretation of this surrection transformation of the sets $P_{1}^{\prime}$ and $G_{j}^{\prime}$ into $P_{1}$ and $G_{j}$ respectively is the replacement of the intensities (gradients) of the
corresponding pixels -elements of $P_{1}{ }^{\prime}$ and $G_{j}$ ' by their averagedvalues for some small local areas $l \times l$. Then the sets $P_{1}$ and $G_{j}$ comprise these average values, the last being named potential and gradient markers:

$$
\begin{aligned}
& P_{1}=\left\{p_{1}, p_{2}, \ldots, p_{n i}\right\}, \quad p_{1}, p_{2}, \ldots, p_{n i}-\text { potential markers (PM) ; } \\
& \mathrm{G}_{j}=\left\{g_{1}, g_{2}, \ldots, g_{n i}\right\}, g_{1}, g_{2}, \ldots, g_{n i}-\text { gradient markers (GM) },
\end{aligned}
$$

where $n i$ is the number of PM for the $i$-th potential area, $n j$ is the number of GM for the $j$-thgradient area.

The selection of the local areas $l \times 1$ in order to get the average values, depends on the character of the image, i.e., whether it consists of smaller or larger details, which has to be known apriori. For the examples fromFig. 12 and Fig. 13, $1=3$ is chosen.

———imageplain

-     -         - cuttingplain $A(x, y, z)=0$
-     - potential markers (PM)
$\times$-gradient markers (GM)
b -intensity
Ai -object average intensity
Ao -background average intensity

Fig. 2.
Fig. 3 shows an example, in which the cuttingplain $A(x, y, z)=0$ is constructed in such a way that it passes across the gradient markers $g \in\left(G_{1} \cup G_{2}\right)$ and is at a distance $+h$ from the potential markers (PM) $p \in P_{2}$ and at a distance -h from the PM $p \in\left(P_{1} \cup P_{3}\right)$.

The orientednon-plain graph $\Gamma=(P, G)$ is withnodes $P_{i}$ and $\operatorname{arcs} G_{i}$, the orientation of the arcs being assumed conditionally fromblack towards white level (down-up) Fig. 4.

The average potential $P_{i}$ (node Pi respectively) is computed for every potential area $B_{i}$ :
(2)

$$
B_{i}=\left(\sum_{k=1}^{n} b_{k}\right) / n,
$$

where $n$ is the number of pixels in the area $P_{i}$ and $b_{k}$ is the intensity (potential) of each one of them.


Fig. 3


Fig. 4
3.4. Selection of the gradient (GM) and potential (PM) markers

## 3. 4. 1. Selection of PM.

The selection of a PM for a potential area $P_{i}$ is done separating the area $P_{i}$ by a grid $m \times n$. For example if $m=3$, the potential (average intensity) bp of the potential marker $p_{k} \in P_{i}$ is computed as the mean arithmetic of the intensities of the neighbouring pixels:
(3)

$$
b p_{k}=\left(\sum_{t=1}^{9} b_{t}\right) / 9,
$$

The average potential $B_{i}$ of the region $P_{i}$ accordingto (2), is:

$$
\begin{equation*}
B_{i}=\left(\sum_{k=1}^{N_{i}} b p_{k}\right) / N_{i}, \tag{4}
\end{equation*}
$$

where $N_{i}$ is the number of PM in the area $P_{i}$.

### 3.4.2. Selection of gradient markers (GM)

The selection of gradient markers is realized defining the points with maximal slope of the intensitytransition from the gradient areas $G_{j}$ according to the following algorithm:

Step 1. The image is scannedby rows, until apixel pix $(x, y) \in G_{j}, j=1,2, \ldots, N_{G}$ is detected.

Step 2. Aprocedure for tracing a route with direction $\vec{g}$ is started, where $\vec{g}$ is a vectorperpendicular to the intensity transition.

Step 3. The tracing of the route from the initial pixel pix $(x, y)$ is realized, continuing in one of the eight possibledirections $\alpha=0,1, \ldots, 7$ ( 0 being North, 1 -NorthEast, 2-East, 3-South-East, 4-South, 5-South-West, 6-West, 7-North-West). If $d_{n}$ denotes the direction of $\vec{g}$ in the pixel pix ( $x, y$ ) (which is initial or next point in the route), and d $d_{b}$-the direction searched for, in which the tracing has to continue, then $d_{b}$ is found according to the rule:

$$
\begin{aligned}
& \mid\left(d_{a}-1\right)_{\bmod 8}, \text { if }\left|\vec{g}_{a-1}\right|=\max \left(\left|\vec{g}_{a-1}\right|,\left|\vec{g}_{a}\right|,\left|\vec{g}_{a+1}\right|\right) \\
& d_{b}=\left|d_{a}, i f\right| \vec{g}_{a} \mid=\max \left(\left|\vec{g}_{a-1}\right|,\left|\vec{g}_{a}\right|,\left|\vec{g}_{a+1}\right|\right) \\
& \quad \mid\left(d_{a}+1\right)_{\bmod 8}, \text { if }\left|\vec{g}_{a v+1}\right|=\max \left(\left|\vec{g}_{a-1}\right|,\left|\vec{g}_{a}\right|,\left|\vec{g}_{a+1}\right|\right)
\end{aligned}
$$

where $\left|\vec{g}_{a-1}\right|,\left|\vec{g}_{a}\right|,\left|\vec{g}_{a+1}\right|$ denote the gradients in three from the eight neighbouring topix $(x, y)$ points, whichare reached, starting frompix $(x, y)$ into one the three directions $\left(d_{a}-1\right)_{\text {modi }}, d_{a},\left(d_{a}+1\right)_{\text {moci }}$ respectively.

Step 4. The route tracing is terminated in case the condition: $\left(\right.$ pix $\left._{\alpha_{-1}} \in P_{k}\right) \cup\left(\right.$ pix $\left._{a} \in P_{m}\right) \cup\left(\right.$ pix $\left._{a+1} \in P_{r}\right)$ is satisfied, where $P_{k^{\prime}}, P_{m^{\prime}}, P_{r}$ are potential areas and pix $_{\alpha-1}$, pix $_{a}$, pix $_{\alpha+1}$ are thepixelsbeing reachedifstartedfrompix $(x, y)$ intoone of the three directions $\left(d_{a}-1\right)_{\text {mod } 8}, d_{a},\left(d_{a}+1\right)_{\text {mod } 8}$ respectively.

Step 5. The point with $\max |\vec{g}|$ must be selected among all the points belonging to the route traced, and it is the gradient marker.

Step 6. The process goes to step 1 if not all the elements of the scene have been examined.

The advantage of the procedure above described is in finding of "representative" points from the gradient areas $G$ (the so called gradient markers-GM), in which the slope of the intensity transition is maximal, since it is most correct that the border between the potential areas must past across the points with maximal value of the intensitytransitiongradient.

### 3.5. Building of the graph $\Gamma=(P$, $G$ )

In order to determine the areas $P_{i} \in \mathbf{P}, G_{j} \in \mathbf{G}$, it is necessary to apply for the potential and gradient markers twofold the procedure, described in [8], but with the following differences:

1. The connection type 8 is modified for the potential markers (PM) as follows:

| $y p_{b}$ | If $\left\|b p_{a}-b p_{b}\right\|<\Theta$, then $p_{a}$ and $p_{b}$ are connected; |
| :--- | :--- |
| $p_{a}$ | If $\left\|b p_{a}-b p_{b}\right\| \geq \Theta$, then $p_{a}$ and $p_{b}$ are not connected. |

The purpose of this modification is to set apart the areas, that have splitted contours (weakly connected areas) as a border between them, as the regions $P_{1}$ and $P_{2}$ fromFig. 5. The hypothesis considered true, is that for the locations of slashing the inequality $\left|b p_{a}-b p_{b}\right| \geq \Theta$ is satisfied and hence the areas $P_{1}$ and $P_{2}$ are treated as separated. The threshold $\Theta$ is chosen lower that the threshold $\Theta_{G}$. For example the choice for the image in Fig. 11 is $\Theta=14, \Theta=8$.

2. The adjacent gradient markers $g_{a}$ and $g_{b}$ are not connected if:
a) $\left(d_{a}-d_{b}\right)_{\text {mod }}>1$, where $d_{a}$ and $d_{b}$ are directions of the gradient in points $g_{a}$ and $g_{b}$.
b) If $L_{a}-L_{b}>\Theta_{L}$, where the threshold $\Theta_{L}$ is:

$$
\Theta_{L}=K\left(\left(L 2_{a}-L 1_{a}\right)+\left(L 2_{a}-L 1_{a}\right)\right) / 2,
$$

i.e., the averaged levels of neighbouringtransitions differby anyvalue greater than the threshold $\Theta_{L}$, defined by the coefficient K. For the examples in Fig. 10 andFig. 11 $K=0$, 3 is experimentally chosen
$L 2_{a}$ and $L 1_{a}$ are the intensity levels of the upper and lower end of the gradient transition passing through point $g_{a}$, and $L_{a}$ is the intensity level in the point of the gradient marker $g_{a}$ (Fig. 6).


Condition 2a) means that transitions (the gradient areas) $G_{a}$ and $G_{b}$ have to be separated. They are topologically adjacent, but theirdirections donot coincide (Fig. 7) . The meaning of condition 2 b is the detection of transitions which are toplogically adjacent, but at different levels, i.e., theydo not differ along axis $z$ (Fig. 8) .


Fig. 7.
After the areas $P_{i}$ and $G_{j} \quad i=1,2, \ldots, N_{p^{\prime}} j=1,2, \ldots, N_{G^{\prime}}$ havebeen obtained, an oriented nonplanar graph $\Gamma=(\mathbf{P}, \mathbf{G})$, has to be constructed, where $\mathbf{P}=\left(P_{1}, P_{2}, \ldots, P_{N p}\right)$ is the set of the potential areas (PA) and $G=\left(G_{1}, G_{2}, \ldots, G_{n G}\right)$ is the set of gradient areas (GA).

Themain rule to be observed in the formation of the graph $\Gamma$ is that its arcs have tobe orienteddown toup, i.e., fromblack to white level, if it is assumed that theblack level is down. Fig. 9 demonstrates an example of graph building.




Fig. 9
3. 6 . Aprocedure for sectioning surfaces construction

In order to build the surface $A_{i}(x, y, z)=0$, it is necessary to know the initial approximation $A_{i}(x, y, z)=0$ and the points, through which the plain will pass, i.e., a problem for approximation of the surface with respect to given points is formulated.

### 3.6.1. Determination of the initial approximation

a) The plains $A_{i}(x, y)=C_{i}$, where $C_{i}=$ const, are accepted as initial approximations, i.e., theseplains areparallel to theplain $(x, y)$.
b) The values $C_{i}, i=1,2, \ldots, n$, and the number $n$ of the sectioning surfaces are determined.by the modified intensity histogram $h(b)$, which is obtained fromhistogram $h\left(b_{p}\right)$, but on itshand it is a histogramof the intensities of thepointspix $(x, y)$, belonging to the set $\boldsymbol{P}^{\prime}:$ pix $(x, y) \in \boldsymbol{P}^{\prime}$. On the other hand $h(B)$ has still better expressedextremums, since $h(B)$ is obtained from $h\left(b_{p}\right)$ replacing the intensities $b_{p}$ ofPM, $p \in P_{i}$, by the average intensities $B_{i}$ of the areas $P_{i}$. As a result the intensity-gradient regions $P_{i}$ areequalized, and this equalization intensifies the differences between theextremum of $h(B)$, which on itsturn is favourable for the process of determining the borders between the areas.

The coefficients $C_{i}$ are definedto be equal to those values of $B$, for which $h(B)$ has a minimum.

In case the number of the maximums of $h(B)$ is equal to $n+1$, the number of the cuttingsurfaces is $n$.

### 3.6.2. Determination of the approximation points

A structuredgraph $\Gamma=(P, G)$ is applied for thepurpose, using the so called iterative algorithmwith dominating areas. This means that a sectioning surface is built at each iteration step, which has to separate (segment) the areas that are the brightest in a certain local region, i.e. "dominating". The procedure determining the final successor (or node) in graph $\Gamma$ is used.

The iterative procedure with n steps canbe describedby its $i$-th step as follows:
If for the i-th step as initial is assumed the graph $\Gamma^{i-1}=\left(P^{i-1}, G{ }^{i-1}\right)$ from the (i-1) -st step, it serves for the construction of the subgraph $\Gamma^{i}=\left(\mathbf{P}{ }^{i}, \mathbf{G}{ }^{i}\right)$, which is a subset of $\Gamma^{i-1}: \Gamma^{i} \subset \Gamma^{i-1}$ and $\mathbf{P}^{i} \subset \mathbf{P}{ }^{i-1}$ and $G{ }^{i} \subset G{ }^{i-1}$ respectively. The initial (the first) graph $\Gamma$ is then denoted by the index ${ }^{0}$, i.e., $\Gamma^{0}=\left(P^{\circ}, G{ }^{\circ}\right)$, where $P^{0} \equiv \mathbf{P}$ and $G^{0} \equiv G$ are the complete sets of the potential and gradient areas correspondingly.

In order tomake clear the process of graph $\Gamma^{i}$ construction as a subset (subgraph) of the graph $\Gamma^{i-1}$, it is enough to explain the way of obtaining $P^{i}$ and $G{ }^{i}$.
a) The subset $\mathbf{P}^{i}$ of $\mathbf{P}{ }^{i-1}\left(\mathbf{P ~}^{i} \subset \mathbf{P}{ }^{i-1}\right)$ is obtainedas adifference: $\mathbf{P}^{i}=\mathbf{P}{ }^{i-1} \mathbf{P}_{b_{b}}^{i-1}$, where $\mathbf{P}{ }_{b}^{i-1}$ is obtained as a subset of $\mathbf{P}{ }_{a}^{i-1}$ and on its hand $\mathbf{P}{ }_{a}^{i-1}$ is a subset of potential areas with average intensities $B_{j}$, for which $B_{j}>C_{i}$. This inequality means that the areas of average intensities $B_{i}$, locatedabove the plain $\boldsymbol{A}_{i}(x, y)=C_{i}$ are "dominating". On the other hand the obtaining of $\mathbf{P}{ }_{b}^{i-1}$ from $\mathbf{P}{ }_{a}^{i-1}\left(\mathbf{P} \underset{b}{i-1} \subset \mathbf{P}{ }_{a}^{i-1}\right)$ is realizedby the following procedure.

Procedure findingthe final successors (nodes) of the graph $\Gamma^{i}=\left(\mathbf{P}{ }^{i}, \mathbf{G}{ }^{i}\right)$
The problem solved with the help of this procedure can be formulated as:
To obtain the subset $P{ }_{b}^{i-1}$ of those nodes that have incoming arcs only and none outgoing, from the subset $\mathbf{P} \underset{b}{i-1}$ of the node of the grapg $\Gamma^{i-1}$ for which $B_{j}>C_{i}$.

Two methods are proposed:
First method. The final successors must be detected for all possible oriented elementary routes passing through nodes from the set $\mathbf{P}^{i-1}$, and they will form the subset P ${ }_{b}^{i-1}$. According to [13] the notion "oriented elementary route" means a route in the graph, each of nodes and arcs in it being used more than once.

Secondmethod. The rows, all theelements of which are zeroes, are separated from the matrix of he oriented graph. The nodes corresponding to the rows thus separated are the searched elements of $\mathbf{P}{ }_{b}^{i-1}$.

Special attention has to be paid to the special case (Fig. 9), when cycles are obtained, i.e., one and the same arc is simultaneously going in and out of a given node. In this case the node $P_{2}$ is considered again as a final node, but the potential markers belonging to $P_{2}$ are not taken into consideration in the construction of this sectioning surface. The broken contour $G_{5}$ is closed by the intersection of the cutting surface $A(x, y, z)$ and the surface of the image $F(x, y, z)=0$.
b) The subset of the arcs $G{ }_{b}^{i-1}$ is obtained fromP ${ }_{b}^{i-1}$ as a subset of arcs, entering the nodesbelongingto $P{ }_{b}^{i-1}$. Afterthat in an analogouswaytoa), $G{ }^{i}=G{ }^{i-1}-G{ }_{b}^{i-1}$ is defined.

Having the subsets ${ }^{b}{ }^{i}$ and $G{ }^{i}$ for the $i$-th surface $A_{i}(x, y, z)=0$, the points that approximate $A_{i}$ are determined as follows:

1. The gradient markers $g_{i 1}$, crossed by the surface $A_{i}$ are elements of the set $G{ }_{b}^{n}$ , which is an element of the set $G{ }_{b}^{i-1}$.

$$
\begin{gathered}
G^{b}{ }^{{ }_{i}-1}=\left\{G_{b}^{11}, G^{12}{ }_{b}, \ldots, G^{L R}\right\}, \\
G_{b}^{I 1}=\left(g_{i 11}, g_{i 12}, \ldots, g_{i l 2}\right) \text { for } L=1,2, \ldots, R,
\end{gathered}
$$

where $R$ is the number of the gradient areas $G^{\prime 1}{ }_{b}$, belonging to the subset $G{ }^{\prime l}{ }_{b}$ and $Q$ is the number of the gradient markres $g_{i 1}$ belonging to $G^{l 1}{ }_{b}$.
2. The potential makers $p_{i 1}$, that are at a distance $+h$ from the surface $A_{i}$, are elements of the set $P^{\prime l}$, which on its turn is an element of the set $\mathbf{P}{ }_{b}^{i-1}$ :

$$
\begin{gathered}
\mathrm{P}{ }^{i-1}=\left\{P_{b}^{11}, P^{12}, \ldots, P_{b}^{\prime K}\right\}, \\
P_{b}^{u l}{ }_{b}=\left(p_{i 11}, p_{i 12}, \ldots, p_{i 1 T}\right) \text { for } l=1,2, \ldots, K,
\end{gathered}
$$

where $K$ is the number of potential areas $P^{l l}{ }_{2}$, belonging to the subset $P \underset{b}{i-1}$, and $T$ is the number of the potential markers $p_{i 1}$ belonging to $P_{b}{ }^{i}$.
3. Thepotential markers, locatedat a distance -h fromthe surface $A_{i}$, are elements of the set $P_{b}^{i l}$, which is an element of the set $P{ }_{b}^{i}$.

$$
\begin{gathered}
\mathbf{P}{ }_{b}^{i}=\left\{P_{b}^{n}, P_{b}^{Q}{ }_{b}, \ldots, P_{b}^{M M}\right\}, \\
P_{b}^{l l}=\left(p_{i 11}, p_{i 12}, \ldots, p_{i l S}\right) \text { for } l=1,2, \ldots, M,
\end{gathered}
$$

where Mis the number of potential areas $P^{l l}{ }_{b}$, belonging to the subset $\mathbf{P}{ }_{b}^{i}$, and $S$ is the number of the potential markers $p_{i 1}$ belonging to $P_{b}^{l 1}$.

The distance $h$, called distance of separation, is experimentallydefined. ForFigs. 10 and 11, $h$ is assigned $\Theta_{G}$.

After the initial approximation and approximation points have been determined, a second iterationalgorithm (4-steps) is appliedto builda "smooth" sectioning surface $A_{i}$. This iteration algorithm consists in successive application of the operation programrecursive filtration on the surface of initial approximation $A_{i}(x, y)=C_{i}$ and themarker points (PM and GM) for the four different directions.

### 3.6.3. Analgorithm forprogram-recursive filtration

Step 1. For the points fixed $a_{i}$, it is assumed $z_{i}=a_{i}$, where $a_{i}$ is the value of the corresponding marker (PM or GM).

Step 2. The following operation is applied for all the points outside $a_{i}$ :

$$
b=\left(\sum_{j=1}^{2} b x_{j}\right) / g .
$$

The segmentation implemented on the image $F(x, y, z)=0$ by $n$ sections with the help of the cutting surfaces $A_{i}(x, y, z)=0, i=1,2, \ldots, n$, can be regarded as a local threshold operation, applied ntimes for each pint of thepicture pix $(x, y)$ :

$$
\text { If } b(x, y) \geq z_{i}(x, y) \text {, thenpix }(x, y) \in O_{i} \text {, }
$$

$$
\text { If } b(x, y)<z_{i}(x, y) \text {, thenpix }(x, y) \in \Phi_{i} \text {, }
$$

where $z_{i}$ is a point from the $A_{i}$-th surface, and $O_{i}$ and $\Phi_{i}$ denote conditionally the object and the background for the $i$-th section.


Fig. 10

## 4. Experimental results

Fig. 10 andFig. 11 show two possible applications of the method. Fig. 10a and Fig. 11a show the initial images of printed documents (a passport and text) with uneven illuminance, unequal background-texture and bad quality. Fig. 10b and 11b show the images after segmentation in three levels (with two sectioning surfaces respectively), andFig. 10c and 11c - the final results, i.e., segmentation in two levels - text and background, andFig. 10d andFig. 11e demonstrate the results from another method - a threshold one, optimizing the threshold according to an ent ropy method [8].

Fig. 10d, 11d and 11e show the cutting surfaces of the two scenes. A tendency is not iced that the sectioning surfaces "go round" the irregularities of the background, which helps the exact separation (without false areas and contours) of the images and the background.


Fig. 11

## 5. Conclusion

The application of the method proposed is various - for analysis of tridimensional scenes with arbitrary location of the illuminating source, for coding of the image homogenizing areas, for analysis of printed documents wit irregular background and poor quality, for reducing the number of the intensity levels and removing the information redundancy, etc.

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## Адаптивньй порогово-прадиентный метод для сегментирования областей и объектов в полутоновых изображениях

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Описывается новьй адаптивньй порогово-градиентный метод для яркостного сепментирования, при котором изображение рассматривается как совокупность границ и областей. Для их описания исполь зуется структурньй граф.

Сепментирование осуществляется с помощью трехмерных поверхностей, рассекающие граф. Их число определяется в зависимости от минимумов модифицированной гистограммы яркостей. Введены понятия потенциальный маркер и прадиентньй маркер, которые используются для построения секущих поверхностей.

Метод можно применять как для обработки документов плохого качества, так и для кодирования изображений через гомогенные области.

