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An Optimization Motivated Interactive Algorithm Solving a Discrete Multicriteria Choice Problem

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Introduction

The multicriteria decision problems are divided into two groups: multiple attribute decision making (MADM) problems (discrete multicriteria problems) and multiple objective decision making (MODM) problems. In MODM problems the criteria (called objective functions) and the constraints are specified by mathematical functions and the purpose is to choose the "best" alternative. In MODM problems an infinite number of possible alternatives exists which have to be generated by the solution process. MADM problems are characterized by the fact that after defining a set I of n (>1) of deterministic alternatives and a set J of k (>2) criteria that define a $n\times k$ decision matrix A one wishes to:

1. determine the best alternative with respect to the set J (the choice problem),
2. divide the set I into subsets (the sorting problem), or
3. rank the set I of alternatives from the best to the worst (the ranking problem).

A number of algorithms have been developed to solve the multiple attribute decision making problems. The basic part of these algorithms may be grouped into two major classes: the multiattribute utility and the analytical hierarchy process (see also Fishburn [3]), Keeney and Raiffa [5], Farquhar [2] and Saaty [11] and the outranking and related algorithms (see also Roubens [9], Roy [10] and Brans and Mareschal [1]).

The first group of algorithms is used to solve MADM problems and is based on the well known value or utility function which is used to express the DM's preferences to aggregate the criteria. The second group of algorithms is based on the DM's preference relation between a pair of alternatives to outrank the alternatives. The methods developed are based on restrictive assumptions concerning the DM's preference structure and DM's abilities, as well as the type and the structure of the problems solved. The methods leave several issues unresolved and some of them are

– at the beginning, the DM has to make too many pairwise comparisons,

– there is not enough interaction between the DM and the solution process in order to permit the DM learn much about the problem or the solution space,

– difficulties connected with the intercriteria information such as the compensation between the criteria, the choice of criteria weights and independence on the criteria,

– difficulties connected with the elimination of the scaling effects and difficulties connected with consideration of the amplitude of deviation among the criteria values, and

– difficulties connected with developing procedures which can easily be understood by the DM.

For some classes of the multiple criteria problems (mainly for the choice problem) "optimization motivated" interactive algorithms have been developed, in which a part of these disadvantages are avoided on the account of the greater tension, but also the more intensive use of the DM (Koksalan, Kartvan and Zionts [6], Marcotte and Soland [8], Korhonen [4], Lotfi, Steward and Zionts [7]. Special attention deserves the reference direction algorithm of Korhonen [4] designed to solve the choice problem with a few quantitative criteria and many alternatives. In this algorithm the DM estimates a given set of alternatives at each iteration. In some cases these alternatives may be spread on the whole set of the non-dominated alternatives. The DM selects one alternative from this set either as the solution best preferred or as a basis for further search. The search is realized in a direction defined by the last alternative and an alternative preferred at the moment, determined by the aspiration levels of all the criteria set by the DM.

The paper proposes an approach for solving the choice problem with several quantitative criteria and many alternatives. It is similar in its idea to Korhonen's approach with respect to the DM engaging in an interactive mode which overcomes the difficulties above described. But in it the DM's tension is decreased much more. This is achieved with the help of two basic alterations. The first one is that in many cases when a current preferred alternative is present, it is easier for the DM to set the desired alterations in the criteria and still more easier to set the desired directions of criteria alteration than to set the criteria aspiration levels. The second one is that the DM has the possibility to estimate neighbouring and widely spread alternatives. It is easier for the DM, especially in the learning process, to estimate and choose one alternative from the set of "neigbouring" alternatives than from the set of spread alternatives. In this case the DM is able to consider more confidently and realistically the importance of the criteria, their correlation and possibilities for compensation among them, as well as the better estimation of the criteria values in the different alternatives.

Scalarizing problem

The discrete multicriteria choice problem is defined as follows:

A set I of n (>1) of deterministic alternatives and a set J of k (>2) criteria be given which define a $n \times k$ decision matrix A. The element a_{ij} of the matrix A denotes the evaluation of the alternatives $i \in I$ with respect to the criterion $j \in J$. The evaluation of the alternative $i \in I$ with respect to all the criteria in the set J is given by the vector $(a_{i1}, a_{i2}, \ldots, a_{ij})$. The assessment of all the alternatives in the set I for the criterion $j \in J$ is given by the column vector $(a_{1j}, a_{2j}, \ldots, a_{nj})$.

The solving of this problem is the search for a non-dominated alternative, satisfying the DM to a larger extent with respect to all the criteria.

The alternative $i \in I$ is called non-dominated if there is no other alternative $p \in I$ for which $a_{pj} \geq a_{ij}$ for all $j \in J$ and $a_{ij} > a_{ij}$ for at least one $j \in J$.

Since it is comparatively simple to separate the dominated alternatives, further on we shall assume that the matrix A contains non-dominated alternatives only.

A preferred alternative is a non-dominated alternative the DM chooses as the best one at the current iteration with respect to the set of the criteria. The alternative preferred best (a compromise alternative, a final alternative) is a preferred alternative that satisfies the DM to the greatest degree.

Desired changes of the criteria at every iteration are the values, by which the DM wishes to alter the criteria values of the current preferred alternative in order to obtain a better one.

A reference alternative is an alternative (it cannot exist in reality) obtained from the current preferred alternative and the desired changes of its criteria values. A reference direction, defined on the basis of the preferred alternative and the reference alternative describe a desirable change in the criterial space.

At each iteration the DM is presented a set $M=\{m_1, m_2, ..., m_p\}$ of alternatives, the first alternative being the preferred alternative. The DM has to estimate the alternatives of this set and to choose one of them either as a current preferred or as the best preferred alternative. In the second case the discrete multiple criteria choice problem is solved. In the first case on the basis of the preferred alternative selected the DM sets the desired changes in the criteria values in order to search for a new better alternative with respect to all the criteria. On the basis of this information a set M is defined with the help of scalarizing problems, which is represented for estimation and choice to the DM. The defining of a set M consisting of alternatives close to the current preferred alternative (with an index h) can be realized on the basis of the following scalarazing problem:

$$A:\ \min_{\substack{i\in I\\ i\neq I}} S(i,h) = \min_{\substack{i\in I\\ i\neq h}} \max_{j\in L_h} \{(a_{ij}-a_{hj})/\Delta_{hj}\},$$

subject to

$$a_{ij} \geq \mathrm{i}\, a_{hj} - (1+\alpha)\Delta_{hj},\ i\in I,\ j\in E_h,$$

where

α is a non-negative parameter and

$$\alpha_{\max} = \min_{j\in E_h}\{(a_{hj} - \min_{i\in I} a_{ij})/\Delta_{ij} - 1\},$$

L_h is the set of indices $j\in J$ of the criteria, for which the DM wishes to increase their values by Δ_{hj} in comparison with their values in the current preferred alternative.

E_h is the set of indices $j\in J$ of the criteria, for which the DM is inclined to deteriorate their values by Δ_{hj} in comparison with their values in the current preferred alternative

$$L_h \cup E_h = J.$$

Solving the scalarizing problem A some alternatives can be defined along the reference direction that can be included in the estimation set M. Such alternatives exist if the corresponding values of the function $S(i,h)$ are positive numbers. In order to obtain these alternatives the multiple solving of problem A is not necessary, but the algorithm for its solving can be built in such a way that it will also give their arrangement with respect to their close allocation to the current preferred alternative. One alternative is closer to the current preferred alternative if its corresponding value of the function $S(i,h)$ is smaller. The first $p-1$ alternatives thus arranged or all the alternatives, if their number is smaller than $p-1$, are included in the set M. If no alternatives with positive values of the function $S(i,h)$ exist, the DM has to choose a new reference direction.

The proposed algorithm

On the basis of the scalarizing problem A an algorithm can be proposed solving the multiple criteria choice problem. With the help of this algorithm the DM has the possibility, setting desired alterations of the criteria values, to estimate iteratively a small subset of alternatives. The alternatives of these subsets are to some extent close to the current preferred alternatives. Thus in the learning process and after that the DM can take into mind such factors that can be hardly formalized. In order to assist the DM in the estimation of the alternatives from the set M it is useful to give additional parameters for each alternative from this set, such as the values of the function $S(i)$, the maximal deterioration of the criteria t_i, the maximal and minimal feasible values of the criteria and others. The visualization of the alternatives is particularly appropriate (bar graphs and so on).

The main steps of the algorithm proposed are as follows:

Step 1. Reject all the dominated alternatives and define the decision matrix A. Ask the DM to choose an initial preferred alternative and sigh h its index.

Step 2. Ask the DM to define the desired alterations of the criteria values Δ_{hj}, $j \in J$.

Step 3. Form the sets L_h and E_h . The set L_h contains the indices $j / j \in J$ of the criteria, which the DM wants to improve by Δ_{hj} value. The set E_h contains the indices $j / j \in J$ of the criteria, that the DM is inlined to be weakened by Δ_{hj}.

Define the set $I' \subset I$ of indices $i \in I$ of the alternatives, for which there exists at least one index $j \in L_h$, for which $a_{ij} \geq a_{hj}$ and $a_{ij} \geq a_{hj} - (1+\alpha)\Delta_{hj}$ for $j \in E_h$ and $\alpha \leq \alpha_{max}$.

For each alternative with an index $i \in I'$ determine the values of the function $S(i, h)$ and the maximal deterioration t_i of the criteria from the set E_h for this alternative with respect to the current preferred alternative

$$S(i) = \max_{j \in L_h} \{(a_{ij} - a_{hj}) / \Delta_{hj}\}, \; i \in I',$$

$$t_i = \max_{j \in E_h} (a_{hj} - a_{ij}), \; i \in I'.$$

Rank the alternatives with indices in the set I' in ascending order of the values of $S(i) / i \in I'$. Include all the first $p-1$ alternatives in the set M if $p \leq |I'|$ or all the alternatives from the set I' if $p > |I'|$. Take also the current preferred alternative as the first alternative in the set M. If the set M contains the current preferred alternative only, pass to Step 4, otherwise – to Step 5.

Step 4. Since there does not exist an alternative, the value of which coincides for at least one criterion with the desired change, the DM has to decide whether to alter his current preferences or to choose the current preferred alternative as the alternative best preferred. In the first case go to Step 2, while in the second – go to Step 6.

Step 5. Show the set M to the DM. If the DM finds one of the alternatives as the most preferred alternative, Stop. Otherwise ask the DM to choose the preferred alternative and sigh h its index. Go to Step 2.

Step 6. Stop.

Remark 1. Any alternative can be selected as an initial preferred alternative in Step 1. One acceptable initial preferred alternative can be found optimizing one criterion.

Remark 2. The defining of the alternatives, comprising the set M in Step 3, is realized solving scalarizing problem A. In case any of the criteria are for minimization, matrix A' is used instead of matrix A. The last one obtained multiplying the elements of the columns of the matrix A, corresponding to the criteria minimized, by (–1).

Remark 3. In DM learning process alternatives close to the current preferred alternative are included in the set M in Step 3. In case the DM feels more confident, he could ask the including of the spread alternatives also in the set M. This can be easily done since in both the steps the alternatives, included in the set M, are selected from the set of alternatives arranged in ascending order.

Illustrative example

In order to illustrate the algorithm suggested, data evaluating enterprises suggested for privatization, have been used. Only 3 criteria and 5 enterprises are considered. The criteria applied are financial relations, computed on the basis of finance-accounting reports of the enterprises. These are indicators, connected with actives convertibility, the enterprise liquidity and the net profit. In order to facilitate the computations, they are scaled and represented in the following table.

j	i=1	i=2	i=3
1	653	11	33
2	996	5	21
3	733	19	16
4	988	8.5	19
5	819	11	13

The DM has to choose the enterprise with the best (maximal) indicators. He selects as an initial alternative the second one, which has maximal value with respect to the first criterion. The DM requires the evaluation of p=3 alternatives and set desired improvements for the 2-nd and the 3-rd criterion $\Delta_{12}=5$ and $\Delta_{13}=5$ and admissible deterioration of the first criterion $\Delta_{11}=200$. The following is computed:

i	1	2	3	4	5
$S(i)$	2.4		2.8	0.9	1.2
t_i	343		263	8	177

The alternatives are ranked according to the values of $S(i,h)$ – 4, 5, 1, 3. M= {2, 4, 5} are included in the set M.

The DM selects the fifth alternative as a preferred one and sets again desired alterations: improvement of the 2-nd and the 3-rd criteria – $\Delta_{22}=5$ and $\Delta_{23}=5$ and possible weakening of the first criterion Δ_{21}=100. The following values are computed:

i	1	2	3	4	5
$S(i)$	4	1.6	0.8	1.2	
t_i	166	–177	86	–169	

The alternatives are ranked – 3, 4, 2, 1 and in the set M, M= {5, 3, 4} are included. The DM makes his final choice from this set for the 3-rd alternative.

Concluding remarks

The algorithm presented belongs to the class of interactive optimization motivated algorithms, intended for discrete multicriteria choice problems solving. It gives the DM the possibility to set his preferences as desired alterations of the criteria values with respect to the current alternative selected. Solving a scalarizing problem at each iteration, a subset of alternatives is defined, among which the DM makes his choice. The algorithm is tested with the help of examples, taken from the references or problems, connected with the privatization evaluation of industrial enterprises. These tests have shown that the algorithm proposes a fast and user-friendly dialogue with the DM, in which he has the possibility to evaluate better the criteria significance and the possibilities for compensation among them.

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Оптимизацинно-мотивированный интерактивный алгоритм для решения дискретных задач многокритериального выбора

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(Резюме)

В работе предложен оптимизацинно-мотивированный интерактивный алгоритм для решения дискретной задачи многокритериального выбора. Алгоритм основляется на использование эталонного направления, определенной на базе текущих значений критериев и желаемые перемены этих значений. Эталонное направление проектировано на множество эффективных альтернатив. Определяется подмножество этих альтернатив, близкие к рассмотренной альтернативе. Это подмножество предоставляется оператору, принимающему решение.

В использованном алгоритме не поставляются ограничения к свойствам функции полезности.