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Direct Adaptive Control of Robots by Means of Auxiliary Signals*

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Introduction

There are certain difficulties in realizing of fast motions with robots. The problems come from the strong couplings between individual joint motions. Non-adaptive compensation of the couplings requires time-consuming computations, since accurate enough dynamic models are bulky. There is some decrease of the sampling rate associated with the increased amount of computations, and too slow sampling may deteriorate robot performance, especially in the case of fast motions. Actually, decentralized fixed-gain control is used with all commercially available robot controllers. Such type of control results in an inherently parallel modular structure of the controller, which is most practical.

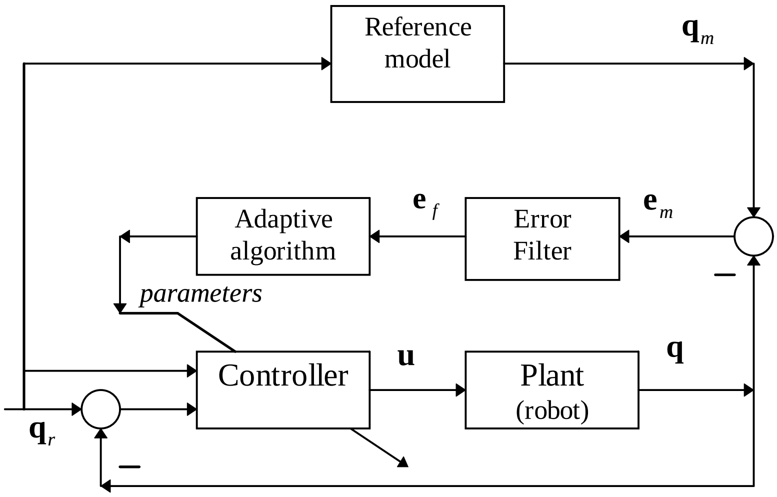

Fig. 1. The general scheme of the direct adaptive control

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Adaptive control is a label assigned to a wide group of approaches, which are based on variations of the control inputs adequately to *a priori* unknown variations of the plant's dynamics [1]. There are two widely recognized groups of adaptive control: direct and indirect ones [2].

Adaptive control schemes based on the *indirect* approach use a controller-embedded model of the plant with a known structure but with unknown parameters. During the normal operation of the system the model parameters are updated on-line (or continuously, for analog systems) in order to give the minimum mismatch between the outputs of the model and the plant to one and the same control input. Subsequently, the control is generated according to the model. This approach is also referred to as *identification-based adaptive control*, as well as *explicit adaptive control.*

Direct adaptive control schemes (see Fig. 1) are based on controllers with previously defined structure but with unknown parameters. During the normal operation the controller parameters are updated *directly* with the goal to minimize the error, or some other qualifier of the closed-loop system performance.

Problem statement and nomenclature

The equation of motion of an n degree-of-freedom rigid-link manipulator is described by

(1) $$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u,$$

where $q \in R^n$ is the vector of joint variables;

$M(q) \in R^{n \times n}$ is the generalized inertia matrix; this matrix is positive definite and bounded [3], i.e.

(2) $$\exists \underline{m} > 0 \quad \exists \bar{m} \geq \underline{m} \text{ such that } \forall q \in Q \Rightarrow \underline{m} I_n \leq M(q) \leq \bar{m} I_n,$$

where Q denotes the allowed joint space, and the matrix inequalities imply positive definiteness rather than component-wise inequalities;

$C(q, \dot{q})\dot{q} \in R^n$ is the vector of Coriolis and centrifugal forces;

$C(q, \dot{q})\dot{q} \in R^{n \times n}$ is the matrix of Coriolis and centrifugal effects; although this matrix is not uniquely defined [5], it can be represented in a unique form [3] such that

(3) $$C(q, \dot{q}) + C^T(q, \dot{q}) = \dot{M};$$

$g(q) \in R^n$ is the vector of gravity terms;

$u(t) \in R^n$ is the vector of generalized forces; it is also the control.

The problem of gross motion robot control is to achieve closed-loop performance that meets some previously defined criteria. The following theorem [3, 5] helps to find such a control.

Theorem 1. The equilibrium state $e(t) = 0$ of the system (1) is globally asymptotically stable under the control

(4) $$u = M(q)\ddot{q} + C(q, \dot{q})\dot{q}_r + g(q) + u_\beta,$$

where $u_\beta = K_p e + K_d \dot{e}$ is the feedback stabilization control, e is the joint error, and the gain matrices K_p and K_d are constant and positive definite.

Proof. Using the control (4), the closed-loop system is represented by the equation

$$M(q)\ddot{e} + [C(q, \dot{q}) + K_d]\dot{e} + K_p e = 0.$$

Obviously, $e(t) = 0$ is an equilibrium state. It has to be proven that this state is stable. The proof is performed using the direct method of Lyapunov [4, 5]. The Lyapunov function candidate can be chosen in the following energy-like form

(5) $$v = (1/2)\{\dot{e}^T M(q)\,\dot{e} + e^T K_p e\}.$$

This quadratic function is positive definite and bounded, since the generalized inertia matrix $M(q)$ is positive definite and bounded by virtue of inequality (2), and the gain matrix K_p is constant. Thus the Lyapunov function candidate is estimated by the following inequalities:

$$\underline{m}\|\dot{e}\|^2 + \underline{k}\|e\|^2 \le v \le \bar{m}\|\dot{e}\|^2 + \bar{k}\|e\|^2,$$

where $\bar{k}$ and $\underline{k}$ ($\bar{k} \ge \underline{k} \ge 0$) are the greatest and the smallest eigenvalues of the gain matrix, respectively. Thus, function (5) is bounded, and therefore it is a ***legitimate*** Lyapunov function candidate. The derivative of (5) is further obtained with the essential use of equation (3):

$$\dot{v} = (1/2)\,\dot{e}^T K_d \dot{e}\ .$$

The derivative is *negative semi-definite*, while the 'standard' technique requires it should be negative definite. However, an equilibrium state $e \neq 0$ is not possible for $\dot{e} = 0$, as can be verified from the equation of the closed-loop system. Hence (5) should decrease and finally (asymptotically) reach zero. Thus, $e(t) = 0$ is the only equilibrium state, and this state is globally asymptotically stable. *QED.*

Remarks:

- Energy-like Lyapunov functions prove to be very suitable for stability analysis of mechanical systems.
- Unlike the computed torque method, the generalized inertia matrix is not used as a gain matrix; this is most suitable for use with direct adaptive control schemes. However, such schemes require that the reference trajectory be available together with its first and second order derivatives. This requirement may be very embarrassing in implementation.
- According to the theorem, the equilibrium state $e(t) = 0$ is proven to be asymptotically stable when arbitrary positive definite gain matrices are used. The equilibrium state can be made globally ***exponentially stable***, provided the velocity feedback is deep enough (refer to [5, 6] for details).

In order to develop the control system that guarantees exponential stability of the equilibrium state, the direct method of Lyapunov is used again, and the associated Lyapunov function is found to be in the form [5]:

(5a) $$v_{exp} = (1/2)\,\{(\dot{e} + \Lambda e)^T M(q)(\dot{e} + \Lambda e) + e^T [\,2K_d - \Lambda M(q)]\,e\},$$

where the matrix $\Lambda \in R^{n \times n}$ is a positive definite constant and diagonal. The function (5a) is a positive definite function of the system state. Its derivative with respect to time is negative definite along the trajectory of the closed-loop system.

The adaptive control

According to all direct adaptive schemes, the control is performed on the base of a chosen set of actual signals in the closed-loop system, such as position and velocity errors, sensor outputs, reference quantities, etc. Generally, the control is obtained in the form [5]

(6) $$u = Y(s)\,\hat{p}\ ,$$

where $\boldsymbol{s}$ is the vector of signals used, $\hat{p}$ is a vector of the controller parameters, and $Y(s)$ is some functional matrix with appropriate dimensions, which is defined by the adopted structure of the controller. It is assumed that there exists a vector of "ideal" parameters, p, such that the performance of the closed-loop system meets some prescribed requirements, provided $\hat{p} = p$. The controller parameters are generally gains [1–6]. The updating of the gains may be performed using a gradient estimator approach [3–5], yielding

(7) $$\dot{\hat{p}} = \Gamma Y^T e_f ,$$

where $\Gamma > 0$ is a user-defined matrix, and e_f is a vector of signals, often referred to as *filtered error*. Stability analysis on the base of energy-like Lyapunov functions shows that the filter can be designed in the form $e_f = \dot{e} + \Lambda e$, where $\Lambda > 0$ [4, 5]. Decentralized versions of direct adaptive control are attractive topics of research, as long as they provide simpler and more practical solution to the problem of robot control in comparison with the centralized control methods [4]. With the decentralized approach, the control (6) can be represented in the component-wise form

$$u_j = y_j^T \hat{p}_j ,$$

where j, $1 \leq j \leq n$, is the joint index, u_j is the corresponding control input, $u_j(s_j)$ is a functional vector of the signals s_j, s_j is the vector of signals used with the j-th control loop, (usually, $y_j = s_j$), and $\hat{p}_j$ is an estimate of the vector of the "ideal" controller parameters. The parameter tuning is performed according to the following law

(8) $$\dot{\hat{p}}_j = \Gamma_j y_j e_{fj} , \text{ where } 1 \leq j \leq n.$$

The adaptation gain matrices, Γ_j, have to be positive definite. Sometimes the parameter tuning is not effective, in particular when the signals used are small in amplitude. The approach here is to use *auxiliary signals* and adjust them on-line, rather than manipulate the gains, in order to avoid such a drawback. The control can be represented as

(9) $$u_j = (y_j + \hat{a}_j)^T p_j , 1 \leq j \leq n,$$

where $\hat{a}_j$ is an estimate of the vector of auxiliary signals associated with the j-th control loop. The dimension of this vector depends on the structure of the controller. With proper choice of the auxiliary signals any control input can be obtained, provided the gains are not zero, i.e. $p_j \neq 0$. The adaptation law is synthesized in accordance with

(10) $$\dot{\hat{a}}_j = \Gamma_j y_j e_{fj} ,$$

which is analogous to (8).

Stability issues

Detailed consideration of the system stability is not possible here because of lack of space. Since the control is in decentralized form, *practical stability* is the control design goal rather than asymptotic stability. It is well known, that asymptotic stability does not necessarily guarantee good closed-loop performance, e.g. refer to [7]. It is assumed further that there exists an "ideal" auxiliary input, a_j^*, such that the equilibrium of the closed-loop system $e(t) = 0$ is exponentially stable with the control (9), provided there are no disturbances, i.e. if $\tilde{a}_j \overset{def}{=} a_j^* - \hat{a}_j = 0$, ($j = 1, 2, \ldots , n$) everywhere along the trajectory of the closed-loop system. We will assume that the associated Lyapunov function is in the form (5a). These assumptions are very natural, since the robotic system is *controllable*, the energy-like Lyapunov functions are most suitable for the analysis, and it is easy to show that arbitrary control can be obtained through application of (9), provided $p_j \neq 0$. As far as the undisturbed system is exponentially stable, the derivative of (5a) with respect to

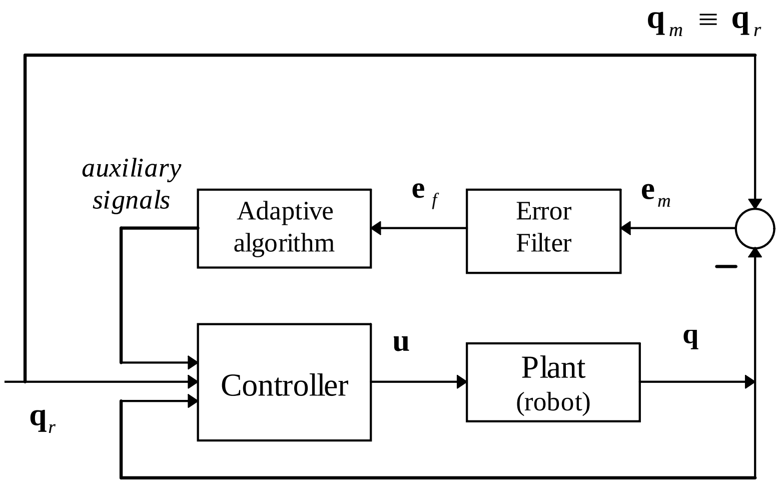

Fig. 2. The scheme of adaptive control with auxilary signals

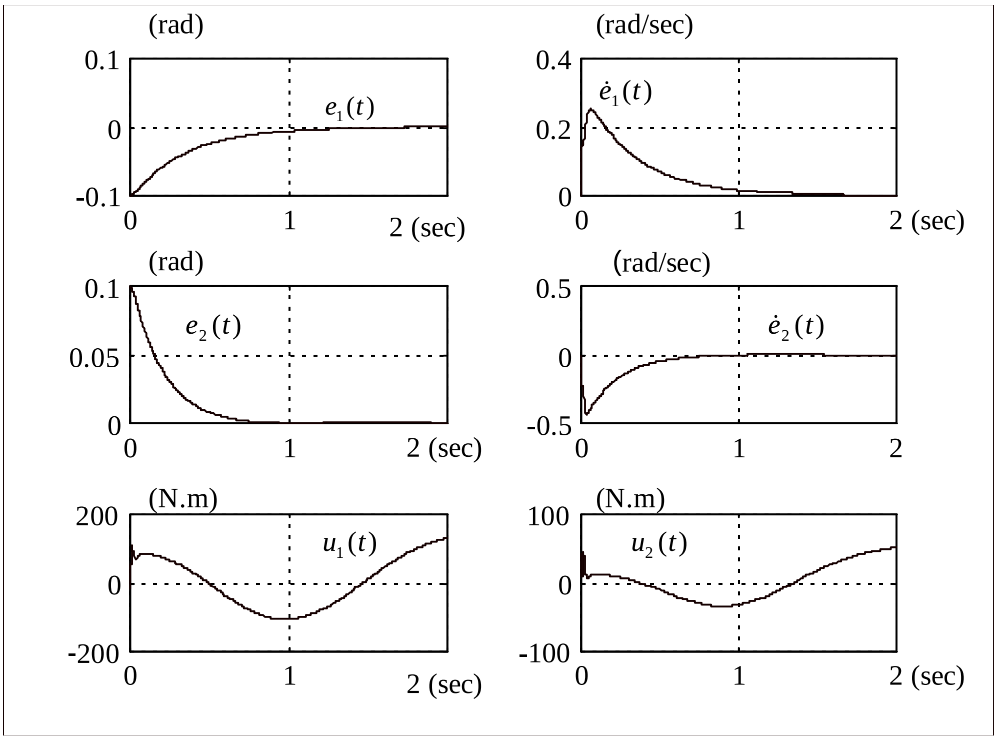

Fig. 3. Results of numerical simulations along a fast trajectory – the position errors (left), the velocity errors (right) and the controls (bottom)

time is negative definite. Following the scheme of proof for theorem 1, and with the essential use of the direct method of Lyapunov, the following theorem can be established.

Theorem 2. Consider the nonlinear system (1) with the control (9). Suppose that there exist vectors a_j^*, $j = 1, 2, \ldots, n$, with bounded elements and bounded derivatives with respect to time, such that the equilibrium $e(t) = 0$ of the undisturbed closed-loop system is exponentially stable. Then the control (9), together with the adjusting (10), guarantees global uniform ultimate boundedness of the closed-loop system errors.

Proof. For lack of space the proof is omitted here. However, the proof is very similar to the proof of theorem 3.2, given in [5], and it uses the statements of Theorem 1 given above.

Remarks:

- The actual values of the "ideal" vectors are unknown; however, they are not needed for implementation.
- The approach falls in the group of direct adaptive control schemes, and it is based on the direct method of Lyapunov.

The scheme of the adaptive control with auxiliary signals is given in Fig. 2. The reference model (see Fig. 1) is assumed to be *unity*, as long as such a choice simplifies the control and does not require integration of the reference model equations.

Numerical simulations

Numerical simulations with a two-degrees-of-freedom robot with revolute joints are carried out and quoted here to demonstrate the method efficiency. The equations of motion are highly coupled and they include gravity terms. The reference motion is smooth, though it is relatively fast, with world velocity and acceleration of some 10 m/s and 10 m/s^2, respectively. For sake of comparison, the reference motion is the same as the one used in [4] and [5]. The standard PD control does not supply for sufficiently good results, since the estimated tracking error is 0.3 [rad] for each joint [4, 5]. In comparison, the adaptive control with tuning of the feedback gains is quite a success [5]. However, the adaptive control based on auxiliary signals as reported in this paper is a considerably better choice.

The control is adopted in the form

$$u_j = (t_d +1) = k_{pj}(e_j + a_{1j}) + k^{+}_{dj}(\dot{q}_{rj} + a_{2j}) - k^{-}_{dj}(\dot{q}_j + a_{3j}) + u_j(t_d)_j(b_j + a_{4j}),$$

where t_d denotes the discrete time, and the velocity gains relate to position gain in accordance with $k^{+}_{dj} = k^{-}_{dj} = \lambda_j k_{pj}$ (friction is not considered). The control is generated using past experience, and it is assumed that $b_1 = b_2 = 0$. Initial joint errors are forced in order to clarify the tendency in decaying of the joint errors. The error filter is designed using the values $\lambda_1 = 3$ s, $\lambda_2 = 5$ s.

The time history of the auxiliary signals is given on Fig. 5. The time histories of the auxiliary signals a_{j4}, j=1, 2, are not reported, as long as they coincide but for the sign with a_{j3}, j=1, 2, respectively.

Obviously, both the auxiliary signals and their derivatives are sufficiently small in amplitude (see Fig. 5). Actually, they depend on the reference motion and the control actions required for the motion. The experiments show that the frequencies present in the reference trajectories induce similar frequencies in the ideal auxiliary signals. However, the amplitudes of these induced frequencies are much smaller.

"Wild" initial values of the auxiliary signals have been assigned during simulation. The initial values do not appear to be of great significance in a long run, since the auxiliary inputs are adjusted in due time. The closed-loop transients can be improved through appropriate choices of the initial values. However, we do not pay attention to these choices here, as long as the accent is put on the adaptation process.

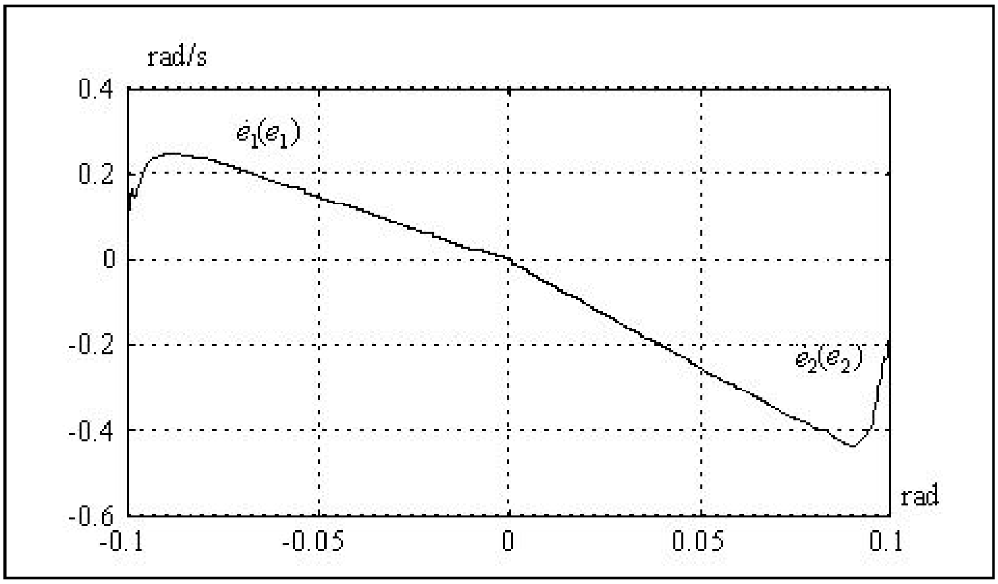

Fig. 4. Phase-plane plots (left – $\dot{e}_1(e_1)$, right – $\dot{e}_2(e_2)$)

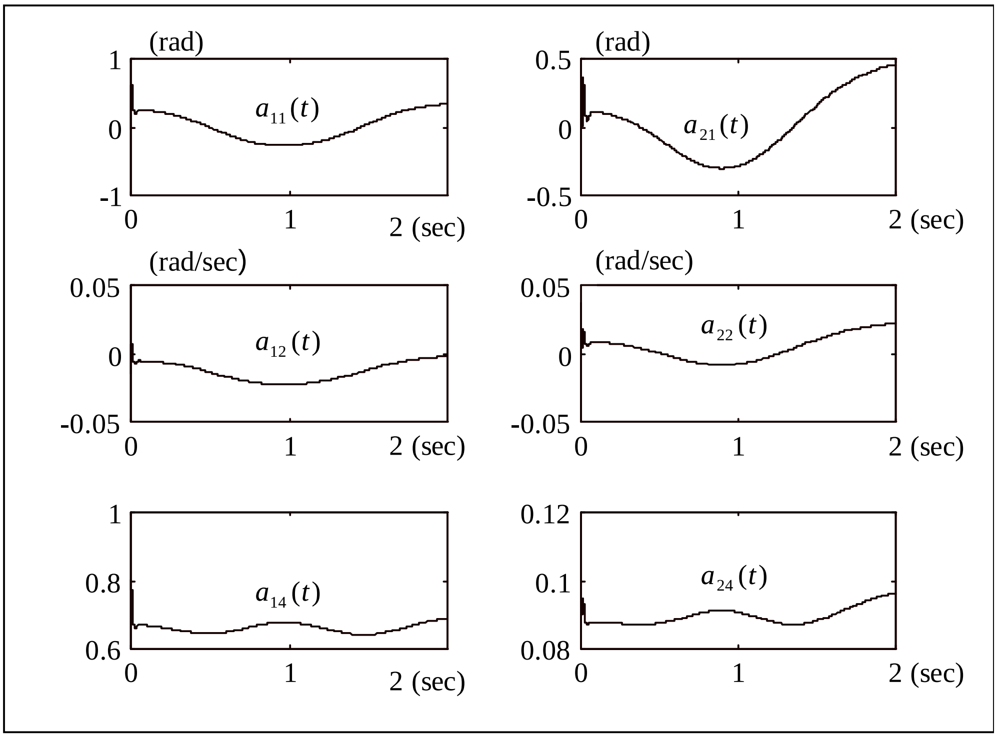

Fig. 5. Time histories of the auxilary signals of the 1st (left) and the 2nd (right) joint.

Conclusion

The article puts forward a new direct adaptive control technique. It is based on decentralized fixed-gains control, as the one used with all commercially available robot controllers. Such type of control results in an inherently parallel modular structure of the controller, which is most practical. The technique falls into the group of *simplified adaptive control* [4], and it can be easily implemented, since there is no need of extensive computational capacity and the sampling rate can be easily kept sufficiently high. Feedback control is used only, in contrast to the well known adaptive control approaches with proven asymptotical stability [1–3] of the equilibrium state, where the feedforward control component is essential. The experimental investigations show that the adaptive control based on auxiliary signals is a much better choice, when compared to the control with variable adaptive gains.

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Прямое адаптивное управление манипуляционными роботами на основе дополнительных сигналов

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(Резюме)

Предлагается подход для управления манипуляционными роботами на основе адаптивной настройки сигналов в обратной связи. Подход отличается простотой реализации и требует динамической модели механической системы. Управление можно приложить и по отношению к постоянным коэффициентам усиления. Результаты моделирования иллюстрируют эффективность управления.